

bridgeshire County Council. The scheme was thrown out—on financial grounds—by the Senate, and here it seemed likely that agricultural education would come to a standstill, had it not been for the action of the County Councils of the Eastern Counties,¹ who, with the help of certain University professors, organised the Cambridge and Counties Agricultural Education Committee, an arrangement by which the counties supply the funds, while the University members supply the teaching. Under this scheme agricultural students are now receiving at Cambridge instruction in a number of subjects bearing directly on agriculture. The students are not necessarily members of the University, nor is agriculture a recognised department of University study; but it has now been practically sanctioned by the appointment of a University syndicate, whose duty it is to superintend the examinations on which the new diploma is to be granted. This procedure has a precedent in the successfully established diploma in State Medicine, and cannot fail to exert—both as a check and a stimulus—a wholesome effect on the unofficial agricultural department.

The first examination will be held in July. It consists of two parts: Part i. embraces botany, chemistry, physiology, entomology, geology, engineering, and book-keeping, in so far as each subject bears on agriculture. Part ii. comprises practical agriculture and surveying. The examinations are open to all who present themselves, and who pay the moderate fee demanded. Intending candidates may, it seems, obtain information from Prof. Liveing (who has taken the chief share in the work from the University side of the question) or from Mr. Francis Darwin.

ON HOMOGENEOUS DIVISION OF SPACE.

§ 1. THE homogeneous division of any volume of space means the dividing of it into equal and similar parts, or cells, as I shall call them, all someways oriented. If we take any point in the interior of one cell or on its boundary, and corresponding points of all the other cells, these points form a homogeneous assemblage of single points, according to Bravais' admirable and important definition.³ The general problem of the homogeneous partition of space may be stated thus:—Given a homogeneous assemblage of single points, it is required to find every possible form of cell enclosing each of them subject to the condition that it is of the same shape and someways oriented for all. An interesting application of this problem is to find for a crystal (that is to say, a homogeneous assemblage of groups of chemical atoms) a homogeneous arrangement of partitional interfaces such that each cell contains all the atoms of one molecule. Unless we knew the exact geometrical configuration of the constituent parts of the group of atoms in the crystal, or crystalline molecule as we shall call it, we could not describe the partitional interfaces between one molecule and its neighbour.

Knowing as we do know for many crystals the exact geometrical character of the Bravais assemblage of corresponding points of its molecules, we could not be sure that any solution of the partitional problem we might choose to take would give a cell containing only the constituent parts of one molecule. For instance, in the case of quartz, of which the crystalline molecule is probably $3(\text{SiO}_4)$, a form of cell chosen at random might be such that it would enclose the silicon of one molecule with only some part of the oxygen belonging to it, and some of the oxygen belonging to a neighbouring molecule, leaving out some of its own oxygen, which would be enclosed in the cell of either that neighbour or of another neighbour or other neighbours.

§ 2. This will be better understood if we consider another illustration—a homogeneous assemblage of equal and similar trees planted close together in any regular geometrical order on a plane field either inclined or horizontal, so close together that roots of different trees interpenetrate in the ground, and branches and leaves in the air. To be perfectly homogeneous

¹ The scheme is now carried on by funds supplied by the County Councils of Cambridgeshire, the Isle of Ely, Essex, Norfolk, Northants, Leicestershire, Hunts, East and West Suffolk, and by a grant from the Board of Agriculture.

² A paper read before the Royal Society on January 18, by Lord Kelvin, P.R.S.

³ *Journal de l'École Polytechnique*, tome xix. cahier 33, pp. 1-128 (Paris, 1850), quoted and used in my "Mathematical and Physical Papers," vol. iii. art. 97, p. 400.

every root, every twig, and every leaf of any one tree must have equal and similar counterparts in every other tree. So far everything is natural, except, of course, the absolute homogeneity that our problem assumes; but now, to make a homogeneous assemblage of molecules in space, we must suppose plane above plane each homogeneously planted with trees at equal successive intervals of height. The interval between two planes may be so large as to allow a clear space above the highest plane of leaves of one plantation and below the lowest plane of the ends of roots in the plantation above. We shall not, however, limit ourselves to this case, and we shall suppose generally that leaves of one plantation intermingle with roots of the plantation above, always, however, subject to the condition of perfect homogeneity. Here, then, we have a truly wonderful problem of geometry—to enclose ideally each tree within a closed surface containing every twig, leaf, and rootlet belonging to it, and nothing belonging to any other tree, and to shape this surface so that it will coincide all round with portions of similar surfaces around neighbouring trees. Wonderful as it is, this is a perfectly easy problem if the trees are given, and if they fulfil the condition of being perfectly homogeneous.

