

are so analogous, that if the distance of the contacts, the duration of the change at the seat of excitation (monophasic variation), and the rate of propagation are known, it is easy to forecast the curve of the diphasic variation.

By a similar method to that employed in the study of muscle, the effect at the distal contact can be partially or

and this change spreads from the excited spot to parts at a distance at a rate which varies with temperature. The interval of time between the culmination of the electrical response and that of the change of form is much more obvious in the leaf than in the heart, because the mechanism by which the latter manifests itself works very slowly, as compared even with cardiac muscular fibres. This contrast, however, affords no ground for doubting that the two processes are, as regards their intimate nature, analogous.



FIG. 7.—Response of veratrinised muscle to instantaneous stimulation.

entirely cancelled. All that is necessary is to destroy by heat the surface under the distal electrode. The result of this operation is that, as in muscle, the devitalised area becomes, while unexcited, negative to all uninjured parts, and that if the surface is excited in the neighbourhood of the uninjured contact, the photographic curve assumes characters which correspond with those of the monophasic curve of muscle, with this noteworthy difference that its duration is that of the ventricular beat.

This can be best understood from the photographic curves reproduced in Fig. 8, with reference to which it is to be noted that the rate of movement of the plate on which the movement of the mercury column is projected is *ten times* as slow as the slowest rate of movement used in observations of muscle. Had the excursion been projected on a plate moving at the same rate, the first half of the curve would have had a contour similar to the veratrine curve. It expresses a sudden coming into existence of a difference of potential between the two contacts which may be maintained (in the heart) for more than two seconds.

In the second curve of Fig. 8 the curve begins as in the first, but the effect on the electrometer of the change which is taking place at the proximal electrode is immediately afterwards counteracted and balanced by the similar change at the distal contact, and is followed by a period of indifference, the end of which is marked by a descent of the column. This (as was explained by the lecturer many years ago) means that the effect at the distal electrode over-lasts that which occurs at the proximal.

The lecture concluded with a comparison of the electromotive properties of the leaf of the fly-trap with those of muscle. If the same method of exploration is applied to the surface of the leaf as to the ventricle of the heart of the frog, it is easy to show that the phenomena observed after excitation in the two structures are essentially analogous. In both an electrical change is the immediate result of a localised instantaneous excitation,

Conic Sections will be required, which will show the practical utility of the study of this elegant subject, as presented in Taylor's "Geometry of Conics."

In the first place we can connect up the notation of Darboux and Routh by taking

$$(28) \frac{a, b, c, h}{m} = \frac{HV, HT, HP, HQ}{OD} = \left(\frac{D}{A}, \frac{D}{B}, \frac{D}{C}, 1 \right) \frac{HQ}{OD},$$

Darboux's a, b, c, h being of the same dimensions as m , an angular velocity estimated in radians/second.

From the fundamental property of the herpolhode as the trace of the points of contact of a quadric surface, rolling about its centre O on a fixed plane GH , namely,

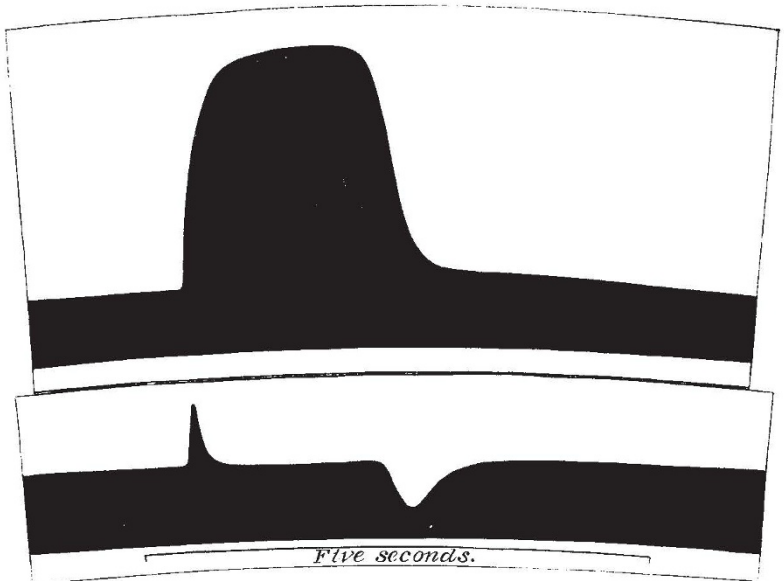


FIG. 8.—Monophasic and diphasic photographic curves of the ventricle of the heart of the frog.

