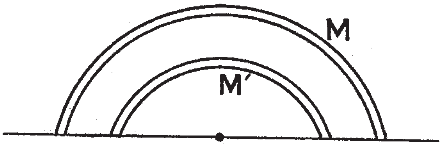
made here and at University College indicate that its specific gravity is about 9, and this figure agrees fairly well with that required for a mineral containing 75 per cent. of thoria.

T. A. HENRY.

Scientific and Technical Department, Imperial Institute, S.W., April 11.

Attraction between Concentric Hemispherical Shells.

BY the usual method of Legendre's functions I have arrived at the following result. If two thin attracting hemispheres, masses M and M′, radii a and a' ($a > a'$), are placed so that the rims lie in one plane and the centres coincide, the resulting attraction is $\frac{1}{2}$ M.M′/a^2.



From this result we conclude that we may replace M′ by any number of thin hemispherical shells (radii $< a$) subject to the conditions that the density of any shell is uniform, and that the total mass of all the shells is M′.

The result is so remarkable and simple that one looks for an elementary proof.

Perhaps some of your readers may be able to suggest one.

GEORGE W. WALKER.

Physical Laboratory, The University, Glasgow, March 28.

MR. G. W. WALKER tells me that he has sent to NATURE his interesting problem of the mutual attraction between two uniform concentric hemispherical shells, bounded by a common diametral plane. The following elementary solution has occurred to me. Call the outer shell A and the inner B. Now let another hemisphere A′ be added to A so that instead of the hemisphere A we have a complete and uniform spherical shell surrounding B. The attraction between the complete sphere and B is zero, if, as is here understood to be the case, the attraction between the particles follows the Newtonian law. Hence the attraction F of A on B is equal and opposite to the attraction of A′ on B. But the force exerted by A′ on B is obviously equal and opposite to the attraction which would be exerted by A on a hemisphere added to B so as to convert it into a complete spherical shell. Hence the force exerted by A on the inner sphere thus completed would be 2F, and this attraction is the same as that which would be exerted on a particle of double the mass of B placed at the centre. The attraction F of A on B is therefore that which would be exerted by A on a particle of mass equal to B placed at the centre, and the same thing holds for the reaction of B on A. Mr. Walker's result is therefore established.

We may go a step beyond the problem as proposed. Let the diametral plane bounding B, the shells remaining concentric, make any angle with the diametral plane bounding A. Then, by the same process of completing the sphere by adding A′ to A, we see that the attraction exerted by A′ on B is equal, and opposite in direction, to that which would be exerted by A on a hemisphere added to B to complete it in its new position. But the attraction of A on the inner sphere thus completed is equal to that which would be exerted by A on a particle of mass equal to twice that of B situated at the centre, and therefore the whole pull exerted on B by A, in any direction, is equal to the force, in that direction, exerted by A, on a particle of mass equal to that of B situated at the centre.

A. GRAY.

The University, Glasgow, April 6.

Curious Formation of Coal.

IN NATURE of January 14 (p. 250) Mr. Henry Hall describes a vertical deposit of a carbonaceous mineral in a wooden trough into which water from a coal mine had been delivered for three years. This interested me very much, as many years ago I described a similar carbonaceous mineral lining vertical cracks in a sandstone near Whangarei, in New Zealand (*Trans.* N.Z. Institute, vol. iii. p. 250, 1871).

I hope that Mr. Hall will make further observations and experiments on this singular phenomenon to see whether he is right in his explanation.

F. W. HUTTON.

Museum, Christchurch, New Zealand, February 25.

Photographic Effect of Radium Rays.

IT is interesting to note how pictures of the portions in relief on coins, medals, &c., can be obtained by means of radium rays. The coin or other object is placed directly in contact with a photographic plate which is enclosed in an envelope opaque to light. A few milligrams of radium bromide, contained in the usual mica-covered box, are placed some distance above the plate, and the whole left for several days. After development it is found that a clearly defined picture is obtained of the portions in relief on the under sides of the coins. Pictures have thus been obtained of the portions in relief on silver coins (half-crown, sixpence, threepence), also of a name engraved on a mother-of-pearl seal. Ten days was the time of exposure when ten milligrams of radium bromide were placed six inches above the plate, and the coin was a threepenny bit. Ten days also in the case of a half-crown when five milligrams were placed 1½ inches above the plate.

