

approached us, they all became naturally easier to make out; but until October no change apparently occurred in any of them, except those in the region about the Lake of the Sun. These by September were already dark. In October they began to show symptoms of growing lighter again. At the next presentation, in November, they showed further signs of change, though not differing as yet very unmistakably in tint. Meanwhile, when the Sinus Titanum region came round in November, I found that its canals had begun likewise to darken. The canals were not only darker relatively to the Mare Cimmerium and the Mare Sirenum than they had been, but actually darker themselves. In the next few nights the more northern canals about Ceraunius had followed suit. They had darkened relatively to the southern ones about the Lake of the Sun.

Now, on looking at a map of Mars, it will be seen that the Solis Lacus region is that part of the continental areas which lies nearest the south pole. Similarly, that the region about Sinus Titanum is the next farthest south. The matter of latitude therefore affects the point.

The canals and so-called lakes share, therefore, in the annual metamorphosis, with a season change dependent in a general way upon their latitudes. A wave of deepening tint passes successively through the blue-green regions from south to north, timed to the seasonal wave that travels from pole to pole. From being pale in winter, their colour comes with the spring, deepens through the summer, and dies out again in the autumn. In any given locality the change comes early or late, in proportion as the place lies, other things equal, distant from the pole.

That this change of tint is due indirectly to water, and directly to the vegetation that water induces, seems probable. For just as there is great difficulty in disposing of the water on the first supposition, so the second would lead us to expect just the phenomena observed. It may therefore be concluded that the formations known as the seas of Mars are probably midway in evolution between the seas of Earth and the seas of the Moon. That is to say, they are not barren ocean beds, but are in that half-way stage of the process when their low-level helios then catch what water still voyages upon the planet's surface, though they have long since parted with their own.

Throughout all these interesting changes that follow the seasons across the face of Mars, there is but one feature approaching permanence—the great continental areas. Except for a possible variation in brightness here and there, this great area has remained unchanged. Like the reddish desert regions of our Earth, its colour and immutability point to like character for cause. It does not change because it is already past such possibility. It is one vast desert waste.

#### UNIVERSITY AND EDUCATIONAL INTELLIGENCE.

A NEW post has been created under the Education Department for the purpose of obtaining special information and issuing special reports, from time to time, in relation to educational work at home and abroad. The frequent demand for fuller information on many educational subjects, and the great increase of purely administrative work, both at the Education Department in Whitehall and at the Science and Art Department, have made it desirable to have a separate officer in charge of a small additional branch for the above-named purpose, who will be designated "Director of Special Inquiries and Reports." This appointment has been accepted by Mr. M. E. Sadler, Student of Christ Church, and Secretary of the University Extension Delegacy at Oxford.

THE Technical Education Board of the London County Council will be prepared early in July, 1895, to award not more than five Senior County Scholarships of the annual value of £60 in addition to the payment of college fees, tenable for three years, and subject to annual renewal. The scholarships are intended to provide the means of obtaining advanced technical training in a university, university college, or technical institute of university rank for students (young men or women) of exceptional ability who would otherwise find it impossible to secure such training. Candidates must, as a rule, be under nineteen years of age on September 1, 1895, but the Board is prepared to consider very special cases in which the candidates are above that age. The scholarships are offered with the view of encouraging the study of science or art, with

special reference to industrial requirements, and will be tenable at such institutions giving appropriate instruction within the statutory definition of technical instruction as may be selected by the scholars and approved by the Board. In the selection of scholars the Board will have regard, in the first instance, to the past achievements of the candidates, but the Board reserves the right to require any or all of the candidates to undertake an examination if it think fit. No candidate will be eligible whose parents have an income from all sources of more than £400 per annum.

SINCE November 1893, the Technical Education Board of the London County Council have awarded 721 Junior County Scholarships, each of the value of £20 and two years' free schooling. More than three thousand candidates presented themselves in competition for the scholarships, which are restricted to children whose parents are in receipt of not more than £150 a year. A detailed analysis of the occupations of the persons whose children competed for these scholarships is given in the *London Technical Education Gazette*. It indicates that the highest percentage of candidates who received scholarships is to be found in the leather trades, and next to these in the printing trades and jewellery and fine instrument trades. After these come the artistic trades and crafts, but the most remarkable feature is the comparatively poor results obtained by the children of clerks, agents and warehousemen, and the very poor success achieved by the professional classes. The time is not far distant when the scholarships granted by the Board will amount to the value of £30,000 per annum.