In fact we may begin with the actual bounding surface of leaves, bark, and roots of each tree. Wherever there is a contact, whether with leaves, bark, or roots of neighbouring trees, the areas of contact form part of the required cell-surface. To complete the cell-surface we have only to swell out¹ from the untouched portions of surface of each tree homogeneously until the swelling portions of surface meet in the interstitial air spaces (for simplicity we are supposing the earth removed, and roots, as well as leaves and twigs, to be perfectly rigid). The wonderful cell-surface which we thus find is essentially a case of the tetrakaidekahedron, which I shall now describe for any possible homogeneous assemblage of points or molecules.

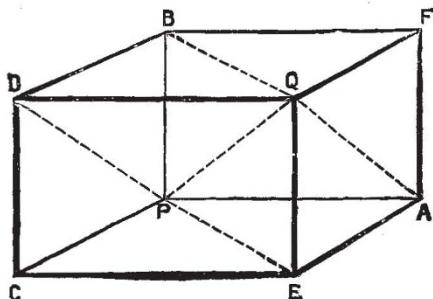
§ 3. We shall find that the form of cell essentially consists of fourteen walls, plane or not plane, generally not plane, of which eight are hexagonal and six quadrilateral; and with thirty-six edges, generally curves, of meeting between the walls; and twenty-four corners where three walls meet. A cell answering this description must of course be called a tetrakaidekahedron, unless we prefer to call it a fourteen-walled cell. Each wall is an interface between one cell and one of fourteen neighbours. Each of the thirty-six edges is a line common to three neighbours. Each of the twenty-four corners is a point common to four neighbours. The old-known parallelepipedal partitioning is merely a very special case in which there are four neighbours along every edge, and eight neighbours having a point in common at every corner. We shall see how to pass (§ 4) continuously from or to this singular case, to or from a tetrakaidekahedron differing infinitesimally from it; and, still continuously, to or from any or every possible tetrakaidekahedronal partitioning.

§ 4. To change from a parallelepipedal to a tetrakaidekahedronal cell, for one and the same homogeneous distribution of points, proceed thus:—Choose any one of the four body-diagonals of a parallelepiped and divide the parallelepiped into six tetrahedrons by three planes each through this diagonal, and one of the three pairs of parallel edges which intersect it in its two ends. Give now any purely translational motion to each of these six tetrahedrons. We have now the 4×6 corners of these tetrahedrons at twenty-four distinct points. These are the corners of a tetrakaidekahedron, such as that described generally in § 3. The two sets of six corners, which before the movement coincided in the two ends of the chosen diagonal, are now the corners of one pair of the hexagonal faces of the tetrakaidekahedron. When we look at the other twelve corners we see them as corners of other six hexagons, and of six parallelograms, grouped together as described in § 15 below. The movements of the six tetrahedrons may be such that the groups of six corners and of four corners are in fourteen planes as we shall see in § 14; but if they are made at random, none of the groups will be in a single plane. The fourteen faces, plane or not plane, of the tetrakaidekahedron are obtained by drawing arbitrarily any set of surfaces to constitute four of the hexagons and three of the quadrilaterals, with arbitrary curves for the edges between hexagon and hexagon and between hexagons and quadrilaterals, and then by drawing parallel equal and similar counterparts to these surfaces in the remaining four hexagonal

¹ Compare "Mathematical and Physical Papers," vol. iii. art. 97, § 5;

and three quadrilateral spaces in the manner more particularly explained in § 6 below. It is clear, or at all events I shall endeavour to make it clear by fuller explanations and illustrations below, that the figure thus constituted fulfils our definition (§ 1) of the most general form of cell fitted to the particular homogeneous assemblage of points corresponding to the parallelepiped with which we have commenced. This will be more easily understood in general, if we first consider the particular case of parallelepipedal partitioning, and of the deviations which, without altering its corners, we may arbitrarily make from a plane-faced parallelepiped, or which we may be compelled by the particular figure of the molecule to make.