This radium effect was first shown at my last lecture on radium at the College of Science, Newcastle, on January 16, and has been shown at my subsequent lectures.

HENRY STROUD.

Durham College of Science, Newcastle-on-Tyne, April 9.

ON THE MEASUREMENT OF CERTAIN VERY SHORT INTERVALS OF TIME.

ACCORDING to the discovery of Kerr, a layer of bisulphide of carbon, bounded by two parallel plates of metal and thus constituting the dielectric of a condenser or leyden, becomes doubly refracting when the leyden is charged. The plates, situated in vertical planes, may be of such dimensions as 18 cm. long, 3 cm. high, and the interval between them may be 0·3 cm., the line of vision being along the length and horizontal. If the polarising and analysing nicols be set to extinction, with their principal planes at 45° to the horizontal, there is revival of light when the leyden is charged. If the leyden remain charged for some time and be then suddenly discharged, and if the light under observation be sensibly instantaneous, it will be visible if the moment of its occurrence be previous to the discharge; if, however, this moment be subsequent to the discharge, the light will be invisible. The question now suggests itself, what will happen if the instantaneous light be that of the spark by which the leyden is discharged? It is evident that the conditions are of extraordinary delicacy, and involve the duration of the spark, however short this may be. The effect requires the simultaneity of light and double refraction, whereas here, until the double refraction begins to fail, there is no light to take advantage of.

The problem thus presented has been very skilfully treated by MM. Abraham and Lemoine (*Ann. de Chimie*, t. xx., p. 264, 1900). The sparks are those obtained by connecting the leyden with a deflagrator and with the terminals of a large Ruhmkorff coil fed with an alternating current. It is known that if the capacity be not too small, several charges and discharges occur during the course of one alternation in the primary, and that while the charges are gradual, the discharges are sudden in the highest degree. If, as in the present case, the capacity is small, it is necessary to submit the poles of the deflagrator to a blast of air, otherwise the leyden goes out of action and the discharge becomes continuous. Under the blast, the number of sparks may amount to several thousands per second of time. In this way the intensity of the light is much increased and the impression upon the eye becomes continuous, but in other respects the phenomenon is the same as if there were but one spark.

In order to obtain a measure of the double refraction, which is rapidly variable in time, somewhat special arrangements are necessary. At the receiving end the light, after emergence from the trough containing the bisulphide of carbon, falls first upon a double image prism, of somewhat feeble separating power, so held that one of the images is extinguished when the leyden is out of action. The other image would be of full brightness, but this, in its turn, is quenched by an analysing nicol. When there is double refraction to be observed, the nicol is slightly rotated until the two images are of equal brightness. This equality occurs in two positions, and the angle between them may be taken as a measure of the effect. A full discussion is given in the paper referred to.

The finiteness of the angle, which in my experiments amounted to 12°, is a proof that the light on arrival at the CS_2 still finds it in some degree doubly refracting. To obtain the greatest effect the leads from the leyden to the deflagrator should be as short as the case admits, and the course of the light from the sparks to the CS_2 should not be unnecessarily prolonged. The measure of the double refraction, and in an even greater degree the brightness of the light as received, are favoured by connecting a very small leyden directly with the spark terminals, but the advantage is hardly sufficient to justify the complication.

The observations of Abraham and Lemoine bring out the striking fact that if the course of the light be prolonged with the aid of reflectors so as to delay by an infinitesimal time the arrival at the CS_2, the opportunity to pass afforded by the double refraction is in great degree lost, and the angular measure of the effect is largely reduced. There is here no change in the electrical conditions under which the spark occurs, but merely a delay in the arrival of the light.

