Cal. This gives a preponderance of weight to the determination of the coordinate Y (X passing through Greenwich), but a station in Portugal, which may possibly be secured later, would essentially increase the accuracy of X. Tschardjui, in Russia, and Ukiah, in California, are nearly opposite, and Mizusawa, in Japan, is in the only remaining unoccupied quadrant. The scheme proposed is, therefore, a favourable one for the study of the motion of the pole. No one knows as yet how long it will be desirable to continue the observations. The period now provided for is five years, but it is proposed to buy the land upon which the observatories will be located, or lease it for one hundred years. It is evident that at least twenty-one years would be desirable, because during the seven years of observations already made the pole has returned nearly to its mean position, and three of these cycles should be completed before any definite idea can be had as to its mean path. The cost of the entire work will be about 2000*l.* annually. The visual method is to be followed regularly without, however, excluding the possibility of employing later the photographic one, which has already given excellent results. Twelve groups of stars, each comprising eight pairs, will be selected. Six pairs in each group are destined for the latitude determinations proper, while the two remaining pairs, having great zenith distances (about 60°), will, it is hoped, throw light on the question of refraction. The observing period for each night is four hours, and will vary from 7 p.m. to 3 a.m., depending on the situation of the group. The instrumental outfit will consist of a zenith telescope and astronomical clock for each station, except that of Japan. Here a chronometer will be substituted for the clock, on account of the frequency of earthquakes.

Although the object of the general conference was scientific discussion, a faithful historian cannot ignore the social and humanitarian side of the function. From our entrance into the beautiful capital of Wurtemberg until the time of our departure we were the recipients of the most cordial hospitality.

Before closing the present paper, attention should be called to a few points of interest noted during the trip to Stuttgart and return. A flying visit was made to the Royal Observatory at Berlin, the Reichsanstalt at Charlottenburg, and the Geodetic Institute at Potsdam. At Paris the offices of the geographic service and the International Bureau of Weights and Measures were examined, and part of one day was devoted to the English Ordnance Survey at Southampton.

An interesting object at the Berlin Observatory is the instrument with which Küstner discovered the variation of latitude; not alone because of the splendid result achieved, but on account of the conditions under which the work was done. It is mounted on a pier more than twenty feet above ground, on a subsoil of sand, in the middle of a city, with bad atmospheric conditions and about one hundred feet from the public thoroughfares. In spite of these adverse circumstances a new fact was added to science, which had baffled the efforts of larger telescopes under immeasurably better conditions. There is much encouragement in this to investigators with scanty means at their disposal.

At the Aichungs-Kommission a balance was shown which easily determines the weight of a kilogram with an error of 1/200 of a milligram, being 1/200,000,000 part of the quantity sought. They have also a complete series of weights in quartz from 1/2 gram to one kilogram, and thermometers giving the temperature by estimation to 1/1000 of a degree Centigrade.

At Charlottenburg the most striking feature was the extension and perfection of the organisation. Nine buildings in all, of which the two larger are devoted, one to theory and the other to practice, have cost, together with the running expenses since 1887, 3,000,000 marks. The annual outlay is at present about 18,000*l.*

The Geodetic Institute at Potsdam has been much less expensive, and presents many admirable points of arrangement and administration. Among the details may be cited: the clock room, always maintained at a temperature between 20° and 21° Centigrade; the pendulum room, artificially heated on all sides, including the floor; a pillar over fifty feet high, and correspondingly thick, with meridian marks several miles away, to study changes in azimuth and the movement of the earth's crust; and finally a small photographic instrument, by means of which the occupation of a station only requires eight minutes, and gives a determination of the geographical position in latitude within two seconds of arc. The subsoil, as at Berlin, is nothing but sand.

At Sevres, near Paris, several interesting instruments were seen, among which may be especially mentioned that designed for the comparison of the metre with the wave-length of light following Michelson's method, and the apparatus for the determination of coefficients of expansion according to the method of Fizeau. Some recent experiments have been made on a composition containing 36 per cent. nickel and 64 per cent. steel. It appears that the expansion from heat is thus reduced to about 1/50 of what we should expect from the individual components. This discovery will simplify enormously the solution of problems where the temperature question has thus far been the great difficulty. It will, for example, be a comparatively easy matter to make pendulum clocks run with a daily correction of about 1/10 of a second per day under varying temperature conditions.

