LETTERS TO THE EDITOR.

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The Life of a Star.

THE letter of Prof. Perry on "The Life of a Star," published in your issue of July 13, is of interest to astronomers; and as the author of it evidently aims to be fair, I think it worth while to set right a misconception into which he has fallen. His reference to my paper in the *Astronomical Journal* (No. 455) shows that he has misconstrued the meaning of the symbol K in the formula $T = \frac{K}{R}$. That paper was unfortunately very much abbreviated, and as I was not concerned with the analytical investigation of K, this constant was not sufficiently explained. Yet in my first note on this law in the *A. J.* (453), which he probably overlooked, it was announced that "K is a constant different for each body." Thus the constant K is not, as Prof. Perry supposes, the same for the whole universe, but varies from star to star, being a function of the mass, specific heat, emissive power, &c., into which we need not go at present.

At the time of writing the paper in the *A. J.* (455) I had not seen the early paper by Ritter in *Wiedemann's Annalen*, 1878, s. 543,[1] where he has reached by a different process a similar formula

$$T_0 r_0 = T r,$$

and anticipated a number of the conclusions to which I came independently. Ritter even applied this law to the temperature of the solar nebula when its periphery extended to the orbit of Neptune, and sagaciously observes that his conclusions do not agree with current views, but are yet uncontradicted by known facts.

As I have prepared for the St. Louis Academy of Sciences a paper in which this whole matter is discussed with some detail, I will here merely summarise a few of the chief results. Suffice it to say that the formula $T = \frac{K}{R}$ is shown to express a law of the utmost generality, for masses composed of one kind of gas; and that even when the body is of heterogeneous constitution, made up of interpenetrating globes of different gases, the law suffers no essential modification except at very long intervals, when it would take the form

$$T = \frac{K (1 + \beta t)}{R},$$

where β is a certain small secular coefficient, and t the time. For a great epoch the term depending on β might be wholly neglected.

The only hypothesis underlying the investigation is that of convective equilibrium, the validity of which is generally recognised by physical investigators. In order that the reader may see how far from metaphysical my argument really is, I add an elementary derivation of the law of temperature.

Suppose a gaseous globe of radius R_0 to be held in convective equilibrium by the attraction, pressure and temperature of its particles (the density and temperature decreasing from the centre to the surface), and let the temperature beneath the surface layer be T_0. Let P_0 be the gravitational attraction exerted upon the thin layer of matter covering a unit surface of the globe, which may be regarded as the base of an elemental cone extending to the centre. Then suppose the globe to shrink by loss of heat to a radius R. If the original element of mass still covered a unit area, the pressure exerted upon its lower surface would thereby become $P = P_0 \left(\frac{R_0}{R}\right)^2$. But since the area of the initial sphere surface has shrunk to $S = S_0 \left(\frac{R}{R_0}\right)^2$ the area of the elemental conical base has diminished in the same ratio. As the force of gravity is increased, while the area upon which the element presses is decreased correspondingly, it follows that in the condensed condition of the globe, the gravitational pressure exerted upon a unit area is $P = P_0 \left(\frac{R_0}{R}\right)^4$. The forces counter-

[1] Prof. Nipher, in preparing the excellent papers which he has contributed to the St. Louis Academy of Sciences, first drew attention to this reference.

acting this increased pressure are obviously the resistance due to the increase in the mean density, and a possible change in temperature which might affect the elasticity of the gas. But the density of the original mass was σ_0, and hence $\sigma = \sigma_0 \left(\frac{R_0}{R}\right)^3$. By hypothesis the equilibrium of the globe is maintained by the elastic force of the gas under the heat developed by the gravitational shrinkage of the mass. If therefore the globe was in equilibrium when the gas just beneath the surface layer had a mean temperature T_0, to remain in equilibrium in the condensed condition, T_0 must be multiplied by $\frac{R_0}{R}$. As $T_0 R_0 =$ constant, we may write the law of temperature.

$$T = \frac{K}{R}.$$

This law of course applies to each layer of the globe, and thus to its mean temperature, and is obviously general for all gaseous celestial bodies condensing under the law of gravitation. Some persons who do not fully understand the problem under consideration have asserted that the functions which express the distribution of density and temperature with respect to the radius, are altered by shrinkage, so that the law then breaks down, or rather is not proved to hold true. It is perhaps worth while to show the error of this view.

Lord Kelvin has shown (*Phil. Mag.*, 1887, p. 287) that the temperature distribution throughout the globe must satisfy the differential equation

$$\frac{d^2\theta}{dx^2} + \frac{\theta^\kappa}{x^4} = 0,$$

where θ is the temperature, x a function of the radius, and κ a constant. If $\theta = \phi(x)$ be a particular solution of this equation, the second differential coefficients

$$\phi''(x) = - \{\phi(x)\}^\kappa x^{-4},$$

and

$$\phi''(\mu x) = - \{\phi(\mu x)\}^\kappa \mu^{-4} x^{-4};$$

and the general solution is shown to be of the form

$$\theta = C\phi\{x C^{-\frac{1}{4}}(\kappa^{-1})\},$$

where C and κ are constants.

Under convective equilibrium the mass will contract in such a way that the particles in any concentric sphere surface do not penetrate those surfaces adjacent, that is, the new ordinate ξ of any particle is defined by the equation $\xi = \alpha x$, where α is a numerical coefficient smaller than unity; and hence

$$\theta = C\phi\{\xi C^{-}(\kappa^{-1})\}$$

will be a solution of exactly the same form as the first. A curve defined by the equation

$$y = \psi(r^{-1})$$

will give the absolute temperature from the centre to the surface. In like manner another curve

$$\eta = \{\psi(r^{-1})\}^\kappa$$

will give the distribution of density with respect to the radius. Shrinkage by which the variables become $\rho = \alpha r$ will not change the character of these two functions; and hence the distribution of density and temperature is rigorously the same after contraction as before. This result continues to hold so long as the body is wholly gaseous and obeys the laws of convective equilibrium.

Prof. Perry has examined at some length the question of radiation, and he deserves our thanks for the interesting suggestions he has advanced. Yet I have considered our knowledge based on terrestrial experiment too limited to furnish any conclusion which can be confidently applied to the conditions existing in the heavens, except that the masses are always in convective equilibrium, and that all shrinkage is determined by this condition. Accordingly the foregoing conclusions would seem to be valid generally. It seems fair to conclude that there are few branches of physical science which offer such an unexplored field as the one which relates to the life-history of stars. And though it may be assumed that forces are at work in space, of which we have little or no experimental analogy up to this time, yet it is always safe to apply known laws to the phenomena of the heavens with a view of explaining observations, and of suggesting unknown causes which may become the subject of future research.   T. J. J. SEE.

Washington, D.C., August 5.