PROJECTIVE GEOMETRY.

Premiers Principes de Géométrie Moderne. Par E. Duporcq. Pp. viii + 160. (Paris: Gauthier-Villars, 1899.)

IT is a curious fact that while projective geometry is becoming better appreciated in England it seems to be going out of favour in France. M. Duporcq, in his introduction, pathetically deplores the predominant place assigned to analysis in the syllabuses of the official examinations; and in France, as with ourselves, most teachers are compelled to neglect a subject that does not pay. It will be sad indeed if, in the fatherland of Monge, Poncelet and Chasles, pure geometry is to be deposed from her former high estate, and made a kind of Cinderella, called in to do odd jobs for Her Serene Highness the Princess *Analyse*, or to amuse the children with tricks of the triangle.

M. Duporcq's book itself helps us to realise the danger that is threatened. One cannot help feeling that his attitude is apologetic, and that his exposition is a half-hearted one. At the very outset we are confronted with homogeneous coordinates; homography is based on an algebraic relation; points at infinity lie in a plane "by definition"; imaginary elements have no real existence, and the introduction of them, due to analysis, is a mere *façon de parler*, vaguely justified by the "Principle of Continuity." With all respect to Poncelet, it may be doubted whether his "principle of continuity," apart from algebraical considerations, has any real working value; on the other hand, von Staudt elaborated, forty years ago, a theory of imaginary elements which, so far as curves and surfaces of the second order are concerned, gives a consistent geometrical theory (quite independent of analysis) in which the principle of continuity has a real meaning, and is at the same time practically self-evident, as one would expect it to be. Von Staudt's name does not appear to be mentioned in M. Duporcq's book, and the reader might not unreasonably infer that the author was ignorant of v. Staudt's existence.

It would, of course, be absurd to advocate the exclusive use of pure, as opposed to analytical, geometry, even in problems of a strictly geometrical character. The ideal geometrician should be equally expert in both methods, and apply one or the other or both combined according as circumstances may require. But it may fairly be urged that a treatise on the *first principles* of projective geometry should avoid the introduction of coordinates except by way of illustration, and for the purpose of showing the points of contact between the two methods. It is right to teach an apprentice the use of a saw as well as that of a plane; but you will not attain this end by giving him a tool that is neither a saw nor a plane, but contains something of both.

Thus to give an explicit example, M. Duporcq frequently infers homography from a one-to-one relation established, not from an equation, but from the inspection of a figure. Thus (p. 49):

"Si donc m et m' désignent les deux points où une droite quelconque Δ coupe une conique circonscrite au quadrangle $a\,b\,c\,d$, on voit qu'à tout point m de Δ ne correspond ainsi qu'un point m'. Comme, d'ailleurs, ces points sont évidemment réciproques, ils déterminent donc une involution sur Δ," &c.

These statements are doubtless correct, but are they sufficiently justified? How is the beginner to distinguish the argument from the following:

"Two points S, H are taken on a tangent to an ellipse, and any ellipse with foci S, H cuts the given ellipse in the points M, M': then to each point M corresponds one point M' and *vice versâ*, hence we have a system in involution, and MM' goes through a fixed point"?

It is not a sufficient answer to say that M, M' are only a pair of four associated points, because this is not geometrically evident. Again, we have cases of Cremona correspondence with the fixed points imaginary: how is the untrained student to distinguish them from homographic correspondences?

We are far from wishing to suggest that M. Duporcq's work is devoid of interest and value. Considering its size it is remarkable for the range and variety of its contents; it comprises a very attractive and, indeed, brilliant sketch of homography, poles and polars, involution, quadratic transformation (including inversion), together with an outline of Lie's line-sphere correspondence. For a reader prepared by previous study, it affords an excellent and suggestive *résumé*; it is rather when it is examined as a methodical text-book for students that it seems to us to fall short of perfection. To the student we would still say: Read Reye, work his exercises, and then, if you like the subject, gird up your loins and tackle von Staudt. For it is a truth past gainsaying that v. Staudt's "Geometrie der Lage" and the immortal "Beiträge" contain, as no other books do, the essentials of projective geometry. G. B. M.

A SYSTEM OF PHYSICS.

Kanon der Physik. By Felix Auerbach. Pp. xii + 522. (Leipzig: Viet and Co., 1899.)

SCIENTIFIC books may be divided into two groups, those which are written because the author has something to teach, and those which are written because he has something to learn. It is no reproach to a writer if his book is classed with the second group, for there may be as much originality in learning as in teaching, and his autodidactic efforts will often prove a source of instruction to others. It is not possible to say whether Prof. Auerbach has been consciously writing his "Kanon" of physics to clear up his own ideas on scientific principles, but the book he has produced gives the impression that this has been one of his principal motives; and I would even go a step further and say that, if life were long enough, every physicist ought, when he gets to the age of fifty, to spend three years in putting his ideas into shape and write a similar treatise. It would serve as a kind of "Abiturienten Examen" to his state of crystallisation.

It is easier to talk about this book in vague and general terms than to give an account of what it is and what it contains. I am afraid of becoming definite in my own words, for fear of giving a wrong impression, and must content myself with the translation of a few sentences taken out of the preface.

"A comprehensive book is still wanting—and not only in Germany—in which the conceptions, principles,