

in the 1 to 8 per cent. alloys, but differing from them in never occurring hollow. On oxidation it becomes very dark and is easily distinguished from the other two constituents of the alloy. In form it is plate-like, and around it crystallises out the bright constituent characteristic of the 1 to 8 per cent. alloys, either as a rough envelope of fairly uniform thickness or as projecting crystals. Stead was the first to draw attention to the fact that the crystals of this division of the series were composite. As the copper approaches 40 per cent., the plate-like crystals are grouped together in parallel bunches. Casting masks the composite character of the crystals, if, in the lower percentages, it does not destroy it; for under 20 per cent. Cu, the crystals cannot be resolved into two components under high powers.

41 to 61·7 per cent. Copper.—At 41 per cent. Cu the crystals are small, lath-shaped, and arranged more or less in groups. The alloy is brittle, and this brittleness increases with the percentage of copper. With each addition of copper, the groups of composite crystals become more and more compact and the amount of eutectic diminishes until at 56 per cent. Cu it disappears (Stead, *Journal of the Society of Chemical Industry*, June, 1897), and the bright constituent of the crystals forms the groundmass. When 61·7 per cent. Cu is reached, the bright constituent disappears and we have a homogeneous mass of SnCu_3 , probably a definite compound. Casting tends to harden and toughen these alloys. Seeing that these alloys up to about 56 per cent. Cu show four breaks in their cooling curves, one would naturally expect to find four different constituents in each. Only three, however, can be distinguished. Quenching below the first and second breaks gives a difference in structure only. As in the alloys containing 61·7 per cent. Cu and onwards, branch *e* of the curve (Fig. 1) corresponds to a rearrangement in the solid, and as the difference between the 40 and 41 per cent. Cu alloys is one of structure only, we may assume that the second retardation in the cooling curve (*e*) is one of rearrangement also.

61·7 to 68·28 per cent. Copper, SnCu_3 to SnCu_4 .—The changes which take place between these two points can only be observed when the alloys are very slowly cooled. The alloys set as a whole at the first break and tend to rearrange themselves subsequently in the solid, on branch *e* (Fig. 1). Each addition of copper to SnCu_3 brings in more and more of a bright constituent, probably SnCu_4 . Quenching and casting produces structures entirely new. Figs. 2-5 show the 66 per cent. Cu alloy differently cooled. Fig. 2 was quenched on the first break. There is a cell-like structure with light-coloured walls or boundaries. In places the change has gone further, and we get the fine cross-hatching characteristic of Fig. 3, which has been quenched below the first break. The cell-like structure has entirely disappeared. Fig. 4 has been quenched below the second break and resembles a slowly-cooled alloy, except that in the latter there are distinct traces of a eutectic structure. Fig. 5 has been cast on an iron plate, and the "schiller" structure is well developed. At 68·2 per cent. Cu the alloy is homogeneous, has a conchoidal fracture and is extremely brittle.

68·28 to 75 per cent. Copper.—Immediately the copper is increased above 68·3 per cent., the second eutectic makes its appearance. As the copper increases, the grains of SnCu_4 split up into bright veins and dendrites, surrounded by the eutectic. The veins and dendrites decrease and disappear at 75 per cent. Cu, where the mass is made up entirely of the eutectic. The alloys are best studied when furnace-cooled; their surfaces above 71 per cent. Cu are seen to consist of a network of dendrites or skeleton crystals resembling those seen on the surface of a pure metal. This surface structure continues right up to the copper end of the series. It was soon noticed that the internal structure of the alloys from 70 to 75 per cent. Cu showed no trace of these dendrites, and so the surfaces of several were rubbed down and polished. In each case their structure was the same as that of the centre of the alloy, which shows that these dendrites have split up and rearranged themselves after solidification, and all that remains of them is this surface structure. Casting makes the structure very minute, and about 73 per cent. Cu traces of the skeleton crystals can be seen in the centre of the ingot. They appear dark and structureless, as if they had been unable to resolve themselves into their two constituents.

