"Whatever can be divided, and has parts, possesses some thinghood, and must, therefore, contain two ultimate units, the whole namely, and the smallest element possessing thinghood."

The *mathematical continuum* contains no "smallest element," and there is, accordingly, no necessity for a thing which can be divided, and which has parts, to contain such an element. This remark may perhaps offer the key for the solution of the problem set by Mr. Russell, the problem namely of determining the properties of a *form of externality*. It is conceivable that, in arriving at the axioms of projective geometry as constituting a statement of these properties, he has assumed the solution of a problem in the *theory of manifolds* just as Helmholtz, in arriving at the axiom of constant *space-curvature* as necessary to congruence, assumed the solution of a problem in the *theory of groups*. In the latter case the weapon needed to attack the problem was forged at a much later date by Lie. In the case of Mr. Russell's problem the appropriate engine of discovery is still undeveloped, the mathematics of the manifold being at present limited to numerical aggregates. No one has yet done for the science of space what Dedekind did for the science of number.

Mr. Russell is happier in his treatment of the axioms of metrical geometry, and he has done real service to mathematics in pointing out the essential weakness of the Riemann-Helmholtz method. This method started from the consideration of space as a numerical aggregate, whose points are determined by coordinates, and then sought for the condition of the possibility of measurement. This condition was found in the uniformity of the measure of space-curvature, and it was shown, on the one hand, to imply the possibility of the straight line, and, on the other, to be equivalent to the statement that figures which can be brought to congruence are equal. The argument, as Mr. Russell shows, really involved a vicious circle. For space can be regarded as a numerical aggregate only if we have the means of assigning to points coordinates which have some spatial import, and coordinates which have such import presuppose measurement. The conclusion arrived at by Mr. Russell is that the essential postulate of metrical geometry is the *axiom of free mobility*, or the assertion of the possibility of equal figures in different places, and he has shown that the denial of this axiom would lead to logical and philosophical absurdities. In this connection it is only fair to Riemann to remember that his essay "Ueber die Hypothese, welche der Geometrie zu Grunde liegen" remained unpublished until after his death, a fact which points to the belief that he was not satisfied with it.

Leaving to philosophers by profession the task of appreciating and criticising Mr. Russell's philosophy of space, we may attempt to estimate the value of his book for mathematics. It has already been pointed out that in his criticism of Riemann and Helmholtz he has brought forward considerations which are mathematically important, and this is not the only place where he has had occasion to point to examples of the special philosophical vice of the mathematician, the tendency namely to mistake the sign for the thing signified (*cf.* Couturat "De l'Infini mathématique," p. 331). To mathematicians also his book should be interesting on account of its acute and novel treatment of familiar topics: thus—projective coordinates are numbers arbitrarily but systematically assigned to points of space "like the numbers of houses in a street" (p. 119). The ambiguity in the definition of distance, which is unavoidable on projective principles, does not show that distance is ambiguous, but that projective methods cannot adequately deal with distance (p. 35). The distinction between real and imaginary points is the distinction between quantities to which points correspond and quantities to which no points correspond (p. 44). The book is throughout well written, and is for the most part free from obscurity, and it may be recommended to all who wish to have clear ideas on matters of fundamental importance in mathematics. A. E. H. L.

OUR BOOK SHELF.

A Bibliography of Gilbert White, the Natural Historian and Antiquarian of Selborne. By Edward A. Martin, F.G.S. Pp. xiii + 274. (Westminster: The Roxburghe Press, 1897.)

THERE are many places in England prettier than the little Hampshire village of Selborne, but none of them are so full of interest to the outdoor naturalist as the home of Gilbert White. Though more than a century has passed away since the simple student of nature's ways in the sleepy hollow of Selborne first gave the world the benefit of his observations and impressions, the book in which these notes are published is as fresh now as ever it was. The reason for this is, it seems to the writer, that Gilbert White was usually content to record facts as he found them, and he did not regard nature from the point of view of a pre-conceived theory. Accurate observations of natural objects and phenomena live for ever, but the explanation of such facts must alter from time to time as wider knowledge of the laws of nature is obtained.

The success of White's "Selborne" has had two unfortunate effects: it has made every country clergyman who can distinguish a martin from a swallow think that he is a Gilbert White, and it has caused the literary world to be deluged with so-called popular natural history works, which are often more remarkable for thoughts about nothing than for observations of something. We can, however, forgive the authors of such rhapsodies for inflicting their musings upon a busy world, because of the real naturalists which White's "Selborne" has created.

How large and widespread is the public to which the book appeals may be seen by the volume before us. Mr. Martin has found no less than seventy-three separate editions of our natural history classic; so the aggregate number of volumes published must be very great. The features of each of these editions are described in detail; hence Selbornites are now provided with interesting particulars of the various volumes which have refreshed the mind and administered to the intellectual enjoyment of thousands of nature-lovers the world over. Mr. Martin has not, however, confined his work to a mere list of editions of the "Natural History of Selborne"; he describes the naturalist himself and the main facts of his life, points out some of the chief observations and discoveries, gives a chapter on the village of Selborne, and devotes another to White's old house, "The Wakes." The work is thus more than a bibliography; it is a guide to the study of Gilbert White and his natural history, and as such will be prized by many of his disciples.

Reference is made on p. 71 to a suggestion of White's that entomology required some "neat plates" for its advancement, and it is stated that the idea has been carried out by the Science and Art Department. Surely there is a mistake here.