#### SCIENTIFIC SERIALS.

*The Mathematical Gazette*, No. 3, December, 1894.—The eccentric circle of Boscovich. In this continuation the editor considers a special case in which the centre of the eccentric circle lies on the straight line whose points of intersection with the conic are required. He then discusses the method as one of transformation, and finally points out a connection between reversion and perspective projection.—Dr. Mackay, in Greek Geometers before Euclid, writes upon Pythagoras and the Italic school.—Cajori's "History of Mathematics" is an all too short notice, by Dr. G. B. Halsted, of a book that has come in for a fair amount of praise and blame. There are some very interesting problems, solutions of examination questions, and questions for solution.—Prof. A. Lodge supplies an addition to his previous article on approximations and reductions.—We note, with pleasure, that in future the *Gazette* is to be enlarged to twelve pages. This additional space will greatly help to increase the use of this journal, which has so quickly made its way in school circles.

*Bulletin of the American Mathematical Society* (2nd series, vol. i. No. 3, December, 1894).—The group of Holoedric Transformation into itself of a given group, by Prof. E. H. Moore, is a paper read before the Society at its November meeting. The remaining article is by Dr. G. A. Miller, on the non-primitive substitution-groups of degree ten. A list of these was given in the *Quarterly Journal of Mathematics*, vol. xxvii. pp. 40-42. A result of the article before us is that the following six groups should be deleted from that list, viz. 200<sub>2</sub>, 200<sub>7</sub>, 200<sub>8</sub>, 100<sub>3</sub>, 50<sub>2</sub>, 50<sub>3</sub>.—In the *Briefer Notices* short accounts are given of H. Hertz, "Gesammelte Werke," Band iii. This volume, the first one as yet issued, contains a memoir on the principles of theoretical mechanics and mathematical physics, which was composed during the last three years of the author's life. The next notice gives a sketch of a new edition of Grassmann's mathematical works. It is to be hoped that, as was recently suggested in NATURE by Prof. Genese, a translation into English of the *Ausdehnungslehre* may soon be made, for the convenience of many students in this country. The other notices summarise the contents of the *Jahresberichte der Deutschen Mathematiker-Vereinigung* (vol. iii. 1893), of the *Proceedings of the American Association*—for the forty-second meeting, held at Madison, Wis. (August, 1893); of "Le Livret de l'étudiant de Paris" (Paris, 1894).—The *Notes* comprise accounts of the November meetings of the American and London Mathematical Societies. By the way, the reporter, who is a member of the latter Society, gives one of the names of the Council incorrectly. There is also an ac-

count of the meeting, in September last, at Vienna, of the German Mathematical Association. The *Bulletin* well maintains its position, and closes with its useful lists of new publications.

SOCIETIES AND ACADEMIES.

LONDON.

Mathematical Society, Dec. 13, 1894.—Major Macmahon, F.R.S., President, and subsequently Mr. A. E. H. Love, F.R.S., Vice-President, in the chair.—The following communications were made:—On Maxwell's law of partition of energy, by Mr. G. H. Bryan. In his recent report to the British Association, the author had shown that if a large number of dynamical systems of any kind be taken, all similar in every respect, it is always possible to distribute their coordinates and momenta so that the distribution shall remain permanent, and shall satisfy Maxwell's law of partition of energy. By this is meant that if the kinetic energy of each system be reduced to a sum of squares, the mean values of these squares are equal. But the author had doubted whether it was possible in any case to infer that the *time averages* of the squares forming the kinetic energy of a single system were equal. In the pre-ent paper the connection between *time averages*, and *averages taken over a large number of different systems*, is examined more fully by means of an artifice suggested by Prof. Boltzmann's paper "On the application of the determinantal relation to the theory of polyatomic gases" (published as an appendix to Mr. Bryan's report). Instead of a vessel containing gas (as taken by Prof. Boltzmann), any single dynamical system is taken whose coordinates and momenta return to their original values after a long time T. If the time be divided into an infinitely large number (*n*) of infinitely short intervals (*i*), we can derive a stationary distribution by taking *n* systems and starting them, the first at time 0, the second at time *i*, the third at time 2*i*, and so on, giving every system the same coordinates and momenta at the time of starting it. At the end of the time T, we shall have the systems distributed according to a permanent or stationary law, and at any subsequent instant the mean value of any function of the coordinates and momenta for the different systems will be equal to the time average of the corresponding function for the original single system. If, however, we start with a number of systems distributed according to a permanent law, we cannot pass back to the original single system unless we can show that the law of permanent distribution is unique. Now in any simple test case, such as that afforded by rigid bodies movable about fixed points or particles moving in planes after the manner of a Lissajou's curve tracer, a stationary distribution exists satisfying Maxwell's Law of Partition, but other stationary distributions are possible which do not satisfy the law. Hence the author concludes that the time averages of the squares into which the kinetic energy of a single system can be divided, are not in general equal, at any rate independently of initial circumstances.—The Spherical Catenary, by Prof. A. G. Greenhill, F.R.S. The pseudo-elliptic integrals developed in the *Proc. L.M.S.* xxv. are applied in this paper to the construction of solvable cases of the spherical catenary, given by the relation