§ 5. Consider, for example, one of the trees of § 2, or if you please a solid of less complex shape, which for brevity we shall



(FIG. 7, of § 9.)

call s , being one of a homogeneous assemblage. Let P be a point in unoccupied space (air, we shall call it for brevity), which, for simplicity we may suppose to be somewhere in the immediate neighbourhood of s , although it might really be anywhere far off among distant solids of the assemblage. Let PA, PB, PC be lines parallel to any three Bravais rows not in one plane, and let A, B, C be the nearest points corresponding to P in these lines. Complete a parallelepiped on the lines PA, PB, PC , and let QD, QE, QF be the edges parallel to them through the opposite corner Q . Because of the homogeneity of the assemblage, and because A, B, C, D, E, F, Q are points corresponding to P which is in air, each of those seven points is also in air. Draw any line through air from P to A and draw the lines of corresponding points from B to F , D to Q , and C to E . Do the same relatively to PB, AF, EQ, CD ; and again the same relatively to PC, AE, FQ, BD . These twelve lines are all in air, and they are the edges of our curved-faced parallelepiped. To describe its faces take points infinitely near to one another along the line PC (straight or curved as may be); and take the corresponding points in BD . Join these pairs of corresponding points by lines in air infinitely near to one another in succession. These lines give us the face $PBDC$. Corresponding points in AE, FQ , and corresponding lines between them give us the parallel face $AFQE$. Similarly we find the other two pairs of the parallel faces of the parallelepiped. If the solids touch one another anywhere, either at points or throughout finite areas, we are to reckon the interface between them as air in respect to our present rules.

§ 6. We have thus found the most general possible parallelepipedal partitioning for any given homogeneous assemblage of solids. Precisely similar rules give the corresponding result for any possible partitioning if we first choose the twenty-four corners of the tetrakaidekahedron by finding six tetrahedrons and giving them arbitrary translatory motions according to the rule of § 4. To make this clear it is only now necessary to remark that the four corners of each tetrahedron are essentially corresponding points, and that if one of them is

in air all of them are in air, whatever translatory motion we give to the tetrahedron.

§ 7. The transition from the parallelepiped to the tetrakaidekahedron described in § 4 will be now readily understood if we pause to consider the vastly simpler two-dimensional case of transition from a parallelogram to a hexagon. This is illustrated in Figs. 1 and 2; with heavy lines in each case for the sides of the hexagon, and light lines for the six of its diagonals which are sides of constructional triangles. The four diagrams show different relative positions in one plane of two equal homochirally similar triangles $ABC, A'B'C'$; oppositely oriented (that is to say, with corresponding lines $AB, A'B'$ parallel but in inverted directions). The hexagon $AC'BA'CB'$, obtained by joining A with B' and C' , B with C' and A' , and C with A' and B' , is clearly in each case a proper cell-figure for dividing plane space homogeneously according to the Bravais distribution of points defined by either triangle, or by putting the triangles together in any one of the three proper ways to make a parallelogram of them. The corresponding operation for three-dimensional space is described in § 4: and the proof which is obvious in two-dimensional space is clearly valid for space of three dimensions, and therefore the many words which would be required to give it formal demonstration are superfluous.

§ 8. The principle according to which we take arbitrary curved surfaces with arbitrary curved edges of intersection, for seven of the faces of our partitional tetrakaidekahedron, and the other seven correspondingly parallel to them, is illustrated in Figs. 3, 4, 5, and 6, where the corresponding thing is done for a partitional hexagon suited to the homogeneous division of a plane. In these diagrams the hexagon is for simplicity taken equilateral and equiangular. In drawing Fig. 3, three pieces of paper were cut, to the shapes kl, mn, uv . The piece kl was first placed in the position shown relatively to AC' , and a portion of the area of one cell to be given to a neighbour across the frontier $C'A$ on one side was marked off. It was then placed

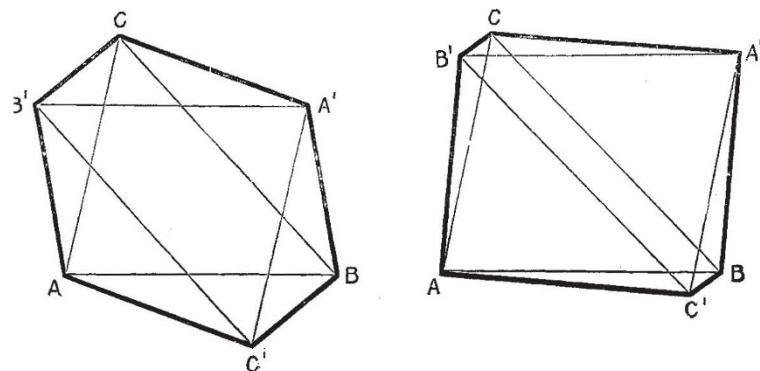


FIG. 1.