that the radius vector GH and the tangent HK are conjugate on the rolling surface, combined with the properties of conjugate diameters, we can deduce the

¹ "Ueber die Theorie des Kreisels." F. Klein und A. Sommerfeld. Heft 1, ii. Pp. 196 and 197 to 512. (Leipzig: Teubner, 1897-8.) (Continued from p. 322.)

analytical theorems of the curve. Thus, if OE and OF are the conjugate diameters parallel to the tangent HK and the vector GH, and if OK is the perpendicular on HK, then, with Darboux's notation,

$$a + b + c = P, \quad bc + ca + ab = Q, \quad abc = R,$$

and putting

$$\begin{aligned} \frac{OG^2}{OD^2} &= \frac{h}{m}, \\ (29) \quad \frac{OE^2 + OF^2 + OH^2}{OD^2} &= \frac{P}{m}, \\ (30) \quad \frac{OE^2 \cdot OF^2 \sin^2 EOF + OK^2 \cdot OE^2 + OF^2}{OD^4} &= \frac{Q}{m^2}, \\ (31) \quad \frac{OG^2 \cdot OE^2 \cdot OF^2 \sin^2 EOF}{OD^6} &= \frac{R}{m^3}. \end{aligned}$$

The elimination of OE^2 , OF^2 , and $\sin^2 EOF$ between these equations gives the relation connecting OH^2 and OK^2 in the herpolhode; it is linear in OK^2 and quadratic in OH^2 . The geodetic radius of curvature of the polhode on the polhode cone is readily found by a differentiation by exactly the same formula as the $rdr/d\phi$ formula of a plane curve; and the radius of curvature of the herpolhode in the plane of G is the projection of this geodetic radius of curvature.

At a point of inflexion on the herpolhode this geodetic radius of curvature is infinite, and now the polhode is a bit of a geodesic on the polhode cone; this shows that the osculating plane of the polhode is now perpendicular to OK.

But to find whether the herpolhode can have points of inflexion, we merely require to find where the value of OK is stationary, and this is found by solving the quadratic in OH^2 , and examining its discriminant; in this way we shall find that the discriminant vanishes, and OK is at a turning point, when

$$(32) \quad m \frac{GK^2}{OD^2} = 0, \quad \text{or} \quad \frac{4(a-h)(b-h)(c-h)R}{\Omega^2 h} = 0,$$

where, in Darboux's notation,

$$\Omega^2 = Q^2 - 4R(P-h).$$

The value $GK=0$ is excluded when the rolling surface is an ellipsoid; it will be found that the other value makes

$$\begin{aligned} m \frac{OF^2}{OD^2} &= \frac{Q}{2h}, \\ m^2 \frac{OF^2 \cdot OG^2}{OD^4} &= \frac{1}{2}Q; \end{aligned}$$

and the maximum value of this being ab , it follows that

$$ab - \frac{1}{2}Q = \frac{1}{2}R \left(\frac{1}{c} - \frac{1}{a} - \frac{1}{b} \right) = \frac{1}{2}m^2 \frac{HQ^2 \cdot D^2}{OD^2 ABC} (C-A-B)$$

is positive, so that the rolling quadric cannot be the momental ellipsoid of real positive matter for points of inflexion to exist, in accordance with the theorems of Hess and de Sparre.

