herbarium were saved, passing into the hands of Nicola Cirillo (1671–1734), a physician and botanist who possessed a private botanical garden and was a Fellow of the Royal Society of London, for which Society he collected data on the climate of Naples, and wrote a treatise on the application of cold in the treatment of fevers. Remaining in the Cirillo family, the herbarium was finally bequeathed to the celebrated botanist Domenico Cirillo, who preserved these volumes as the most precious treasure in his collections. In 1783, Martin Vahl, a friend of Linnæus, saw Imperato's herbarium in Cirillo's house, and it is said that he fell on his knees in reverence before the ancient relic. In 1799, when the royalist mob sacked Cirillo's house and Cirillo himself was hanged, all his collections were dispersed, including the herbarium of Imperato. Of the nine volumes only one was saved, and finally came into the hands of Camillo Minieri-Riccio, who in 1863 published a short account of this botanical relic (C. Minieri-Riccio: "Breve notizia dell' Erbario di Ferrante Imperato," *Rendiconti dell' Accademia Pontaniana*, xi., 1863). Minieri says that Imperato's name is written in the volume.

The collections of Minieri-Riccio were finally sold to the National Library at Naples, where the volume of Imperato's herbarium may now be seen.

The volume, of 268 pages, is bound in parchment and is labelled "Collectio Plantarum Naturalium." It contains 440 plants, glued to the paper, each with one or more names. There is an alphabetical index, probably written by Imperato himself.

The authorities in the Naples library do not seem aware of the importance of the relic they possess, for the herbarium is kept as an ordinary book and the plants are exposed to inevitable damage and decay. Several of the specimens have already been eaten up by insects. ITALO GIGLIOLI.

R. Stazione Agraria Sperimentale, Rome, January 8.

A Curious Projectile Force.

I AM able to corroborate B.A. Oxon.'s letter (p. 247). In my case, the screw stopper of the bottle (inverted) rested at an angle against some books on a table. When the pressure of the gas was sufficient to force out the stopper, the bottle sprang three or four feet into the air and fell some distance off on the floor of the room. NORMAN LOCKYER.

The Principle of Least Action. Lagrange's Equations.

WHETHER good mathematicians, when they die, go to Cambridge, I do not know. But it is well known that a large number of men go there when they are young for the purpose of being converted into senior wranglers and Smith's prizemen. Now at Cambridge, or somewhere else, there is a golden or brazen idol called the Principle of Least Action. Its exact locality is kept secret, but numerous copies have been made and distributed amongst the mathematical tutors and lecturers at Cambridge, who make the young men fall down and worship the idol.

I have nothing to say against the Principle. But I think a great deal may be said against the practice of the Principle. Truly, I have never practised it myself (except with pots and pans), but I have had many opportunities of seeing how the practice is done. It is usually employed by dynamicians to investigate the properties of mediums transmitting waves, the elastic solid for example, or generalisations or modifications of the same. It is used to find equations of motion from energetic data. I observe that this is done, not by investigating the actual motion, but by investigating departures from it. Now it is very unnatural to vary the time integral of the excess of the total kinetic over the total potential energy to obtain the equations of the real motion. Then again, it requires an integration over all space, and a transformation of the integral before what is wanted is reached. This, too, is very unnatural (though defensible if it were labour-saving), for the equation of motion at a given place in an elastic medium depends only upon its structure there, and is quite independent of the rest of the medium, which may be varied anyhow. Lastly, I observe that the process is complicated and obscure, so much so as to easily lead to error.

Why, then, is the P. of L. A. employed? Is not Newton's dynamics good enough? Or do not the Least-Actionists know that Newton's dynamics, viz. his admirable Force = Counter-force and the connected Activity Principle, can be directly applied to construct the equations of motion in such cases as above referred to, without any of the *hocus pocus* of departing from the real motion, or the time integration, or integration over all space, and with avoidance of much of the complicated work. It would seem not, for the claim is made for the P. of L. A. that it is a commanding general process, whereas the principle of energy is insufficient to determine the motion. This is wrong. But the P. of L. A. may perhaps be particularly suitable in special cases. It is against its misuse that I write.

Practical ways of working will naturally depend upon the data given. We may, for example, build up an equation of motion by hard thinking about the structure. This way is followed by Kelvin, and is good, if the data are sufficient and not too complicated. Or we may, in an elastic medium, assume a general form for the stress and investigate its special properties. Of course, the force is derivable from the stress. But the data of the Least-Actionists are expressions for the kinetic and potential energy, and the P. of L. A. is applied to them.

