ties of the sediment, or mud, which I made, seemed to indicate that the organic matter was condensed hydrocarbon gases, or condensed volcanic vapours (such as one might expect to be evolved unburnt in a very large volcanic outburst). The sediment seems to be terrestrial, as the large amount of organic matter, coupled with the small amount of iron found, prohibits the theory of a meteoric origin.

The rain water contains 37·0 grains of suspended matter, or mud, to the gallon.

The analysis of the suspended matter, dried at 100° C., is as follows:—

Organic matter (loss on ignition) ...	36·4	per cent.
Silica	45·6	,,
Alumina and oxide of iron... ...	13·6	,,
Magnesia	2·4	,,
Unclassified	2·0	,,
	100·0	,,

Buckfastleigh, March 2. ROWLAND A. EARP.

Proof of Lagrange's Equations of Motion, &c.

IN your issue of January 29, Mr. Heaviside put forward a demonstration of Lagrange's equations of motion which appears invalid. As neither his interpretation of Newton nor his argument based thereon was stated with sufficient clearness to enable a critic to locate the weak spot without running serious risk of misinterpreting him, it seemed better in the first instance to point out a well-known case in which precisely similar reasoning would lead to Lagrange's equations of motion where they are known to be untrue (the reason, and a proper remedy, being also generally known). This I did in your number of February 19; his reply, in the same number, is to the effect that he does not intend to uphold the truth of Lagrange's equations in such a case. It is not, however, logically permissible for anyone to escape the inconvenient consequences of his own argument in such a fashion.

Possibly Mr. Heaviside has not grasped my point. If the argument he puts forward on p. 298 is valid, I am unable to see any point at which the following can without inconsistency be alleged to fail:—"In the case of a rigid body rotating round a fixed point with angular velocities ω_1, ω_2, ω_3 about its principal axes the kinetic energy T is a homogeneous quadratic function of the ω's, with coefficients which are constants. This makes

$$2\mathrm{T}=\omega_1\frac{d\mathrm{T}}{d\omega_1}+\omega_2\frac{d\mathrm{T}}{d\omega_2}+\omega_3\frac{d\mathrm{T}}{d\omega_3} \qquad (8)$$

therefore

$$2\dot{\mathrm{T}}=\omega_1\frac{d}{dt}\left(\frac{d\mathrm{T}}{d\omega_1}\right)+\dot{\omega}_1\frac{d\mathrm{T}}{d\omega_1}+\ .\ .\ . \qquad (9)$$

But also by the structure of T,

$$\dot{\mathrm{T}}=\dot{\omega}_1\frac{d\mathrm{T}}{d\omega_1}+\dot{\omega}_2\frac{d\mathrm{T}}{d\omega_2}+\dot{\omega}_3\frac{d\mathrm{T}}{d\omega_3} \qquad (10)$$

So, by subtraction of (10) from (9)

$$\dot{\mathrm{T}}=\omega_1\frac{d}{dt}\left(\frac{d\mathrm{T}}{d\omega_1}\right)+\omega_2\frac{d}{dt}\left(\frac{d\mathrm{T}}{d\omega_2}\right)+\omega_3\frac{d}{dt}\left(\frac{d\mathrm{T}}{d\omega_3}\right) \qquad (11)$$

and therefore, by Newton, the torque about the first axis is the coefficient of ω, *i.e.* $\mathrm{A}\dot{\omega}_1$, and similarly for the rest."

There is no step in his demonstration which requires that the coordinates should be "proper Lagrangian coordinates within the meaning of the Act"; in the proof usually given there is such a step.