Fig. 1 has been drawn with the idea at first of giving the graphical representation of the numerical case discussed in VI. § 6; so that

$$f = \frac{a-b}{\omega'} = -0.0068, \quad f' = \frac{a+b}{\omega'} = 1.421$$

(these numbers appear to show that s and s' on p. 481 must be interchanged).

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But as these numbers bring the point P inconveniently near to A, fresh dimensions are chosen; we can take a scale such that $OA = 10$ cm., $OB = 5$, so that κ is reduced from 0.521 to 0.5; and the points P and P' were so placed as to make $\theta_3 = 45^\circ$, $\theta_2 = 30^\circ$; and now, by measurement, $OD = 17.73$ cm.,

$$\begin{aligned} HQ &= 16.4 \text{ cm.} & HQ' &= 8.9 \text{ cm.} \\ HT &= 5.4 \text{ cm.} & HT' &= 21.6 \text{ cm.} \\ HV &= 15.0 \text{ cm.} & HV' &= 8.3 \text{ cm.} \\ HP &= 6.2 \text{ cm.} & HP' &= 10.4 \text{ cm.} \end{aligned}$$

The angle AOQ or $\omega = 33^\circ$ by measurement, so that from Legendre's Table IX., to the co-modular angle 60° ,

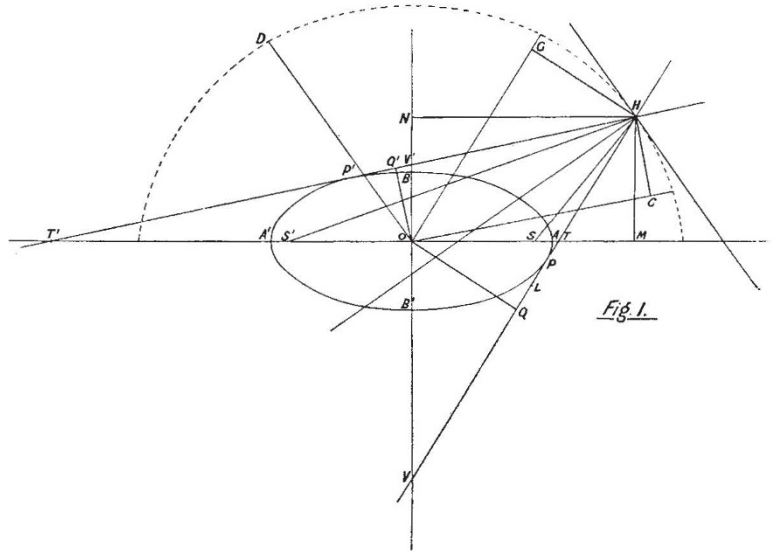
$$\begin{aligned} F\omega &= 0.6009, & K' &= 2.1565, \\ E\omega &= 0.5528, & E' &= 1.2111; \end{aligned}$$

and thence

$$f = -0.2787,$$

and

$$QL = 2.153 \text{ cm.}$$



Thence the apsidal angle, as a fraction of a right angle, is given by

$$(33) \quad \frac{\Psi}{\frac{1}{2}\pi} = \frac{HL}{OA} \frac{K}{\frac{1}{2}\pi} + f = 1.245.$$

As θ diminishes from θ_3 to θ_2 , the deformable hyperboloid opens out from the plane of the focal ellipse and flattens again in the plane of the focal hyperbola. As a typical intermediate position choose that of half-time; with our dimensions this will make

$$\begin{aligned} \sin^2 \phi &= \frac{1}{1+\kappa'} = 0.5359, \\ \cos \theta &= 0.792, \quad \theta = 37^\circ 37'. \end{aligned}$$

Project the figure on to the plane of the generating lines HQ, HQ', which are now drawn inclined at $37^\circ 37'$, and place the points Q, T, V, P, &c., on these lines as before; the confocal quadrics project into confocal conics.

The lines TT', VV', PP' form a triangle XYZ, the sides of which are the traces of the principal planes: Ω , the orthocentre of this triangle, is the projection of the origin O.

