

### Polar Plotting Paper

MAY I be allowed to direct the attention of all interested in mathematical teaching in our schools and colleges to the polar plotting paper recently prepared by Mr. Ellice Horsburgh, lecturer on technical mathematics in Edinburgh University?

The special feature of this paper is that it is ruled radially with lines which subdivide the region about a point into aliquot parts of a radian. There are two forms of sheets now in the market. In one the origin is at the centre, and the radial subdivision is carried right round through four right angles. In the other, a reduced copy of which is here reproduced, the origin is taken near one corner, and the graduation is carried through a little more than a quadrant. Dotted radial lines show the backward continuation of the axis from which the radians are measured, and also the axis perpendicular to it. These dotted lines do not, of course, belong to the system of lines dividing the region into aliquot parts of a radian.

The radius of the fiftieth orthogonal circle is taken as the unit, and on the margins just outside the proper radian subdivisions small radial lines are drawn giving the usual division into degrees. The two circles drawn, the one on the axis as diameter and the other on the dotted perpendicular of unit length, serve to give by in-

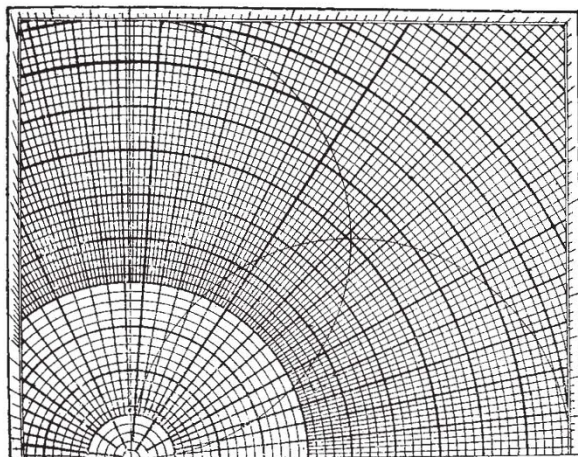


FIG. 1

spection the sines and cosines of the angles given in radians.

Thus the paper contains on its own surface the means of plotting with great ease the polar equations of curves involving radians, sines and cosines, and a little calculation will enable the student to take account of other functions.

The first important use in the hands of the student is obviously to get a clear idea of the radian as the true scientific measure of angle; but a great many other important uses will at once occur to the teacher of practical mathematics, such, for example, as finding reciprocals, geometric means, mean proportionals, fourth proportionals, squares, square-roots, &c.

Another use is the evaluation of the integrals  $\int r^2 d\theta$  and  $\int r d\theta$ . The former is got by simply counting the elements included in the area, and the latter by multiplying the total angle between the initial and final radius by the mean radius, the value of which may be obtained by a method similar to Simpson's rule.

From these few statements and indications the purpose of Mr. Horsburgh's patent will be readily appreciated. It is doubtful if the average student, taught along the usual lines, ever gets an accurate working knowledge of the radian or circular measure of an angle, indispensable though that is for all higher trigonometrical and analytical

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work. A few hours' systematised exercise with the polar paper will do more than days of arithmetical transformations in the usual academic style. C. G. KNOTT.

### Lissajous's Figures by Tank Oscillation.

THE oscillation of a rectangular water basin may be utilised for the illustration of the composition of two simple harmonic motions in two directions, perpendicular to each other.

A light pendulum was constructed of a thin aluminium rod,  $r$  (Fig. 1), 10 cm. long. The bob  $B$  was made of a disc of wood. On the upper end of the rod a light mirror  $M$  was attached. The rod could be supported at any desired point by a small gimbal  $G$ , so that the rod could oscillate as a spherical pendulum. A small brass weight  $w$  was attached to adjust the period of oscillation by raising or lowering it to a proper position.

The bob is sunk into the middle part of a suitable rectangular basin, filled with water to a proper depth. If the basin be tilted suddenly, and then let stand, the water is set in an oscillation which consists of two simple harmonic motions in perpendicular directions, the ratio of the periods varying as the ratio of the corresponding sides. The amplitudes of two component oscillations may be varied at pleasure. If the natural period of the pendulum is considerably shorter than that of the basin, the bob follows very nearly the motion of water, as judged by the motion of fine dust particles suspended in water. Now, if a beam of strong sun-light be sent as shown in the figure, the motion is projected on the ceiling of the room.

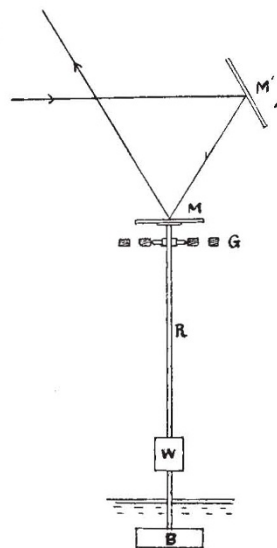
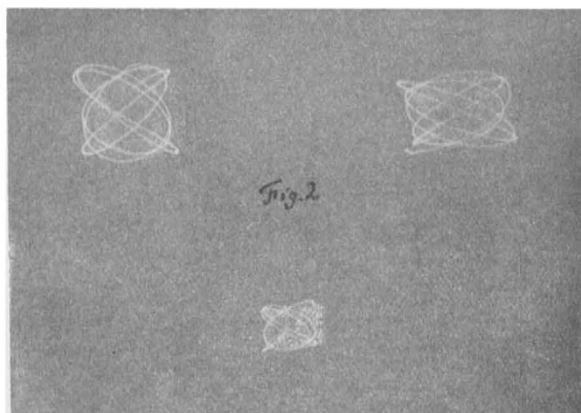


Fig. 1

I have also obtained a photographic record of the motion of a small bead attached to the upper end of a small needle erected on the rod. By making the illumination



intermittent by means of a perforated rotating disc, the difference of velocities at different phases may be shown.

The motion of a kaleidophone may be projected in a similar manner.

T. TERADA.

Physical Laboratory, Tokyo, December 19, 1904.