75 to 100 per cent. Copper.—When 76 per cent. copper is present, two new constituents make their appearance and the alloy assumes a yellow tint. In section we find yellow grains, surrounded by a bright white border, set in the second eutectic,

in which small bright white grains also occur. As the total copper is increased, the yellow grains increase, forming dendrites and skeleton crystals, the white borders and grains merge together and the eutectic decreases till at about 90 per cent. it disappears. The yellow grains become darker and darker (contain less and less tin in solid solution). The light borders diminish and disappear, about 95 per cent. leaving copper dendrites behind. These dendrites vary in composition from centre to outside, and so the centre etches a darker colour. They darken with increase of copper until 100 per cent. is reached. Casting tends to make the copper grains solidify, containing a considerable quantity of tin. In this way the eutectic can be made to disappear considerably below 90 per cent. Cu. Quenching shows that the upper break corresponds to the solidification of the copper; break 2 to the solidification of the groundmass which splits up into a eutectic when branch *e* is reached. Fig. 6 contains 80 per cent. Cu furnace-cooled, whilst Fig. 7 shows the surface of the same and also that with this percentage of copper the dendrites of copper have directed the formation of the surface skeletons. Fig. 8 is the same alloy quenched below first break. The dendrites of copper are seen set in a structureless matrix. Fig. 9 is the same alloy quenched below the second break. The dendrites of copper (light, because of a different etching process) are seen, set in a fibrous matrix—the eutectic of which the formation has been faced. Fig. 10 shows the same alloy cast. As its appearance would indicate, the alloy is very tough and cuts well.

It seems clear then that branch *e* of the cooling curve is one of change in the solid, and this conclusion has been proved beyond doubt by the beautiful work of Heycock and Neville published by the Royal Society. When one considers the many and distinct different structures in the series produced by quenching at different temperatures and by reheating and then quenching, it is quite evident that the changes which take place during the cooling of an alloy of copper and tin, especially in the neighbourhood of the second eutectic, are even more numerous than those of the carbon-irons.

UNIVERSITY AND EDUCATIONAL INTELLIGENCE.

CAMBRIDGE.—The Allen studentship, value 250*l.* for one year, for research in connection with medicine, mathematics, physics and chemistry, biology and geology, or moral science, will be filled up at the end of the present term. It is open to graduates under the age of twenty-eight on January 8.

Principal E. H. Griffiths, F.R.S., of Cardiff University College, has been approved for the degree of Doctor of Science.

The Rede lecture will be delivered next term by Prof. Osborne Reynolds, F.R.S., of Owens College, Manchester.

Mr. W. N. Shaw, F.R.S., will give three lectures, on February 13, 20 and 27, on the physics of the ventilation of buildings.

Prof. Tilden, F.R.S., has been appointed an elector to the chair of chemistry; Lord Rayleigh, F.R.S., an elector to the chairs of chemistry and of mechanism; Dr. Hill, to the anatomy chair; Mr. F. Darwin, F.R.S., to the botany chair; Dr. Hinde, F.R.S., to the geology chair (Woodwardian); Sir G. G. Stokes, F.R.S., to the Jacksonian and Cavendish chairs; Dr. D. MacAlister, to the Downing chair of medicine; Dr. Hugo Müller, F.R.S., to the chair of mineralogy; Prof. E. Ray Lankester, F.R.S., to the chair of zoology and comparative anatomy; Prof. McKendrick, F.R.S., to the chair of physiology; Lord Lister, F.R.S., to the chair of pathology; and Prof. Marshall Ward, F.R.S., to the chair of agriculture.

Dr. J. Reynolds Green, F.R.S., has been elected to a fellowship at Downing College.

THE University College, Bristol, does not receive the generous support given to similar colleges elsewhere, but the report of the council for the session 1900-01 shows that much valuable work has been done in spite of limited means and opportunities. Important papers have been published by various members of the scientific staff and others are in progress. The clinical and bacteriological research laboratory, which has been at work under Prof. Stanley Kent for little more than a year, has, among other matters, been able to afford valuable aid to the Medical Officer of Health in reporting upon the presence of plague in-