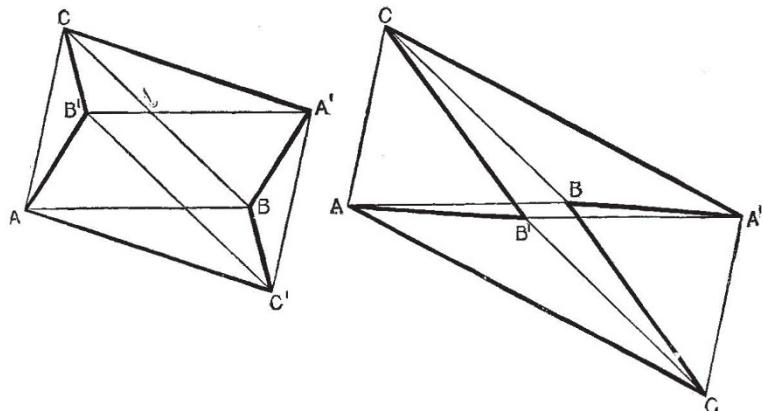


FIG. 2.

in the position shown relatively to $A'C$ and the equivalent portion to be taken from a neighbour on the other side was marked. Corresponding give-and-take delimitations were

marked on the frontiers $C'B$ and $B'C$, according to the form mn ; and on the frontiers BA' , AB' , according to the form uv . Fig. 4 was drawn on the same plan but with one pair of frontiers left as straight lines, and the two other pairs drawn by aid of two paper templets. It would be

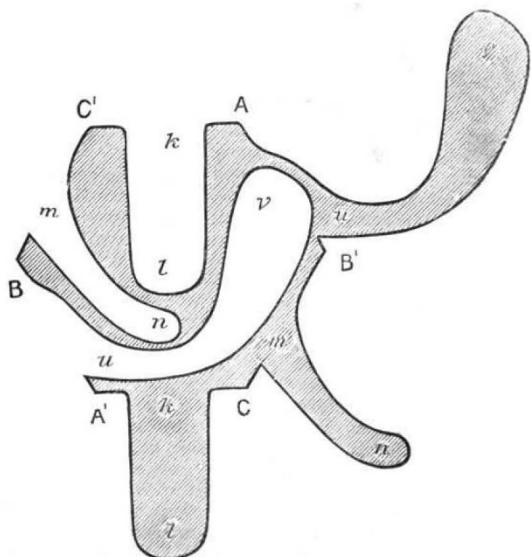


FIG. 3.

easy, but not worth the trouble, to cut out a large number of pieces of brass of the shapes shown in these diagrams and to show them fitted together like the pieces of a dissected map. Figs. 5 and 6 are drawn on the same principle; Fig. 6 showing,

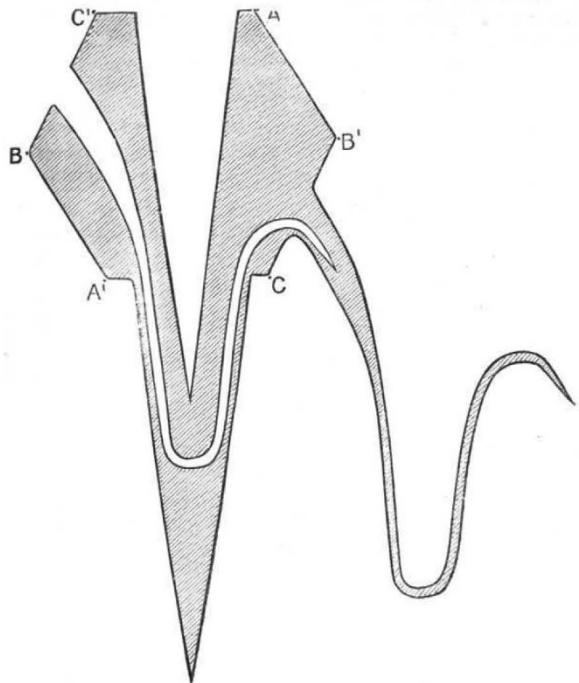


FIG. 4.