We must be content with the mere statement of the following geometrical theorems, required for throwing light upon the analytical theorems of this dynamical problem, with a view of obtaining a clear image of the motion in accordance with Poinso't's ideas; the demonstrations will be found in Salmon's "Solid Geometry,"

or better still, in Mannheim's "Géométrie Cinématique," which has been found valuable in carrying on the Poinset traditions of dynamical presentation.

Join HΩ, cutting the sides of the triangle XYZ in α, β, γ; describe the circle round XYZ, and let XΩ, YΩ, ZΩ cut this circle in d, e, f; then ad, βe, γf intersect the circle in one point F. F is the focus, and HΩ the directrix of the parabola, which is the envelop of the normals of the confocals (Salmon, "Solid Geometry," § 177); this parabola touches the sides of the triangle XYZ; the tangents to the parabola from H are tangents to the projections on the plane of the confocals through H, while the tangents to the parabola from Ω are the axes of these plane confocals; also HΩ and HF are equally inclined to HQ and HQ'.

If Hc, Hc' are the tangents to the parabola from H, the points of contact c and c' are the centres of curvature of the plane confocals through H; while C and C', the centres of curvature of the normal sections of the confocal surfaces through H made by planes through the line of curvature traced out H, are such that

$$HC = Hc \cos^2 \frac{1}{2}\theta, HC' = Hc' \sin^2 \frac{1}{2}\theta,$$

and thus the point C and C' are easily constructed geometrically.

The line CC (a tangent to the parabola) is Mannheim's axis of curvature, and the plane through H perpendicular to it is the osculating plane of the polhode. Thus in the degenerate case of Fig. 1, when H lies in the plane of the focal ellipse, PP' is the axis of curvature of the polhode, while Z and Ω coincide with O.

The various relations in the conjugate Poinset movements, investigated by Darboux and Routh, and given in VI. § 8, can now receive a geometrical interpretation.

Thus the relation (2), p. 476, written in the form

$$(34) \quad \frac{B}{A} + \frac{B'}{A'} = \frac{C}{A} + \frac{C'}{A'}, \text{ \&c.,}$$

is the equivalent of

$$(35) \quad \frac{HV}{HT} + \frac{HV'}{HT'} = \frac{HV}{HP} + \frac{HV'}{HP'}, \text{ \&c.;}$$

expressing the fact that, if PP' cuts OA in X in Fig. 1, then MX is the harmonic mean of HT, HT' and of Hm, Hm', where m, m' are the feet of the ordinates from P, P' on OA.

Again, if XΩ meets YZ in D, then OD and OV are conjugate diameters in the principal plane OYZ of the rolling quadric

$$(36) \quad \frac{x^2}{HV} + \frac{y^2}{HT} + \frac{z^2}{HP} = HQ,$$

and therefore

$$\frac{HP}{HT} = \tan YOD \tan YO'V = \frac{VY}{VZ};$$

similarly

$$\frac{HV}{HP} = \frac{TZ}{TX}, \quad \frac{HT}{HV} = \frac{PX}{PY};$$

so that, drawing Xx parallel to YZ to meet HQ and HQ' in x and x',

$$(37) \quad \frac{HV}{HT} = \frac{PY}{PX} = \frac{PV}{Px}, \quad \frac{HV}{VT} = \frac{PV}{Vx},$$

$$\frac{VP.VT}{HV^2} = \frac{Vx}{HV} = \frac{V'x'}{HV'} = \frac{V'P'.V'T'}{HV'^2}$$

the geometrical equivalent of the relation

$$(38) \quad \left(1 - \frac{A}{B}\right)\left(1 - \frac{A}{C}\right) = \left(1 - \frac{A'}{B'}\right)\left(1 - \frac{A'}{C'}\right).$$

Similarly for the other relations, which we have not space to develop.

The A and C employed here require to be carefully distinguished from the values referring to the top itself, which ought to be differentiated by a suffix.