MIRAGE.[1]

WHEN a ray of light passes from point to point or a medium which is everywhere similarly constituted, its path is a straight line; when it passes from one medium to another medium of different density, then the ray of light is refracted or bent at the surface which separates the two media. When the ray passes from one medium to another which is denser, the refraction or bending is always towards the normal to the surface separating the two media at the point of incidence; when, on the other hand, the ray passes from a medium of a certain density to one of less density, then the bending is always from the normal to the common surface at the point of incidence. The earth is surrounded with a spherical envelope of air, and if that air were always of the same density everywhere its refractive index would be the same, and there would be no terrestrial refraction. But the spherical envelope which surrounds the earth is not all of the same density, and the refractive index of the air varies with the density. There are two causes, in the main, which militate against the uniform density of the atmosphere; one is barometric pressure, and the other is temperature. Taking no account of temperature for the moment, taking merely as the cause barometric pressure, the density of the air diminishes gradually upwards from the surface of the earth, so that the refractive index of the air diminishes upwards. The diminishing of the refractive index is not absolutely proportional to the decrease of density, but it is found by experiment to be sensibly proportional to the excess of the density over unity. The circumstance of normal refraction in the British Isles, as regards temperature, is that there is a gradual diminution of temperature upwards at the rate of about 1/300° F. for every foot of ascent. As the air gets cooler the density increases, so the tendency is to some extent to counteract the effect of barometric pressure, but it does not altogether do so. The result in the normal refraction of the British Isles is that there is a gradual diminution of density upwards.

We may consider the air to be stratified in horizontal layers; as a matter of fact, it is stratified in spherical layers, but it will simplify matters to consider it stratified in horizontal layers, the more so as the sphericity of the earth, though it is a slight cause of terrestrial refraction, is not by any means the chief cause; terrestrial refraction would still exist if the earth had no sphericity, and if its surface were perfectly plane. I show you here a diagram representing the normal state of the atmosphere, and showing the curvilinear path taken by a ray of light when it passes from one point of such an atmosphere to another point horizontally distant from it. The reason a curved path is taken is this: supposing the ray to have a general direction upwards, and supposing it to have been inclined at incidence at a certain angle with the normal, as it is going from a medium—air—to air which is less dense, it bends away from the normal, and therefore there would be a successive bending away from the normal at each layer until finally the ray would arrive at the highest point in the diagram. Then, if it were to pass downwards, it would be passing from a medium of a certain density to one of a greater density, and it would approach the normal at each surface of separation of the media, and therefore its path would be a curved path presenting concavity downwards. A ray of light will actually take some such path, because by curving upwards it takes the path which it can pass over in the least time. Generally, a ray of light takes the minimum path as regards time, and it is found to curve up into the layers of air which are of less density, because it can traverse them with greater velocity. It is important to notice that a ray of light

[1] A lecture, delivered at the Camera Club, by Major P. A. MacMahon, F.R.S.

always presents the concavity towards the denser layers; and there is another principle, that the amount of curvature of the ray, *i.e.* the total bending that there is in the ray over a given length of ray, is directly proportional to the rate at which the density changes along the normal to the direction of the ray. For instance, if we take a vertical line, the normal to the direction of the ray is horizontal, and there is no change of density in that horizontal direction, and accordingly there is no bending of the ray; if, on the other hand, the general direction of the ray is horizontal, the normal to the ray is vertical, and that is the direction along which there is maximum change of density, and therefore in that case you will get maximum bending. We get, then, the two cases: when the ray is vertical there is no bending, no curvature; when the ray is horizontal you get maximum curvature; and for intermediate directions you get intermediate amounts of curvature.

I was first led to look into this subject by considerations which arise in discussing certain questions respecting artillery firing, and I propose to give you some results which I have obtained in order to show the effect of refraction upon artillery fire. A mile on the earth's surface subtends at the centre of the earth an angle of about 52″; the refraction will have a mean angle of about 4″, a minimum value of about 2½″, and a maximum value of about 8″; and taking a range of three miles these values of the refraction would become 12″, about 8″, and about 24″ respectively; and very exceptionally the refraction would amount to about 1′ 18″. The meaning of this, in regard to artillery fire, is that (taking this exceptional amount of refraction), if the tangent sights of the gun were laid upon an object at that distance they would be really laid at an elevation 1′ 18″ too much, and correction for terrestrial refraction would consist in depressing the gun through an angle of 1′ 18″. That is not a very important matter from an artillerist's point of view, because guns can only be laid properly to within about 5′.