on a reduced scale, the result of putting pieces together precisely equal and similar to that shown in Fig. 5. In these diagrams, unlike the cases represented in Figs. 3 and 4, the primitive hexagon is, as shown clearly in Fig. 5, divided into isolated parts. But if we are dealing with homogeneous division

of solid space, the separating channels shown in Fig. 5 might be sections, by the plane of the drawing, of perforations through

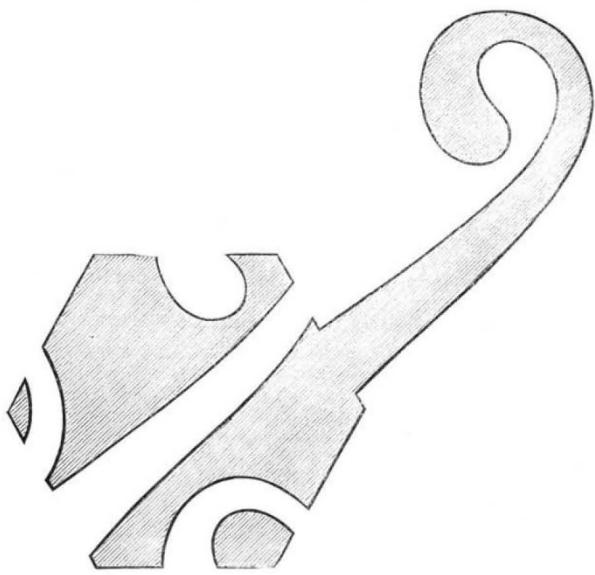


FIG. 5.

the matter of one cell produced by the penetration of matter, rootlets for example, from neighbouring cells.

§ 9. Corresponding to the three ways by which two triangles can be put together to make a parallelogram, there are seven,

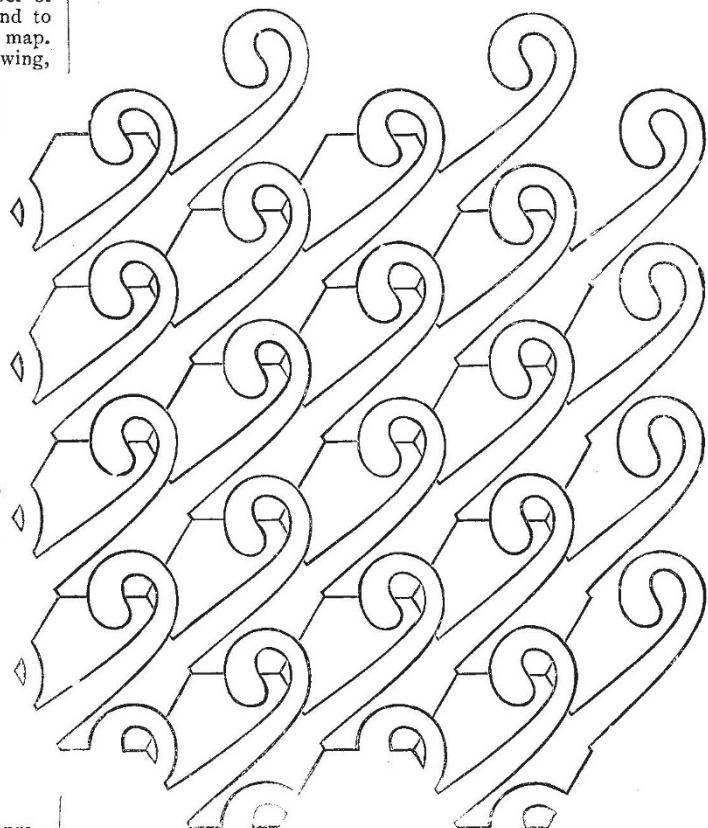


FIG. 6.

and only seven, ways in which the six tetrahedrons of § 4 can be put together to make a parallelepiped, in positions parallel to

those which they had in the original parallelepiped. To see this, remark first that among the thirty-six edges of the six tetrahedrons seven different lengths are found which are respectively equal to the three lengths of edges (three quartets of equal parallels); the three lengths of face-diagonals having ends in P or Q (three pairs of equal parallels); and the length of the chosen body-diagonal PQ. (Any one of these seven is, of course, determinable from the other six if given.)

