The formula (ii.), taken to the fifth central difference, gives

$$\frac{du}{dx} = 18127\tfrac{1}{3},$$

the true value being

$$\frac{du}{dx} = 18127{\cdot}224.$$

The inaccuracy in the ordinary formula is, of course, due to the fact that a table such as the above never gives the exact value of the function tabulated, but only the nearest integral multiple of a certain unit (in this case ·001). If we denote this unit by ρ, each tabulated value differs from the true value by some quantity lying between $-\frac{1}{2}\rho$ and $+\frac{1}{2}\rho$. It may be shown that this makes it possible for $\Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \frac{1}{4}\Delta^4$ to differ from its true value by as much as $\frac{16}{3}\rho$, while $a_0 - \frac{1}{6}c_0$ cannot differ from its true value by more than $\frac{3}{4}\rho$. Hence this latter formula is more accurate than the ordinary one in the ratio of 64 : 9, or about 7 : 1, when fifth differences are negligible. When only seventh differences are negligible, the formula $a_0 - \frac{1}{6}c_0 + \frac{1}{30}e_0$ is more accurate than the ordinary formula, in the ratio of 832 : 55, or about 15 : 1.

(3) The formulæ (ii.) and (iii.) give the first and second differential coefficients for the values of the "argument" shown in the table. It is often more useful to have them for the *intermediate* values. This requires a modification of the method of central differences. Let us write

$$\begin{aligned} \tfrac{1}{2}(u_1 + u_0) &= V \\ \Delta u_0 &= \Delta_1 \\ \tfrac{1}{2}(\Delta^2 u_0 + \Delta^2 u_{-1}) &= \Delta_2 \\ \Delta^3 u_{-1} &= \Delta_3 \\ &\text{\&c.} \end{aligned}$$

Thus for the interval from 5·2 to 5·3, in the above example, we have

V	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
$190804\frac{1}{2}$	19065	$1909\frac{1}{2}$	189	$19\frac{1}{2}$	9

With this notation, it may be shown that, for any value of x from 0 to 1,

$$\begin{aligned} u_x = &\left\{V - \frac{1-2x}{2}\Delta_1\right\} \\ &- \frac{x(1-x)}{2!}\left\{\Delta_2 - \frac{1-2x}{6}\Delta_3\right\} \\ &+ \frac{x(1-x^2)(2-x)}{4!}\left\{\Delta_4 - \frac{1-2x}{10}\Delta_5\right\} \\ &- \frac{x(1-x^2)(4-x^2)(3-x)}{5!}\left\{\Delta_6 - \frac{1-2x}{14}\Delta_7\right\} \\ &+ \ldots\ldots\ldots\ldots \qquad \text{(iv.)} \end{aligned}$$

Or, if we write $x = \frac{1}{2} + \theta$, then for values of θ from $-\frac{1}{2}$ to $+\frac{1}{2}$,

$$\begin{aligned} u_{\frac{1}{2}+\theta} = &\{V + \theta\Delta_1\} \\ &- \frac{1-4\theta^2}{2^2.2!}\left\{\Delta_2 + \tfrac{1}{3}\theta\Delta_3\right\} \\ &+ \frac{(1-4\theta^2)(9-4\theta^2)}{2^4.4!}\left\{\Delta_4 + \tfrac{1}{5}\theta\Delta_5\right\} \\ &- \ldots\ldots\ldots\ldots \qquad \text{(v.)} \end{aligned}$$

Differentiating this last expression twice with regard to θ, and putting $\theta = 0$ we find

$$\left(\frac{du}{dx}\right)_{\frac{1}{2}} = \Delta_1 - \frac{1}{24}\Delta_3 + \frac{3}{640}\Delta_5 - \frac{5}{7168}\Delta_7 + \ldots\ldots \qquad \text{(vi.)}$$

$$\left(\frac{d^2u}{dx^2}\right)_{\frac{1}{2}} = \Delta_2 - \frac{5}{24}\Delta_4 + \frac{259}{5760}\Delta_6 - \frac{3229}{322560}\Delta_8 + \ldots\ldots \qquad \text{(vii.)}$$

Thus for $y = 5{\cdot}25$, in the above example, we find

$$\frac{du}{dx} = 19057{\cdot}17,$$

the true value being

$$\frac{du}{dx} = 19056{\cdot}63.$$

(4) The ormula (iv.) is useıul for constructing tables by means of interpolation. For halving the intervals in a table, it gives

$$u_{\frac{1}{2}} = V - \tfrac{1}{8}\Delta_2 + \frac{3}{128}\Delta_4 - \frac{5}{1024}\Delta_6 + \frac{35}{32768}\Delta_8 - \ldots \qquad \text{(viii.)}$$

