

the kinetic energy of the system is necessarily positive; still more, if the potential energy is a true minimum.

Prof. Bromwich has given an excellent account of the analytical theory, with various geometrical and dynamical illustrations, and he has added a very useful bibliography. As he has pointed out himself, he has selected Kronecker's method of proof of the invariance of the factors of the discriminant of  $S+\lambda T$ ; and he has made no reference to the specially arithmetical form of the problem, where the coefficients of the forms are integers, and the equations of transformation have to be unitary and integral. For this, the student will consult Frobenius, Hensel, and H. J. Smith, whose memoirs, of course, Prof. Bromwich includes in his list of references.

Dr. Whitehead's chapters deal with a subject which, on the one hand, is more ancient than that of Prof. Bromwich, but, on the other, has changed its aspect recently in a much more remarkable way. Dr. Whitehead is one of the company of sappers who are reducing all the mathematical part of geometry to a system of abstract logic applied to a minimum number of undefinable entities, connected by a minimum number of undefinable relations. Put in this bald way, their work seems purely destructive and hateful, but in reality it is not so. In the tract on projective geometry it is shown how, with the help of Dedekind's axiom, and those of order, it is possible to make rigorous von Staudt's proof that all the points on a line are either reached by harmonic constructions starting from three given points, or definable as limiting points of a set of such points. This leads to definitions of numerical cross-ratios and of numerical homogeneous coordinates which are independent of any theory of distance or measurement, a very remarkable and far-reaching result. It is very encouraging to find that the magnificent genius of von Staudt is gradually gaining the recognition that it deserves; the interval between him and his predecessors is at least as great as that between Apollonius and Steiner.

By "descriptive" geometry Dr. Whitehead means "any geometry in which two straight lines in a plane do not necessarily intersect." Besides the discussion of preliminary axioms and definitions, his second tract falls into two principal parts; the first deals with the problem of enlarging a descriptive space into a projective space (the simplest example is that of adjoining the plane at infinity to Euclidean space), the second with the theory of displacements and measurement. The latter is based upon, what is, perhaps, the only satisfactory method—that of Sophus Lie. The last chapter gives the formulæ of metrical geometry in the shape given to them by Cayley and Laguerre, so that, neglecting a constant numerical factor, a distance and an angle are each measured by the logarithm of a cross-ratio. The cross-ratio, of course, must be *projectively* defined, otherwise we should be in a vicious circle, and it is in the avoidance of this circle that the latest perfection of the theory consists.

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#### OUR BOOK SHELF.

*Practical Agricultural Chemistry.* By F. D. S. Robertson. Pp. x+210. (London: Baillière, Tindall and Cox, 1907.) Price 7s. 6d. net.

IN his introduction the author tells us that his book is intended as a course of laboratory practice for the use of students in agricultural chemistry. How much time does he suppose that students can give to agricultural chemistry to allow them to deal with subjects like the valuation of tea and coffee, or the determination of hop resin and glycerin in beer? To what class of readers is a paragraph like the following of use? "*The Bitter Used.* This is necessarily a tedious operation, and for full particulars the reader is referred to such books upon poisons as describe the processes of Dragendorff and others. The prepared and concentrated beer is subjected to a series of extractions with petroleum ether, benzene, chloroform, and amyl alcohol, each of which is examined in turn."

Even in the more properly agricultural parts of the book there is little evidence that the author possesses any working acquaintance with his subject, e.g. the chapter on the analysis of soils is the merest skeleton, possessing small reference to the methods in regular use, and containing actual errors, such as the attempt to estimate humus by solution in ammonia without a preliminary treatment of the soil with acid.

Again, in his description of the Reichert-Wollny process for estimating volatile acids in butter, the author says nothing of the official standardisation of the dimensions of the apparatus and other details, which, however, must be followed if figures are to be obtained comparable with those of other analysts, and indeed are absolutely essential if the analyst is doing public work. We cannot recommend Mr. Robertson's book.

*An Episode of Flatland, or How a Plane Folk discovered the Third Dimension, to which is added An Outline of the History of Unaea.* By C. H. Hinton. Pp 181. (London: Swan Sonnenschein and Co., Ltd.)

THESE plane people live on the edge of a disc which is their world. The third dimension exists only in their mathematics. Their astronomers find that a catastrophe will certainly happen. One cranky philosopher believes that there is a third dimension, and shows a scared people how their world may be tilted and the catastrophe averted. The author's characters act and make love much like three-dimensional people, and they talk of a higher dimension just as Mr. Hinton would himself talk of a fourth dimension. There never was an allegory, not even that of Bunyan, which was consistent with itself for one chapter, but Mr. Hinton's is more inconsistent with itself than any other allegory we have seen. J. P.

*The Bernese Oberland.* Vol. iii. Dent de Morcles to the Gemmi. By H. Dübi. Pp. xxiv+136. (London: T. Fisher Unwin, 1907.) Price 10s.

THIS conveniently arranged pocket-book is the most recent addition to the "Climbers' Guides" edited by Sir Martin Conway and the Rev. W. A. B. Coolidge. The southern limit of the region described is the Rhone Valley from Martigny to Leuk; and the northern is marked by the low passes leading from the Ormonts-Lessus glen to Kandersteg by the heads of the Grande Eau, the Sarine, the Simme, and the Kander valleys, which mark it off from the foothills. The preface of the book directs attention to the fact that the present is the jubilee year of the formation of the Alpine Club and of the ascents of the Oldenhorn and the Wildstrubel, and we are confident that the increase in the number of climbers during the last fifty years will ensure a wide popular for this workmanlike volume of "marching orders."