The constancy of the perpendicular from the centre

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on the tangent plane of the rolling quadric along the polhode is expressed by

$$(39) \quad \frac{x^2}{HV^2} + \frac{y^2}{HT^2} + \frac{z^2}{HP^2} = 1,$$

and

$$\frac{x}{HV} \cdot \frac{y}{HT} \cdot \frac{z}{HP}$$

are obviously the cosines of the angles which the line HQ makes with the coordinate axes.

The Darboux-Koenig's arrangements by which the polhode, herpolhode, and associated top motion are produced mechanically by articulated movements, are worth mention and study in elucidating the various theorems.

When the parameters a and b (p. 263) are aliquot parts of the period ω', the multiplicative elliptic functions α, β, γ, δ become algebraical functions of u, qualified by exponential functions of the time. Take the simplest case, where a = 1/2 ω', b = 1/3 ω', equivalent to placing P at A and P' at B in Fig. 1, then we shall have

$$e = 0, \quad e' = \kappa, \quad e'' = \frac{1}{\kappa};$$

$$\alpha = \frac{1}{\sqrt{2}} e^{it} \left[\sqrt{(e'' - u \cdot e' - u)} + i \frac{1 + \kappa}{\sqrt{\kappa}} \sqrt{(u - e)} \right]^{\frac{1}{2}},$$

$$\beta = \frac{1}{\sqrt{2}} e^{it} \left[\sqrt{(e'' - u \cdot e' - u)} - i \frac{1 - \kappa}{\sqrt{\kappa}} \sqrt{(u - e)} \right]^{\frac{1}{2}}, \text{ \&c.,}$$

with

$$l = \frac{1 + \kappa}{4\sqrt{2\kappa}} m + \frac{1}{2} \left(\frac{1}{C} - \frac{1}{A} \right) N,$$

$$l' = \frac{1 - \kappa}{4\sqrt{2\kappa}} m - \frac{1}{2} \left(\frac{1}{C} - \frac{1}{A} \right) N,$$

$$\frac{1}{2} i \sin \theta e^{4t} = \alpha \beta = \frac{1}{2} e^{(l+l')it} \left[\sqrt{(1 - \kappa u)} + i \sqrt{(u \cdot \kappa - u)} \right].$$

In the next case, where

$$a = \frac{2}{3} \omega', \quad b = \frac{1}{3} \omega',$$

it is found that we can put

$$e = -1 + c, \quad e' = -\frac{1 - 3c + c^2}{(1 - c)^2}, \quad e'' = \frac{1 - c}{c};$$

$$\alpha = \frac{1}{\sqrt{2}} e^{it} \left[\frac{(1 - c + c^2)u + (1 - c)^2}{(1 - c)\sqrt{c}} + i \sqrt{V} \right]^{\frac{1}{2}}$$

$$\beta = \frac{1}{\sqrt{2}} e^{it} \left[\frac{(1 - 2c)(2 - c)}{(1 - c)\sqrt{c}} \sqrt{(e'' - u \cdot u - e)} + i \left\{ u - 2 \frac{(1 - c)^2}{c} \right\} \sqrt{(e' - u)} \right]^{\frac{1}{2}}, \text{ \&c.}$$

with

$$l = \frac{1}{3} \frac{1 - c + c^2}{(1 - c)\sqrt{2c}} m + \frac{1}{2} \left(\frac{1}{C} - \frac{1}{A} \right) N$$

$$l' = -\frac{1}{3} \frac{(2 - c)(1 - 2c)}{(1 - c)\sqrt{2c}} m - \frac{1}{2} \left(\frac{1}{C} - \frac{1}{A} \right) N.$$

The general form of this solution can now be inferred, but it is evident that the algebraical complexity mounts up very rapidly; with

$$a = \frac{2r\omega'}{n}, \quad b = \frac{r'\omega'}{n},$$

$$\alpha = \frac{1}{\sqrt{2}} e^{it} \left[A_1 + i A_2 \sqrt{V} \right]^{\frac{1}{n}},$$

$$\beta = \frac{1}{\sqrt{2}} e^{it} \left[B_1 \sqrt{(e'' - u \cdot u - e)} + i B_2 \sqrt{(e' - u)} \right]^{\frac{1}{n}},$$

where the A's and B's are rational polynomials of u.