$$\psi = \int_{(1-z^2)} \frac{Adz}{\sqrt{Z}}$$

where

$$Z = (1-z^2)(z-h)^2 - A^2,$$

connecting  $\psi$  the azimuth, and  $z = \cos \theta$ , where  $\theta$  denotes the angular distance from the lowest point of the sphere, the tension being  $w(z-h)$  (Clebsch, *Crelle*, 57). Putting

$$u = \int \frac{dz}{\sqrt{Z}}$$

and

$$\chi = \psi - pu = \int \frac{-p(1-z^2) + A}{(1-z^2)\sqrt{Z}} dz,$$

then  $\chi$  can be identified with the standard form of the pseudo-elliptic integral of the third kind, of order  $\mu$ ,

$$I = \frac{1}{2} \int \frac{\rho(s-\sigma) - \mu \sqrt{(s-\Sigma)} ds}{(s-\sigma)\sqrt{S}}$$

where

$$S = 4(s+x)^2 - \{(y+1)s + xy\}^2$$

by putting

$$\chi = \frac{I}{\mu}, \quad p = \frac{1}{2} \left( \frac{M\rho}{\mu} + A \right), \quad A = M(y+1), \quad h^2 = A^2 - 2y - 1,$$

where

$$M^2 = -\frac{y+1}{2x},$$

and  $x, y$  are Halphen's functions, defined in his "Fonctions Elliptiques," I. p. 102. If  $u = a$  when  $z = 1$ , and  $u = b$  when  $z = -1$ , then  $u = \frac{1}{2}(a-b)$  when  $z = h$ ;  $a$  and  $b$  are each of the form  $f\omega_3$ , a fraction of the imaginary period  $\omega_3$ . Also

$$M^2 p(a+b) = -\frac{1}{6}(1-h^2) - \frac{1}{4}A^2,$$

or  $12p(a+b) = 8x - (y+1)^2$ , so that  $\sigma = 0$ , and

$$a+b = \frac{4\omega_3}{\mu}.$$

Thus, for instance, when

$$a+b = \omega_3, \quad A = h^2 - 1, \quad p = \frac{1}{2}A;$$

and the corresponding spherical catenary is given by

$$(1-z^2)^{\frac{1}{2}} e^{\chi} = \sqrt{\frac{h-1}{2}} \sqrt{\{z^2 - (h+1)z - 1\}} + i \sqrt{\frac{h+1}{2}} \sqrt{\{-z^2 + (h-1)z + 1\}}.$$

With

$$\mu = 3, \quad a+b = \frac{2}{3}\omega_3, \quad A^2 = h^2 - 1, \quad p = \frac{1}{3}A;$$

and

$$(1-z^2)^{\frac{2}{3}} e^{\chi} = A(z^3 - hz^2 - 2z) + i(hz + 1)\sqrt{Z}.$$

With

$$\mu = 5, \quad a+b = \frac{2}{5}\omega_3,$$

then  $y = x = -c$ , suppose;

$$M^2 = \frac{1-c}{2c}, \quad p = \frac{1}{2}(3-c)M;$$

and

$$(1-z^2)^{\frac{5}{2}} e^{\chi} = H_5 z^5 + \dots + H_3 + i(Lz^3 + \dots + L_3)\sqrt{Z},$$

where

$$H = \frac{2-5c+c^2}{c}M, \quad L = \frac{2-c}{c}h, \quad \&c.$$

With

$$a+b = \frac{1}{2}\omega_3$$

the calculation is rather more complicated, as this case must be derived from  $\mu = 8$ ; but the result is of the form

$$(1-z^2)e^{2\chi} = (Hz - H_1)\sqrt{(z_3 - z - z_0)(z - z_0)} + i(Lz - L_1)\sqrt{(z - z_2)(z - z_1)},$$

with

$$z_3 > z > z_2 > z_1 > z_0 > z > z_0,$$

$$z_0, z_1, z_2, z_3$$

denoting the roots of the quartic  $Z = 0$ .—The Transformation of Elliptic Functions, by Prof. A. G. Greenhill, F.R.S. The function  $z_n$ , introduced by Prof. Klein in his paper on the transformation of elliptic functions (*Proc. L.M.S.* xi. p. 151), and developed in Klein and Fricke's "Modulfunktionen," ii. part v., is shown here to be connected with Halphen's  $\gamma$  function by the relation

$$\rho(-1)z_n = \lambda^n \gamma_n,$$

for a transformation of the  $n$ th order; and for an odd value of  $n$ ,

$$\lambda^{2p+1} = \frac{\gamma_{n-2p+1}}{\gamma_{n+2p-1}}, \quad p = 1, 2, 3, \dots, \frac{n-1}{2};$$

in this manner the relation  $\gamma_n = 0$  is satisfied.

The biquadratic relations satisfied by the function  $z_n$  are now derived from Halphen's formula

$$\gamma_{m+n}\gamma_{m-n} = \gamma_{m+1}\gamma_{m-1}\gamma_n^2 - \gamma_{n+1}\gamma_{n-1}\gamma_m^2$$