Similarly, for subdivision of the intervals into fifths,

$$\left.\begin{aligned} u_{\frac{1}{5}} = V &- {\cdot}3\Delta_1 - {\cdot}08\Delta_2 + {\cdot}008\Delta_3 + {\cdot}0144\Delta_4 - {\cdot}000864\Delta_5 \\ &- {\cdot}0029568\Delta_6 + {\cdot}00012672\Delta_7 + {\cdot}000642048\Delta_8 - \ldots \\ u_{\frac{2}{5}} = V &- {\cdot}1\Delta_1 - {\cdot}12\Delta_2 + {\cdot}004\Delta_3 + {\cdot}0224\Delta_4 - {\cdot}000448\Delta_5 \\ &- {\cdot}0046592\Delta_6 + {\cdot}00006656\Delta_7 + {\cdot}001018368\Delta_8 - \ldots \\ u_{\frac{3}{5}} = V &+ {\cdot}1\Delta_1 - {\cdot}12\Delta_2 - {\cdot}004\Delta_3 + \text{\&c.} \\ u_{\frac{4}{5}} = V &+ {\cdot}3\Delta_1 - {\cdot}08\Delta_2 - {\cdot}008\Delta_3 + \text{\&c.}; \end{aligned}\right\} \text{(ix.)}$$

the terms in $u_{\frac{3}{5}}$ and $u_{\frac{4}{5}}$ being the same as in $u_{\frac{2}{5}}$ and $u_{\frac{1}{5}}$, but with signs alternately alike and different; and the sequence of signs in each case being . . . + + − − + + . . . The corresponding formulæ for subdivision into tenths might be found: but it is simpler to subdivide into halves and then again into fifths.

When several differences have to be taken into account, the above method of direct calculation is less troublesome than the ordinary process of building up the table by calculation of the sub-differences.

In the formulæ (ix.) the terms due to V and Δ_1 have been given in the form $V - {\cdot}3\Delta_1$, $V - {\cdot}1\Delta_1$, . . . ; but in practice these terms would be obtained by successive additions of $\cdot 2\Delta$ to u_0, so that it is not necessary to calculate V.

August 16. W. F. SHEPPARD.

Apparent Dark Lightning Flashes.

ON the evening of the 5th of the present month we were visited by a severe thunderstorm, which passed practically over this place. The lightning was very vivid and at times occurred at intervals of only a few seconds. In order to photograph some of the flashes I placed a camera on my window sill and exposed four films for consecutive periods of 15 minutes each.

During the exposures I was observing the sky, and repeatedly found that after nearly each bright flash I could see distinctly a *reversed image* of each flash *in any part of the sky* to which I turned my head. These apparent dark flashes, or rather the images on my retina, lasted for sometimes 5 to 10 seconds. At the time I wondered whether dark flashes had ever been noticed before, and thought that this phenomenon was not uncommonly observed, but seeing Lord Kelvin's letter in your issue of August 10, I send this note in case it may prove of interest.

Westgate-on-Sea, August 13. WILLIAM J. S. LOCKYER.

Subjective Impressions due to Retinal Fatigue.

IN reading the interesting optical experience as described by Lord Kelvin in NATURE of August 10, it occurred to me that a somewhat similar effect on the eye, as noticed by myself, might be of interest.

Frequently late in the evening, and with a dull cloudy sky, I have seen my own figure, at least in part, apparently projected in gigantic form high up on the cloudy background.

This happened in the following manner. Going to the door of the house, and standing there with the strong light from the lobby or hall lamp shining out upon the gravel-walk in front, I saw my figure in shadow strongly defined upon the illuminated pathway. On raising my eyes quickly to the sky, I there saw the same form marked out on the dark clouds, but in a lighter shade.

The effect on the eye, as in Lord Kelvin's experience, is doubtless that of fatigue: in my experience, however, the form observed being very dark as compared with the illuminated background, I received the complementary impression of a light-coloured figure on a dark background.

The time during which this impression remained when looking at the clouds might be a couple of seconds.

August 14. W. J. MILLAR.

Mathematics of the Spinning-Top.

IT should have been stated on p. 321 that, while θ_3 is the angle between HQ and HQ′ in Fig. 1, p. 347, the angle between HS and HS′ is θ_2. At the same time this opportunity is available for some corrections, for which the printers are not responsible. On p. 321 the values of sin θ_3 and sh θ_1 should be interchanged; on p. 348, after equation (35), read ... "MX is the harmonic mean of MT, MT′ and of Mm, Mm', ..."

August 12. A. G. GREENHILL.