The optical arrangements which I found most convenient in repeating the above experiment differ somewhat from those of the original authors. The sparks are taken at a short distance from the polarising nicol and somewhat on one side, and in both cases they are focused upon the analysing nicol. When the course is to be a minimum, the light is reflected obliquely by a narrow strip of mirror situated in the axial line, and focused by a lens of short focus placed near the first nicol. This lens and mirror are so mounted on stands that they can be quickly withdrawn, and by means of suitable guidance and stops as quickly restored to their positions. In this case the distance travelled by the light from its origin to the middle of the length of CS_2 is about 30 cm.

The arrangements for a more prolonged course are similar, and they remain undisturbed during one set of comparisons. The mirror is larger, and reflects nearly perpendicularly; it is placed upon the axial line at a sufficient distance behind the sparks. The light is rendered nearly parallel by a photographic portrait lens of about 18 cm. focus, the aperture of which suffices to fill up the field of view unless the distance is very long. In all cases the eye of the observer is focused upon the double image of the interval between the plates of the CS_2 leyden.

The earlier experiments were made at home somewhat under difficulties. For the blast nothing better was available than a glass-blowing foot bellows; but nevertheless the results were fairly satisfactory. Afterwards at the Royal Institution the use of a larger coil in connection with the public supply of electricity, and of an automatic blowing machine, gave steadier sparks and facilitated the readings. An increase of about one metre in the total distance travelled by the light reduced the measured angle from 12° to 6°, so that the time occupied by light in traversing one metre was very conspicuous.

It is principally with the view of directing attention to the remarkable results of Abraham and Lemoine that I describe the above repetition of their experiment, but I have made one variation upon it which is not without interest. In this case the spark is placed directly in the axial line and at some distance behind, which involves the use of longer leads, and therefore probably of a lower degree of instantaneity. The additional retardation is now obtained by the insertion of a 60 cm. long tube containing CS_2 between the sparks and the first nicol, and the comparison relates to the readings obtained with and without this column, all else remaining untouched. The difference is very distinct, and it represents the time taken in traversing the CS_2 over and above that taken in traversing the same length of air. It should be remarked that what we are here concerned with is not the wave-velocity in the CS_2, but the *group*-velocity, which differs from the former on account of the dispersion.

In the above experiments the leyden, where the Kerr effect is produced, is charged comparatively slowly and only suddenly discharged. For some purposes the scope of the method would be extended if the whole duration of the double refraction were made comparable with the above time of discharge. This could be effected somewhat as in Lodge's experiments, where a spark, called the B-spark, occurs between the outer coatings of two jars at the same moment as the A-spark between their inner coatings. The outer coatings remain all the while connected by a feeble conductor, which does not prevent the formation of the B-spark under the violent conditions which attend the passage of the A-spark. The plates of the Kerr leyden would be connected with the outer coatings of the jars, or themselves constitute the "outer" plates of two leydens replacing the jars. RAYLEIGH.

ENTROPY.[1]

IN NATURE, April 30, 1903, there is an article entitled "Entropy," describing at some length the great practical use which the engineer now makes of the $t\phi$ diagram. Engineers very ignorant of mathematics are able with clearness and certainty to make quantitative computations such as used to task the powers of mathematicians. The problems so easily worked out are very numerous and of a useful, interesting character, and mistakes are not easily made. On the other side of this question, in a notice of Mr. Donkin's translation of Prof. Boulvin's "The Entropy Diagram and its Applications" (NATURE, May 4, 1899), it was pointed out that such books were doing much harm because they made an illegitimate use of the $t\phi$ diagram. Thus I say:—"Of course we may, if we please, say that when steam is released to the condenser, we may imagine the whole change as occurring in the cylinder itself; only we ought to remember that we are substituting a very simple hypothetical process for a very complicated reality, which has almost nothing in common with it. We ought to remember that the very pretty, beautifully complete, cyclic $t\phi$ diagrams, which we obtain from childish assumptions, may get to be looked upon by students, and even by ourselves, as having a real meaning."

It is evident that this misuse of the $t\phi$ diagram is too prominent in Mr. Swinburne's mind, and that he fails to see the real usefulness of ϕ to engineers.

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[1] "Entropy or Thermodynamics from an Engineer's Standpoint and the Reversibility of Thermodynamics." By James Swinburne. Pp. x+137. (Westminster: Archibald Constable and Co.). Price 4s. 6d. net.