QUATERNIONS.

Anwendung der Quaternionen auf die Geometrie. Von Dr. P. Molenbroek. (Leyden : E. J. Brill, 1893.)

The Outlines of Quaternions. By Lieut.-Colonel H. W. L. Hime. (London : Longmans, Green, and Co., 1894.)

IN these books we have evidence of the growing demand for quaternion literature. Dr. Molenbroek's work is the promised sequel to his first volume on the Theory of Quaternions, and contains many admirable examples of the application of the method to geometry. All who are familiar with Hamilton's and Tait's classics on the subject will recognise many of these examples as old friends, taken almost verbatim from their original sources. In not a few of the applications, however, Dr. Molenbroek ventures into fresh fields, and shows that he can use quaternions with ease and power. It is interesting to notice the occasional effective use of the conjugate quaternion, an invention of the great master which is apt to be lost sight of after the foundations of the calculus have been laid. The treatment throughout is on the familiar Hamiltonian lines, the author's aim being development and not fancied improvements. The book consists of six chapters, in which are taken up—to name a few of the most important applications—spherical trigonometry, the plane and sphere, quadric surfaces, surfaces in general, curves in space, and the theory of rectilinear rays. The elementary properties of the remarkable operator ∇, and the integration of partial differential equations of the first and second orders, are discussed as part of the general theory of surfaces. In the same chapter, Dr. Molenbroek, by means of two new differentiating operators, obtains a simple symbolic representation for the first, second, and higher polars of a point with regard to a given surface. These remarks will indicate sufficiently the scope of a work which, though not altogether above criticism in minor details, is a distinct addition to quaternion literature, and deserves a wide circulation.

Colonel Hime's work is a much more modest production, being intended for the mere beginner. In general scope it might be compared to the first nine chapters of Kelland and Tait's " Introduction to Quaternions." The book contains many good examples in the simpler applications of quaternions to the geometry of triangle, plane, sphere, conic section, cone, &c., but it is less satisfactory in the exposition of the fundamental principles of the calculus. For example, the identification of unit vector and right versor is stated, but the reason for this identification is nowhere distinctly given. Again, the truth that the familiar Hamiltonian symbols i j k may be regarded as in a sense *imaginaries*, because $i^2 = j^2 = k^2 = -1$, is supposed to lead to the equation

$$i = j = k = \sqrt{-1} = -i = -j = -k \text{ (Eq. 8, p. 40)}.$$

This seems to be playing sad havoc with one's very definitions. Then on page 76 we find what is virtually the equation $\delta\beta^{-1}\delta = \gamma$ transformed into $\delta^2 = \gamma\beta$, the non-commutative principle being wholly ignored, and in consequence a quaternion and a scalar equated ! These errors, especially the latter, are very surprising in a book whose author is a true disciple of Hamilton. Of minor blemishes we might refer to the appeal to Cartesian

expansions in order to demonstrate (?) the associative principle in multiplication. Nor do we quite understand Colonel Hime's system of referring to authorities. For example, why should Prof. Hardy be quoted as the authority for the statement that every versor may be represented by a power of a unit vector ; for is it not all in Hamilton (see " Elements," § 309) ? Again, Dr. Odstrčil is credited with a proof that the three angles of a plane triangle are together equal to two right angles, the proof being an obvious particular case of Hamilton's remarkable expression for the product of the versor arcs of a spherical triangle. But surely the theorem regarding the angles of a plane triangle *underlies* the fundamental properties of quaternions and versors ; so that the supposed proof is really reasoning in a circle. Dr. Odstrcil is worthy of higher praise than this. These blemishes apart, however, and leaving out of account the two errors already noted, we find in Colonel Hime's book a serviceable exposition of the elementary applications of quaternions. A careful study of its pages will go far to fit the reader for the arduous task of grappling with the higher and more characteristic developments to be found in the writings of the masters of the quaternion calculus.

OUR BOOK SHELF.

Sir Victor Brooke, Sportsman and Naturalist. By O. L. Stephen. (London : John Murray, 1894.)

THE late Sir Victor Brooke was an excellent example of a combination of sportsman and naturalist. In this book his life as a sportsman predominates ; but a chapter on his researches in natural history, by Sir William Flower, shows that he possessed the keenness of observation required in a man of science. His most important contribution to science was an exhaustive paper, published in the *Proceedings* of the Zoological Society, the subject being the classification of the *Cervidæ*. At one time he was an enthusiastic student of natural history, but the state of Lady Brooke's health having compelled him to live out of England for the greater part of the year, he could not conveniently carry on his researches. From about 1880 his life was chiefly devoted to foreign travel and sport. The extracts from his letters and journals are full of stirring adventures, and contain some interesting observations on animal life and habits. Mr. Stephen prefaces these extracts with a memoir of his dead friend. The book is beautifully printed, and is illustrated by ten fine plates. It appeals particularly to those who were acquainted with Sir Victor Brooke, and who admired his character ; nevertheless, such of the public as read it will find the contents interesting.

A Text-book of Dynamics. A Text-book of Statics. By William Briggs and G. H. Bryan. (The University Tutorial Series.) (London : W. B. Clive, 1894.)

THESE books belong to the elementary class, and a perusal of them shows that they will prove excellent additions to this series of useful text-books.

In both the authors have assumed little or no knowledge of trigonometry, and they have been written so that either may be read first. The treatment is conspicuous for its clearness and conciseness, and is all that a student about to enter a course could desire. The figures are neatly drawn, and many new ones are noticeable in the latter book.

Notwithstanding the fact that these text-books are published to meet the requirements of candidates for certain examinations, they may still be used by others, who are making themselves acquainted with these sub-