There is also the question of determining the range. That can be done from an elevation by an instrument called the depression range-finder. If we take the range of an object by means of this instrument the terrestrial refraction would give a longer range than we ought to have, and the necessary correction would diminish the found range.

I now come to other cases that may present themselves in the density of the air. The case that we have considered already is that of the temperature diminishing gradually upwards at the rate of 1/300° F. for every foot of ascent; in Great Britain the diminution is seldom more than about 1/130° or less than 1/400° F., but in other climates it is different. If the temperature were to fall at a more rapid rate the air would be very much more nearly of uniform density, and that would result in a smaller curvature of the ray. If the fall were at the rate of about 1/30° F. for every foot of ascent it would result in the density of the air being uniform, and there would be no terrestrial refraction at all. A more rapid fall of temperature still would cause a rise of air density upwards, and that would cause a curved ray with the concavity presented upwards. In such a case the refraction is said to be negative, whereas in the case that we had formerly the refraction is said to be positive. On the other hand, a rise of temperature upwards causes a rapid diminution in air density in ascending, and the ray with the concavity presented downwards is then more curved; and in the extreme case in which the rise of temperature upwards is at the rate of 1/16° F. for every foot, the curvature of the ray would be the same as that of the earth, and in that case—which, of course, would never actually present itself—we should be able to see completely round the globe, simply because the ray would encircle it. Again, climatic conditions may lead to another distribution of the density. There might be a gradual rise in density upwards to a certain stratum of maximum density, and then a diminution of density upwards, in which case the ray of light would pass in a curious sinuous path with the concavities presented towards the denser layers.

[Major MacMahon here enumerated conclusions, relating to terrestrial refraction in the plains of India, drawn from the experiments and observations of the late General J. T. Walker, R.E., formerly Superintendent of the Great Trigonometrical Survey of India, from which it appeared that the circumstances mainly affecting terrestrial refraction were in order of importance as follows: (1) The time of the day, (2) the temperature, (3) the aspect of the sky—whether cloudy or sunshine, and (4) the humidity of the air.]

Before proceeding to the next part of my subject, I will mention the well-known case of astronomical refraction—the circumstance that during a total eclipse of the moon the whole disc of the moon is not obscured, the fact being that the rays of the sun in passing through the atmosphere of the earth are refracted towards the normal, and in that way some light does get upon the surface of the moon.

I come now to that effect of extraordinary refraction which is more especially termed mirage, and which was first noticed, so far as we know, by the French army under Napoleon. The conditions favourable to mirage are generally a very hot sun and a sandy soil, which becomes very hot under the influence of the sun, and an almost total absence of wind. Under these circumstances there is an extraordinary amount of negative refraction, the rays are very much curved, and the concavity is presented upwards. An illustration which is familiar to everybody, because it occurs in many books on physics, is that in which the observer sees the top of a tree by means of a negatively-curved ray, and it accordingly appears to him to be in the direction of the tangent to the ray where it enters the eye; the other points in the tree he sees in other directions, and altogether the tree appears with an inverted image below it. Not only so, but where there is a very clear sky that also is refracted down, so that the tree appears in the midst of an inverted image of the sky, which has very much the appearance of water. This phenomenon is very common in Egypt. In Egypt there is very often a great quantity of sand with villages dotted about, these being somewhat raised in order to escape the periodical inundations of the Nile, and the effect to an observer is that of a number of raised villages in the midst of what appears to be an enormous lake. The illusion is increased by two circumstances: first, that by reason of the great heat of the sand there is a considerable convection of heated air upwards, which gives the air that peculiar rippling appearance, causing the refracted sky to look like ruffled water; and second, that the general direction of the ray is very similar to that of a ray reflected by a horizontal mirror placed upon the ground. This phenomenon is also common in Australia and in the plains of India, in the low-lying fen districts of England, and on the shingle ranges at Lydd.