But the Principle of Activity, as understood by Newton, furnishes the answer on the spot. To illustrate this simply, let it be only small motions of a medium like Green's or the same generalised that are in question. Then the equation of activity is

$$\text{div. } \mathbf{q}\mathrm{P} = \dot{\mathrm{U}} + \dot{\mathrm{T}}\,; \qquad (1)$$

that is, the rate of increase of the stored energy is the convergence of the flux of energy, which is $-\mathbf{q}\mathrm{P}$, if $\mathbf{q}$ is the velocity and P the stress operator, such that

$$\mathrm{P}\mathbf{i} = \mathbf{P}_1 = \mathbf{i}\mathrm{P}_{11} + \mathbf{j}\mathrm{P}_{12} + \mathbf{k}\mathrm{P}_{13} \qquad (2)$$

is the stress on the $\mathbf{i}$ plane. Here $\mathbf{q}\mathrm{P}$ is the conjugate of $\mathrm{P}\mathbf{q}$.

By carrying out the divergence operation, (1) splits into two, thus

$$\mathbf{F}\mathbf{q} = \mathrm{T}, \qquad \mathbf{G}\mathbf{q} = \mathrm{U}. \qquad (3)$$

Here $\mathbf{F}$ is a real vector, being the force, whilst $\mathbf{G}$ is a vector force operator. Both have the same structure, viz. $\mathrm{P}\nabla$, but in $\mathbf{F}$ the differentiators in ∇ act on P, whereas in $\mathbf{G}$ they are free and act on $\mathbf{q}$, if they act at all.

Now when U is given, U becomes known. It contains $\mathbf{q}$ as an operand. Knock it out; then $\mathbf{G}$ is known; and therefore $\mathbf{F}$; and therefore the equation of motion is known, viz.

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{q}),$$

where m is the density, or the same generalised eolotropically, or in various other ways which will be readily understood by electricians who are acquainted with resistance operators.

Of course, P becomes known also. So the form of U specifies the stress, the translational force and the force operator of the potential energy. To turn $\mathbf{G}$ to $\mathbf{F}$ is the same as turning $\mathrm{A}\dfrac{d}{dx}$ to $\dfrac{d\mathrm{A}}{dx}$.

If, for example, the displacement is $\mathbf{D}$, the potential energy is a quadratic function of the nine differentiants $d\mathrm{D}_1/dx$, &c., of the components. Calling these r_{11}, r_{12}, &c.;

$$\mathrm{U} = \tfrac{1}{2}r_{11}\frac{d\mathrm{U}}{dr_{11}} + \tfrac{1}{2}r_{12}\frac{d\mathrm{U}}{dr_{12}} + \ldots, \qquad (4)$$

by the homogeneous property. Therefore, since $\dot{r}_{12} = dq_1/dy = \mathbf{i}d\mathbf{q}/dy$,

$$\dot{\mathrm{U}} = \left(\frac{d\mathrm{U}}{dr_{11}}\,\mathbf{i}\,\frac{d}{dx} + \frac{d\mathrm{U}}{dr_{12}}\,\mathbf{i}\,\frac{d}{dy} + \ \ldots\right)\mathbf{q} = \mathbf{G}\mathbf{q}\,; \qquad (5)$$

therefore, writing P_{21} for $d\mathrm{U}/dr_{12}$,

$$\mathbf{F} = \mathbf{i}\left(\frac{d\,\mathrm{P}_{11}}{dx} + \frac{d\,\mathrm{P}_{21}}{dy} + \frac{d\,\mathrm{P}_{31}}{dz}\right) + \ \ldots \qquad (6)$$

$$= \frac{d\mathbf{P}_1}{dx} + \frac{d\mathbf{P}_2}{dy} + \frac{d\mathbf{P}_3}{dz}. \qquad (7)$$

It is clear that the differentiants in (4) (which involve the large number 45 of coefficients of elasticity in the general case of eolotropy) are the nine components of the conjugate of the stress operator. Of course, vector analysis, dealing with the natural vectors concerned, is the most suitable working agent, but the same work may be done without it by taking the terms involving q_1, q_2, q_3 separately.

Another expression for U is $\mathrm{U} = \frac{1}{2}\mathbf{G}\mathbf{D}$, which shows how to find $\mathbf{F}$ from U directly.