fection in material submitted for examination. In one case, plague infection was found to be undoubtedly present, and the report of the laboratory upon it was made immediately. In a second case the appearances presented were suspicious, and the report of the laboratory enabled precautions to be taken to safeguard the city in the event of true plague appearing. More accommodation for this kind of research work is required, as there is no lack of persons willing to undertake it. The same remark applies to other departments of the College. If people of means in Bristol and the neighbourhood took any interest in educational progress, the establishment of the University of Birmingham, and the movement in favour of other universities of a similar kind, would inspire them to action in the same direction. There is room for a University of the West of England, and if Bristol does not rise to its opportunity another city of the west will take its place. The subject has been brought up over and over again, and only a few days ago Mr. Haldane spoke in favour of it at the annual dinner of the University College Colston Society. The Bishop of Hereford also alludes to it at the end of the present report. But the rich citizens of Bristol do not seem to understand what has been done by private persons for higher education in cities like Liverpool, Birmingham and Manchester, or if they know they apparently have no desire to follow the example. There will have to be a complete awakening of the spirit of pride in local resources for education and research before Bristol can make any real movement towards a University of the West of England.

SCIENTIFIC SERIALS.

Transactions of the American Mathematical Society, vol. iii. No. 1, January.—On a class of automorphic functions, by J. L. Hutchinson. In Burkhardt's "Ueber die darstellung einiger fälle der automorphen primformen durch Specielle Thetareihen," the following monodromy group of the Riemann surface, $y^3 = (x - \alpha_1)(x - \alpha_2)(x - \beta_1)^2(x - \beta_2)^2$, is considered, and he shows how a certain prime form which is automorphic for the group can be expressed by a theta series. Further results are here given concerning the group and the functions belonging to it, the chief object being to obtain explicit analytic formulæ by means of which all functions of the group can be represented. To this end the theta-fuchsian functions of Poincaré are introduced, and their expressions in terms of the hyperelliptic theta series deduced.—Concerning the existence of surfaces capable of conformal representation upon the plane in such a manner that geodetic lines are represented by a prescribed system of curves, by H. F. Stecker, is in continuation of a previous paper under nearly the same title (vol. ii. p. 152).—Zur erklärang der Bogenlänge und der inhaltes einer Krümmen fläche, by O. Stoll (cf. the author's "Grundzüge der Differential- und Integralrechnung," Bd. 2, and *Math. Ann.*, Bd. 18).—The groups of Steiner in problems of contact, by Dr. L. E. Dickson, gives an elementary proof of Jordan's ("Traité," pp. 229-249). Reference is given to Steiner and Hesse (*Journal für Math.*, vol. xlix. (1855) and vol. lxiii. (1864)), and to papers by the author (*Bulletin Amer. Math. Soc.*, vol. iv., and the *American Journal of Mathematics*, vol. xxiii. pp. 337-377).—Quaternion space, by A. S. Hathaway, follows up Stringham's work in vol. ii. p. 183, but frequent reference is made to Clifford's paper on biquaternions. Stringham deals analytically with the equations of loci and develops the geometry by the interpretation of those equations; the author uses a more synthetic method, which interprets the quaternion symbols themselves instead of the equations between them. It is this divergence which constitutes the general difference between the methods of Cayley and Tait. Clifford stated the synthetic view in his Further note on biquaternions.—Reciprocal systems of linear differential equations, by E. J. Wilczynski, arrives at interesting results in connection with previous papers (*Transactions*, vol. ii. No. 4; *American Journal of Mathematics*, vol. xxiii.).—On the invariants of quadratic differential forms, by C. N. Haskins, investigates, by means of Lie's theory of continuous groups, the problem of determining the number of invariants of the general quadratic form in n variables. Numerous references occur in the paper.—On the nature and use of the functions employed in the recognition of quadratic residues, by Dr. E. McClintock, refers to Tannery, "Leçons d'Arithmétique," Bachmann, "Elemente der Zahlentheorie," and to Baumgart, "Ueber der Quadratische Reciprocitätsgesetz."—A determination of the number of real