One very curious thing about mirage is that it depends very much upon the position of the eye; a few inches in the height of the eye may make all the difference. I remember myself, on the plains of India, observing a mirage which was only evident when I was at a particular height; there was only a vertical space of two or three inches in which the effect could be seen, so that these phenomena may easily escape notice. A singular effect may sometimes be observed at a particular spot on the south coast, and very likely at other places; when the waves come in on to a very hot beach, if you place the eye within about a foot from the ground and look parallel to the wave-fronts, you can see an image of the wave two or three feet above the real wave. This may conceivably arise in this way: the wave may bring in some cold air, and if the wind were blowing a little off the heated beach there might be some heated air brought in as a layer above that cold air; that would give that rapidly diminishing density upwards which gives a ray with considerable curvature and with concavity presented downwards, and would certainly result in an image of the wave above the real wave.

I should like to mention a case of refraction observed on the artillery ranges at Lydd, which has given rise to some controversy. An officer was firing on a hot sultry day, from an ordinary Maxim machine gun at a target about 600 yards distant, and as the firing went on he saw a great wedge-shaped gap in the top of the target, and he thought that the hail of bullets had beaten down a part of the target. To his astonishment, however, when the firing ceased the gap disappeared, and the target appeared intact and undamaged. I imagine this to be an effect of mirage. I conclude that he was looking over the gun through the heated gases rising from the muzzle, which formed a medium of gradually increasing density upwards; he was observing the upper portion of the target through this medium, and I have no doubt that that caused negative refraction, bending the rays so that the sky above the target was seen in the shape of a wedge, something like the shape of the vertical section of the rising gases. I have discussed the matter with physicists, and some agree with me, but other physicists of note do not accept the explanation which I have given.

I now pass on to another part of the subject, which is called "looming." Distant objects are said to "loom" when they appear in positions much higher than their actual positions, that is to say, when images of them appear considerably raised above their true positions. The effects of looming are very extraordinary, and I have some slides to show you which I have prepared from examples recorded by Commander William Scoresby, who went on his third voyage in his ship the *Baffin* to the Greenland whale fishery in the spring of 1822. The first of these views includes several large irregularly-shaped icebergs, which must cause very unusual distributions of air density, and gave rise to quite remarkable vertical and lateral refractions. In the second there are images of ice which was quite out of sight or quite beyond the horizon. There was extraordinary vertical magnification; small hummocks of ice were drawn out into spires, sometimes of a castellated shape and sometimes having the appearance of naked trees; at other times there appeared to be a city of ice, with public edifices, spires, &c., and Commander Scoresby states in his book that these effects were constantly changing, and were never the same for two minntes together. The first of these drawings, which you saw on the screen, showed a curious inverted image of a ship in the sky, raised considerably above the horizon; that ship was so distant that it could not be seen with a powerful telescope.

During the Crimean War observers on one occasion saw the whole of the British Fleet inverted at a considerable height, an illustration of which appeared in *The Illustrated London News* at the time. Some very interesting cases were recorded by Dr. Vince in the Bakerian lecture of 1798, read before the Royal Society, some of which are delineated in the succeeding slides. He remarks upon these curious phenomena that he thinks that in cases of national emergency certain people should be told off with telescopes to look out for the enemy's ships, and to search the horizon to see if they could detect any ships looming. Dr. Vince mentions another remarkable instance in which he saw Dover Castle from Ramsgate, at a point from which the whole of the keep of the castle cannot be seen, the four turrets only being visible. The most curious case of lateral refraction that I have been able to discover was observed at Geneva in 1818, by M. Jurine; a barque was seen approaching on the left bank of the lake, and at the same time an image of the sails was observed above the water, which, instead of following the direction of the barque, separated from it and appeared to approach Geneva by the right bank of the lake, the image moving from east to west while the barque moved from north to south. This case was brought to the notice of Biot, the physicist, and he, in one of the scientific journals, gave a very long explanation. He came to the conclusion, from the geographical features, and climatology, and the direction in which the sun's rays were passing at the time of the observation, that there would be considerable lateral difference in the temperature, quite sufficient to produce this phenomenon of lateral refraction.

Another case of curious refraction has been noticed by many people—I have seen it myself particularly on the coast of Norway. Low lands, and the extremity of headlands, or points forming an acute angle with the horizon of the sea, and viewed from a distance beyond it, appear elevated above it, with an open space between the land and sea, the effect being proportional to the amount of evaporation taking place at the surface.