Thus for n = 5, we can take

$$e = -\frac{2c}{\sqrt{C+1}}, \quad e' = \frac{c^3 - 3c^2 - c - 1}{4c}, \quad e'' = \frac{2c}{\sqrt{C-1}},$$

where

$$C = c^3 + c^2 - c;$$

and

$$A_1 = P_1 u^2 + P_2 u + P_3, \quad P = \frac{c^3 - c^2 + 7c - 3}{2c^2(c+1)(c-1)^{\frac{1}{2}}}$$

$$P_1 = \frac{5c^3 + 11c^2 + 26c^3 - 10c^2 + c - 1}{2c^{\frac{3}{2}}(c+1)^{\frac{3}{2}}(c-1)^{\frac{3}{2}}}, \quad P_2 = \frac{c^4 - 10c^3 + 2c^2 - 2c + 1}{c^{\frac{3}{2}}(c+1)^{\frac{3}{2}}(c-1)^{\frac{3}{2}}};$$

$$A_2 = u + \frac{5c^2 - 2c + 1}{c(c+1)^2}, \quad l = \frac{P}{5\sqrt{2}} + \frac{1}{2} \left(\frac{I}{C} - \frac{I}{A} \right) N;$$

$$B_1 = Qu + Q_1,$$

where

$$Q = \frac{(c+3)(c^2 - 4c - 1)}{2c^{\frac{3}{2}}(c+1)(c-1)^{\frac{3}{2}}}, \quad l' = \frac{Q}{5\sqrt{2}} - \frac{1}{2} \left(\frac{I}{C} - \frac{I}{A} \right) N,$$

$$Q_1 = -\frac{(c^2 - 4c - 1)(5c^2 + 2c + 1)}{2c^{\frac{3}{2}}(c+1)(c-1)^{\frac{3}{2}}}.$$

$$B_2 = u^2 - \frac{5c^3 + 19c^2 + 7c + 1}{c(c+1)^2(c-1)} u + \frac{-2c^3 + 22c^2 + 10c + 2}{(c+1)^2(c-1)^2}.$$

The same functions $\alpha, \beta, \gamma, \delta$, and their special algebraical forms are suitable for Kirchoff's case of the motion of a solid in infinite liquid, but now V is a quartic function of u , requiring resolution into factors.

In the more general case invented by Clebsch, and developed in Halphen's "Fonctions elliptiques," t. II., the component rotation about OZ is no longer constant, and the solution is more complicated, introducing multiplicative elliptic functions to a parameter corresponding to the infinite value of u .

If the motion of the axis of the top is alone required, we take $\Lambda = \infty$, and investigate the function $\lambda = a/\gamma$; this is a multiplicative elliptic function, with an effective parameter $a-b$, which can be made algebraical when $a-b$ is made an aliquot part of ω' , irrespective of the separate terms a and b . By a further restriction, the exponential function of the time can be made to disappear by making $l + l' = 0$, and then H is at L in Fig. 1; it was in this way that the analysis was prepared of the algebraical cases, represented stereoscopically by Mr. T. I. Dewar, referred to on p. 199.

The authors say they have refrained from utilising these stereoscopic diagrams, because they would not like to assume in the reader the possession of a stereoscope. But our eyes should be drilled into control to pick up the solid appearance without any apparatus; a little quiet practice will suffice. Treatises on Solid Geometry of the future should be profusely illustrated with stereoscopic figures, which the student should see solid at will; and wall diagrams or lantern projections should also be drawn stereoscopically, and the solid effect obtained in the audience by crossing the two lines of sight.

Mr. T. I. Dewar's untimely death, at San Remo last May, has deprived us of any further diagrams from his skill, but the example he set will we trust be followed out completely in mathematical diagrammatic instruction.