It is with great diffidence, lest I may do Mr. Heaviside injustice through misinterpreting him, that I now venture to express the conjecture that in his argument he may possibly have failed, as is sometimes done [by Maxwell, for instance, "Treatise," second edition, § 561, equations (5)], to distinguish between the displacements which a material system actually receives during its motion and displacements which are perfectly arbitrary subject only to the geometrical connections of the system, and have thus confounded the equation

$$\mathrm{X}_1v_1+\ .\ .\ .\ =\left(\frac{d}{dt}\frac{d\mathrm{T}}{dv_1}-\frac{d\mathrm{T}}{dx_1}\right)v_1+\ .\ .\ .$$

which expresses that the rate at which work is done by the forcives is equal to the rate at which the system gains kinetic energy, with the very different one

$$\mathrm{X}_1\delta x_1+\ .\ .\ .\ =\left(\frac{d}{dt}\frac{d\mathrm{T}}{dv_1}-\frac{d\mathrm{T}}{dx_1}\right)\delta x_1+\ .\ .\ .$$

in which δx_1, &c., are arbitrary displacements as above. When the latter equation is established, Lagrange's equations follow at once, but Mr. Heaviside has made out no case for deducing them from the former. In every case, as in the example I cited, the right-hand member of the former equation can be written in the form

$$v_1\varphi_1(x_1,\ v_1,\ \dot{v}_1,\ x_2,\ v_2,\ \dot{v}_2,\ .\ .\ .)+\ .\ .\ .$$

in an infinite variety of ways, and accordingly it is sufficiently obvious that there is no warrant for stating that the force on x_1 is the coefficient of v_1 in any one such form more than in any other. Samples of expressions which might thus be wrongly obtained for the torque about the first axis in the instance alluded to are

$$\mathrm{A}\dot{\omega}_1,\ \mathrm{A}\dot{\omega}_1-(\mathrm{B}-\mathrm{C})\omega_2\omega_3,$$
$$\mathrm{A}\dot{\omega}_1+(\mathrm{B}-\mathrm{C})\omega_2\omega_3,\ \mathrm{A}\dot{\omega}_1-(\mathrm{B}\omega_2^3-\mathrm{C}\omega_3^3)/\omega_1.$$

For the simpler case of a particle moving in a plane, one could thus obtain, for example, the equations,

$$\mathrm{X}=m(\ddot{x}-k\dot{y}),\ \mathrm{Y}=m(\ddot{y}+k\dot{x}),$$

where k is any quantity whatever.

In short, the latter of the two equations compared above differs from the former in being equivalent to a set of independent equations equal in number to that of the coordinates of the system.

Similar remarks apply, of course, to his treatment of the question of an elastic medium, p. 297.

That the Principal of Energy, or of Activity, does not by itself afford a sufficient basis from which to formulate the fundamental equations of dynamics in any form whatever is admitted almost universally; from Mr. Heaviside's letters it appears at least doubtful whether he is willing to agree with this general and well grounded opinion; he has advanced no valid argument against it, however. W. McF. ORR.

February 22.

A FEW weeks ago you published in a letter from Mr. Heaviside a proof of Lagrange's equations of motion of a system of bodies. I must confess that I in common with others swallowed it, but I have now come to the conclusion that the proof, though doubtless admirable as an example of the power of the "Principle of Activity," does not prove Lagrange's equations. In fact, if q be a coordinate, $\dot{q}$ the corresponding velocity, and Q the corresponding force, we have the result

$$\Sigma\dot{q}\left\{\frac{d}{dt}\frac{\partial\mathrm{T}}{\partial\dot{q}}-\frac{\partial\mathrm{T}}{\partial q}-\mathrm{Q}\right\}=0$$

for any possible motion of the system. But we are not entitled to equate the quantities in the brackets to zero, for *these are not independent of* $\dot{q}$. The "proof" is, in fact, merely Maxwell's well-known but fallacious proof, simplified by going direct instead of *viâ* Hamilton.

Cambridge, February 28. R. F. W.

Genius and the Struggle for Existence.

PERMIT me to point out that Dr. A. R. Wallace's statement (p. 296), "the comparatively short lives of millionaires," is not supported by facts, at any rate by those for the last three years.

The following has been obtained from the details concerning estates on which death duties were paid. Nine millionaires died during 1900, leaving in the aggregate 19 millions. The average age of these nine testators is seventy-four—the youngest was fifty-nine and the oldest ninety-one years.

During 1901, we find that the deaths of eight millionaires are recorded, whose joint estates were valued at $10\frac{1}{2}$ millions. In this case too, we find that the average age is above the allotted threescore years and ten, being seventy-two. The