and imaginary roots of the hypergeometric series, by E. B. Van Vleck. Concisely we must refer to Klein (*Math. Ann.*, vol. xxxvii. p. 573) for the number of the roots of the equation considered between 0 and $-\infty$. Mr. Van Vleck claims that he gives, for the first time, the number of imaginary roots. Numerous references and diagrams (six and a page of sixteen) accompany the text.—The second variation of a definite integral when one end-point is variable, by G. A. Bliss. The method which the author applies to the discussion of the case in which one end-point moves on a fixed curve is closely analogous to that of Weierstrass ("Lectures on the Calculus of Variations," 1879). In the present case terms outside of the integral sign are taken into consideration. Then, as a result of the discussion, the analogue of Jacobi's criterion is derived, defining, apparently in a new way, the critical point (Kneser's "Brennpunkt") for the fixed curve along which the end-point varies. Then the relation between the critical and conjugate points is discussed.—On the projective axioms of geometry, by E. H. Moore, contains a consideration of the axioms called by Hilbert ("Grundlagen der Geometrie") the axioms of connection and of order, and by Schur ("Über die Grundlagen der Geometrie") the projective axioms of geometry. There are several citations of authorities, such as Peano, Pasch and Ingrami.

Bulletin of the American Mathematical Society, January.—Note on Mr. George Peirce's approximate construction for π , by E. Lemoine. This article gives four constructions suggested by a discussion of Mr. Peirce's which we have previously noticed (*Bulletin*, July, 1901). The relative theoretic exactness is determined by calculating the true value of the length which in each case approximately represents π . The solutions are worked out by aid of the geometrographic notation. A slight sketch of this method, sufficient for the present purpose, is given. (For a fuller account, reference may be made to M. Lemoine's "La Géométrie" or to the "Traité de Géométrie" in the *Archiv der Mathematik und Physik*, April and May, 1901, vol. i., Gauthier-Villars). There is also appended a close approximation to the trisection of an angle by C. Störmer worked by the same method. There are several diagrams.—Concerning the elliptic $\wp(\xi, 2831\xi)$ functions as coordinates in a line complex, and certain related theorems, by Dr. H. F. Stecker, is an application of the coordinates to the Kummer surface and certain other configurations (cf. Klein, *Math. Ann.* vol. v., pp. 294-5).—A short note on the Abelian groups which are conformal with non-Abelian groups follows, by Dr. G. A. Miller. Dr. S. E. Slocum writes on the infinitesimal generators of certain parameter groups. The paper opens with a *résumé* of the method employed by the author on pp. 97-103 of the *Proceedings* of the American Academy of Arts and Sciences, vol. xxxvi., and then proceeds to give tables in which are enumerated all possible types of structure of two-, three- and four-parameter complex groups as given by Lie, and under each structure are given the symbols of the infinitesimal transformations which generate the parameter group corresponding to that structure, obtained by the method referred to above (cf. Lie, "Continuierliche Gruppen," pp. 565-589; "Transformations Gruppen," vol. iii., pp. 713-730).—Notices follow of the "Einführung in die Theorie der Differentialgleichungen mit einer unabhängigen Variablen" of Dr. L. Schlesinger and of Prof. Hatzidakis's "εισαγωγή εις την Ανωτέραν Αλγέβραν."—The usual information of interest to mathematicians follows in the form of notes and new publications.

Annals of Mathematics, January.—Some applications of the method of abridged notation, by Maxime Bôcher, is an interesting elementary paper the nature of which may be gathered from an illustration. Let the sides of a triangle be $u=0, v=0, w=0$ (where $u \equiv x \cos \alpha + y \sin \alpha - \rho$), then $u-v=0, v-w=0, w-u=0$ are the bisectors of the angles, and as the sum of the sinisters vanishes identically we get the property of the bisectors of the angles countersecting in a point. If, again, $u \equiv x^2 + y^2 + ax + by + c = 0$ and so on, we can show that the common chords of three circles meet in a point. The author then proceeds to the proof of Desargues' theorem and thence to generalisations for four circles and for curves of the n th order, and extends, by suggestion, his results to surfaces.—On the roots of functions connected by a linear recurrent relation of the second order, by M. B. Porter, reproduces in part some unpublished theorems of Sturm (cf. "Liouville," vol. i.), and shows how, by means of the Cauchy-Lipschitz theorem for the existence of solutions of a differential equation, it is possible to establish