The unsymmetrical top, discussed in V. § 9, leads into such great analytical complication, that only a few special degenerate cases have so far received any adequate attention; the next century will have its work cut out for the mathematical treatment of this problem and also of the dynamics of the bicycle. The symmetrical top of the boy, with the point free to wander over a smooth or rough horizontal plane, leads to similar analytical difficulties, and should be discussed in the same place.

On the other hand, the many attempts at a popular explanation of the motion of the top, restricted principally to the case of regular precession, are described in V. § 3. Prof. Perry's interesting little book on "Spinning Tops" comes in for praise, and the authors cite with pleasure the comparison of the top to a wilful beast (*eigensinniger thier*), always ready to move in some other direction to that in which it is pushed; inasmuch that the Irishman can persuade his pig to accompany him on the road only by pretending that his way lies in the opposite direction; and so Bessemer's invention to steady the

motion of a cabin mounted on gimbals, by means of the controlling influence of gyrostats, was a failure.

If the authors are in search of other practical elementary illustrations, they should take the modern centrifugal machine, and examine the practical devices, as in the Weston machine, for controlling the nutations; these devices discovered experimentally without any assistance from theory will serve to elucidate the abstract formulas with advantage.

A third part of this book is still to appear, and we await it with great interest; the work when complete will form an indispensable book of reference for all who wish to make themselves thoroughly acquainted with this complicated problem in Dynamics.

A. G. GREENHILL.

NOTES.

At Osborne, on Wednesday, August 2, the Queen conferred the honour of knighthood upon Sir William Henry Preece and Sir Michael Foster, Knight Commanders of the Order of the Bath, and invested them with the riband and badge of the Civil Division of the Second Class of the Order, and affixed the star to their left breasts.

The Hanbury Gold Medal of the Pharmaceutical Society of Great Britain has been awarded to Prof. Albert Ladenburg, of Breslau, for his work on alkaloids and their derivatives.

MR. J. S. BUDGETT, of Trinity College, Cambridge, who accompanied Mr. Graham Kerr on his expedition in search of *Lepidosiren*, has been successful in obtaining eggs and larvæ of the Crossopterygian Ganoid *Polypterus*. From a short account of his investigations, illustrated by sketches, which Mr. Budgett has sent to this country, it appears that the larva is very minute, and possesses a "cement organ" on the dorsal surface of the head. Mr. Budgett is now on the journey home, and the full account of his work will be looked forward to with much interest.

ON a preceding page we have referred to some of the work performed by the Royal Gardens, Kew. Coincidentally we have received the number for July 21 of our American contemporary *Science*, which contains an elaborate article by Prof. Underwood, headed "The Royal Botanic Gardens at Kew," in which the features of the garden and its position as a scientific institution—"its beautiful lawns, its delightful shade, its historic associations, its immense collections of cultivated plants, and its wonderful activity in the direction of botanical research"—are described and discussed with critical appreciation *apropos* the recent establishment of the Botanic Garden of New York and its capability to become "even more influential in democratic America than Kew has become throughout the length and breadth of the Queen's dominions." It is gratifying to have this acknowledgment of the work of Kew; and the tribute paid to the versatility and ability of Sir William Thiselton-Dyer in promoting its development and widening its influence will be everywhere endorsed. There are some blots on the escutcheon in the eyes of Prof. Underwood, but we imagine there are many who will not see with him in all the instances he mentions. The crowding of the museum collections he notes is an apparent blemish, and one we may hope to see removed by the provision of increased room for the exhibition of the specimens. A somewhat jealous comparison of Kew and Berlin as centres of botanical work is a jarring note in the article; and Prof. Underwood allows, we fear, German bias to weigh with him in making it, for instance, when he writes, "the principles of plant distribution are not so thoroughly grasped at Kew as they have been brought out at the German Botanical Garden through the skill of Prof. Engler and his associates." Yet Kew is the home of Sir Joseph Hooker!