effect such a saving of time and trouble that I think they ought to be regularly taught in schools.

Years ago I had discovered the curious fact that, if you put a "o" over the unit-digit of a given number, which happens to be a multiple of 9, and subtract all along, always putting the remainder over the next digit, the final subtraction gives remainder "o," and the upper line, omitting its final "o," is the "9-quotient" of the given number (*i.e.* the quotient produced by dividing it by 9).

Having discovered this, I was at once led, by analogy, to the discovery that, if you put a "o" under the unit-digit of a given number, which happens to be a multiple of 11, and proceed in the same way, you get an analogous result.

In each case I obtained the quotient of a division-sum by the shorter and simpler process of subtraction: but, as this result was only obtainable in the (comparatively rare) case of the given number being an exact multiple of 9, or of 11, the discovery seemed to be more curious than useful.

Lately, it occurred to me to examine cases where the given number was *not* an exact multiple. I found that, in these cases, the final subtraction yielded a number which was sometimes the actual remainder produced by division, and which always gave materials from which that remainder could be found. But, as it did not yield the quotient (or only by a very "bizarre" process, which was decidedly longer and harder than actual division), the discovery still seemed to be of no practical use.

But, quite lately, it occurred to me to try what would happen if, after discovering the remainder, I were to put it, instead of a "o," over or under the unit-digit, and then subtract as before. And I was charmed to find that the old result followed: the final subtraction yielded remainder "o," and the new line, omitting its unit-digit, was the required quotient.

Now there are shorter processes, for obtaining the 9-remainder or the 11-remainder of a given number, than my subtraction-rule (the process for finding the 11-remainder is another discovery of mine). Adopting these, I brought my rule to completion on September 28, 1897 (I record the exact date, as it is pleasant to be the discoverer of a new and, as I hope, a practically useful, truth).

(1) Rule for finding the quotient and remainder produced by dividing a given number by 9.

To find the 9-remainder, sum the digits: then sum the digits of the result: and so on, till you get a single digit. If this be less than 9, it is the required remainder: if it be 9, the required remainder is 0.

To find the 9-quotient, draw a line under the given number, and put its 9-remainder under its unit-digit: then subtract downwards, putting the remainder under the next digit, and so on. If the left-hand end-digit of the given number be less than 9, its subtraction ought to give remainder "o": if it be 9, it ought to give remainder "1," to be put in the lower line, and "1" to be carried, whose subtraction will give remainder "o." Now mark off the 9-remainder at the right-hand end of the lower line, and the rest of it will be the 9-quotient.

Examples. $9/75309 \begin{smallmatrix} 6 \\ 83677/3 \end{smallmatrix}$ $9/94613 \begin{smallmatrix} 8 \\ 105126/4 \end{smallmatrix}$ $9/58317 \begin{smallmatrix} 3 \\ 64797/0 \end{smallmatrix}$

(2) Rule for finding the quotient and remainder produced by dividing a given number by 11.

To find the 11-remainder, begin at the unit-end, and sum the 1st, 3rd, &c., digits, and also the 2nd, 4th, &c., digits; and find the 11-remainder of the difference of these sums. If the former sum be the greater, the required remainder is the number so found: if the former sum be the lesser, it is the difference between this number and 11: if the sums be equal, it is "o."

To find the 11-quotient, draw a line under the given number, and put its 11-remainder under its unit-digit: then subtract, putting the remainder under the next digit, and so on. The final subtraction ought to give remainder "o." Now mark off the 11-remainder at the right-hand end of the lower line, and the rest of it will be the 11-quotient.

Examples. $11/73210 \begin{smallmatrix} 8 \\ 66555/3 \end{smallmatrix}$ $11/85347 \begin{smallmatrix} 1 \\ 77588/3 \end{smallmatrix}$ $11/59426 \begin{smallmatrix} 3 \\ 54023/10 \end{smallmatrix}$ $11/47568 \begin{smallmatrix} 4 \\ 43244/0 \end{smallmatrix}$

These new rules have yet another advantage over the rule of actual division, *viz.* that the final subtraction supplies a *test* of the correctness of the result: if it does not give remainder "o," the sum has been done wrong: if it does, then either it

has been done right, or there have been *two* mistakes—a rare event.

Mathematicians will not need to be told that rules, analogous to the above, will necessarily hold good for the divisors 99, 101, 999, 1001, &c. The only modification needed would be to mark off the given number in periods of 2 or more digits, and to treat each period in the same way as the above rules have treated single digits. Here, for example, is the whole of the working needed for dividing a given number of 17 digits by 999 and by 1001:—

$$\begin{array}{r} 999/75410836428139 \begin{smallmatrix} 214 \\ 75486322750890/104 \end{smallmatrix} \\ 1001/75410836428139 \begin{smallmatrix} 214 \\ 75335500927212/2 \end{smallmatrix} \end{array}$$

But such divisors are not in common use; and, for the purposes of school-teaching, it would not be worth while to go beyond the rules for division by 9 and by 11.

Ch. Ch., Oxford.

CHARLES L. DODGSON.

Notes on Madagascar Insects.

HAVING recently received a small miscellaneous collection of rather typical Madagascar insects, collected by my son on that island, perhaps a modest, non-technical letter concerning them might interest your readers. The climate is so deadly to Europeans that insect-collecting is hazardous in the extreme, and I think not much is known of the insects. Some of the Orthoptera are highly curious, possessing antennæ five and six times the length of their bodies, so as to be able to detect danger afar. The Longicorn beetles appear very similar to ours, but the markings on their elytræ are brighter. The beetles generally are remarkable for the extreme brilliancy in colouring of their under surfaces and legs, while the upper surface is dull. I apprehend, therefore, they are *not* ground feeders. The dragon flies appear similar to some of ours, both in size, colouring, and shape. There is a lantern fly, or two, and a mole cricket, much resembling ours.

The spiders are *not* large, but as ugly and venomous-looking as nature knows how to make them. But, oh, the Centipedes!—gruesome-looking, plated, mailed, jointed, spiny-tailed, and, corneous horrors, half a foot long, with twenty legs (each side) their rapidity in travel must be great, while the large curved fangs, attached to the "business end," suggest the deadliest of grips. Had Milton ever seen one, "Paradise Lost" might have contained another horror.

I have had mounted with them a praying Mantis (also from the island), to somewhat neutralise their felonious aspect, though I fear the usual attitude of this most hypocritical of insects (with its extended fore limbs) indicates *anything* but prayer, or even reverence.

The brilliancy in colouring of Madagascar butterflies is not remarkable for a sub-tropical region. Many are like our fritillaries in aspect, while the clouded yellows appear identical. A few of the white butterflies are also like ours.

Rottingdean, Sussex.

E. L. J. RIDSDALE.

Protective Colouring.

THE following instance of apparent consciousness of protective colouring in a young bird seems worth recording. On August 14, while walking in my orchard, which being on a steep slope is terraced with low stone walls, I put up a young Nightjar (*Caprimulgus europæus*) which flew straight to the top of one of the walls and flattened itself down on a broad flat stone. As it was within 6 feet of a hedge on one side, and there were gooseberry bushes, &c., on the other, there was no lack of cover if it had wished to hide. I left it there, and coming again two hours later found it in the same spot. Its colouring matched the stone on which it was lying so closely that had not one known that it was there, it would probably have been overlooked. On being closely approached it flew to another of the walls higher up, and crouched down in exactly the same way. I then tried to catch it with a butterfly net, when it flew over the hedge to a rough field on the opposite side of the valley from which it had, no doubt, come.

ALFRED O. WALKER.

Nant-y-Glyn, Colwyn Bay, October 5.