Fata Morgàna is a name given to an optical phenomenon sometimes seen in the Straits of Messina between Sicily and the Italian coast. Minasi says: "When the rising sun shines from that point whence its incident ray forms an angle of about 45° on the sea of Reggio, and the bright surface of the water is not disturbed either by wind or current, the spectator being placed on an eminence of the city with his back to the sun and his face to the sea, on a sudden there appear in the water various multiplied objects, namely, numberless series of pilasters, arches, castles, columns, towers, palaces with balconies and windows, valleys of trees, plains with herds and flocks, &c., in their natural colours and proper action, passing rapidly in succession along the surface so long as the above-mentioned causes exist. If, in addition, the atmosphere be highly impregnated with vapour and dense exhalations not previously dispersed by the action of the wind and waves, or rarefied by the sun—in this vapour, as in a curtain, to a height of 24 or 25 feet, and nearly down to the sea, the observer sees the same objects not only reflected from the sea, but likewise in the air, though less distinct. Lastly, if the air be hazy and slightly opaque and dewy, the objects appear only at the sea surface, but with prismatic colours." He endeavours to prove that they are representations of objects on the two coasts. He considers the sea an inclined speculum, on account of the rapid current which runs through the Straits, and divided into different planes by contrary eddies, and he ascribes the *aerial morgana* to the refractive and reflective power of matter suspended in the air.

Lastly, I would mention the experiments of Wollaston upon the subject of refraction and mirage. First, he says, into a square phial containing a little clear syrup put an equal quantity of water in such a way that it floats without mixing, and after a little time, by mutual penetration, you see effects; if you view through the syrup a card with a written word upon it, you see it, and also above it an inverted and erect image of the same. That is a case in which the density diminishes upwards, and the ray has its concavity presented downwards. Then, above the water he placed rectified spirits of wine, when the inverted and erect images were seen below, these appearances continuing many hours and even days, and he carried out similar experiments with water at different temperatures. Everybody knows the experiment with a red-hot poker; the effects of mirage can be seen by looking along the surface of a red-hot poker, held at a distance of about a foot from a sheet of paper, when there is perceptible refraction. Again, Wollaston looked along a horizontal plate of glass upon which he poured ether, and a line appeared instantaneously upon the opposite wall at an elevation of half a degree, this effect being due to the cold caused by the evaporation of the volatile liquid. Finally, Brewster showed that all the phenomena of unusual refraction might be observed by holding a heated iron over a mass of water bounded by parallel plates of glass, and then substituting a cold bod y for the hot iron.

UNIVERSITY AND EDUCATIONAL INTELLIGENCE.

THE foundation-stone of the Gordon Memorial College at Khartum was laid on Thursday last by Lord Cromer, who remarked that the College would aim at diffusing knowledge of agriculture, engineering, and other practical acquirements useful to all classes.

THE New South Wales Government invite applications for the position of Professor of Physics in the University of Sydney, from University graduates under thirty-five years of age. Particulars of the conditions of appointment, duties, &c., can be obtained from Sir Daniel Cooper, Bart., G.C.M.G., Acting Agent-General for New South Wales, 9, Victoria Street, Westminster, London, S.W.

THE necessity of encouraging scientific investigation, and of providing means for training investigators, is pointed out by Prof. Cleveland Abbe in the U.S. *Monthly Weather Review* (September 1898). He remarks:—A mistaken idea has widely prevailed that the investigator is a genius, born and not made. The history of German science has, however, shown that environment and training are as important as birth and inheritance. The whole system of education in the German universities has for five generations been directed to the development of the investigator as its highest product. Those who discover important new facts, laws, or principles have been rewarded with the highest places in the intellectual world of that nation. Those who feel that they have a desire or calling for scientific research are encouraged to study for the degree of doctor of philosophy, a degree that is only granted when the candidate has, by actual observation, experiment, or exploration, made some important contribution to human knowledge. The professors under whom he studies have, in their turn, made many similar contributions, and are well prepared to judge of the value of *his* work. The German universities have, during the past seventy years, published over fifty-thousand so-called "doctors' dissertations," embodying the results of the works of fifty thousand candidates. The consequence is that to-day Germany easily leads all the world in the amount and value of her contributions to human knowledge and the energy with which her students pursue the study of nature.

SCIENTIFIC SERIALS.

Bulletin of the American Mathematical Society, December 1898.—At the October meeting of the Society seven papers were communicated. Abstracts of the papers not to be published in the *Bulletin* are given.—Prof. Woodward's paper, on