In the diagram, Fig. 7, full lines show the edges of the primitive parallelepiped, and dotted lines show the body-diagonal PQ and two pairs of the face-diagonals, the other pair of face-diagonals (PF, QC), not being marked on the diagram to avoid confusion. Thus, the diagram shows, in the parallelograms QDPA and QEPA, two of the three cutting planes by which it is divided into six tetrahedrons, and it so shows also two of the six tetrahedrons, QPDB and QPEA. The lengths QP, QD, QE, QF are found in the edges of every one of the six tetrahedrons, the two other edges of each being of two of the three lengths QA, QB,

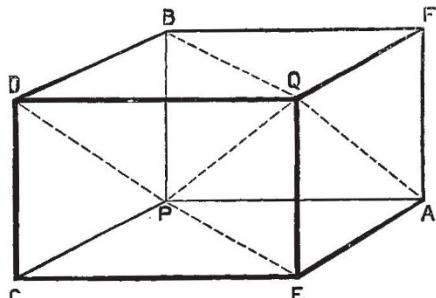


FIG. 7.

QC. The six tetrahedrons may be taken in order of three pairs having edges of lengths respectively equal to QB and QC, QC and QA, QA and QB. It is the third of these pairs that is shown in Fig. 7. Remark now that the sum of the six angles of the six tetrahedrons at the edge equal to any one of the lengths QP, QD, QE, QF is four right angles. Remark also that the sum of the four angles at the edge of length QA in the two pairs of tetrahedrons in which the length QA is found is four right angles, and the same with reference to QB and QC. Remark lastly that the two tetrahedrons of each pair are equal and dichirally¹ similar, or enantiomorphs as such figures have been called by German writers.

(To be continued.)

UNIVERSITY AND EDUCATIONAL INTELLIGENCE.

CAMBRIDGE.—The following is the speech delivered by the Public Orator (Dr. Sandys) on March 1, in presenting for the degree of LL.D. *honoris causa* the Right Hon. the Earl of Kintore, G.C.M.G., M.A. Trinity, Governor of South Australia:—“Quam libenter salutamus ex alumnis nostris unum, qui Britanniae in parte Septentrionali Collegii florentissimi Aberdoniensis a conditore oriundus, inter colonias nostras Australes Academiam Adelaidensem, quam inter filias nostras non sine superbia numeramus, sua sub tutela positam esse gloriatur. Ibi provinciae maximae tota Gallia, tota Germania, plusquam quadruplo latius patenti praepositus, regionem tam immensam audacter peragravit, itineris tanti socium insignem nactus medicum Cantabrigiensem, cuius ipsum nomen Caledoniae suae castellum in memoriam revocat. Quid commemorem proconsulim nostri ductu plusquam quadraginta dies inter loca deserta atque arida fortiter toleratos, rerumque naturae soliditudes reconditas feliciter reclusas? Quid (ne maiora dicam) etiam talpae genus novum, quod *notoryctes* nominatur, e latebris suis in lucem protractum? Quid eiusdem auspicio et imperio etiam beluae antique, quae *diprotodon* vocatur, reliquias ingentes saeculo nostro denuo patefactas? Ipsum Sancti Georgii inter equites illustiores numeratum, non draconem fabulosum vi et armis domuisse dixerim, sed monstrorum haud minus horrendorum vestigia immania sumptu et labore maximo detegimus curavisse. Talium virorum auxilio non modo imperii Britannici provinciae remotissimae vinculis

¹ A pair of gloves are dichirally similar, or enantiomorphs. Equal and similar right-handed gloves are chirally similar.

artioribus nobiscum consociantur, sed etiam scientiarum fines nostris a filiis totiens propagati per spatio indies latiora extenduntur. Duco ad vos scientiarum patronum illustrem, provinciae maxime et proconsulē et investigatorem indefessum, virum et suo et fratri sui nomine nobis coniunctissimum, Algernon Keith-Falconer, Comitem de Kintore.” Lord Kintore was accompanied on his adventurous journey of 2000 miles from Port Darwin to Adelaide by Dr. Edward Charles Stirling, Trinity Lecturer on Physiology in the University of Adelaide.

The following is the speech delivered by Dr. Sandys, on March 6, in presenting for the degree of Sc.D. Dr. S. Ramon y Cajal, Professor of Histology and Pathological Anatomy in the University of Madrid:—“Hodie laudis genus novum libenter auspicati, Hispanae gentis civem nunc primum salutamus. Salutamus virum de physiologia scientia optime meritum, qui inter flumen Hiberum montesque Pyrenaeos duo et quadraginta abhinc annos natus et fluminis eiusdem in ipsis Caesaraugustae educatus, primum ibidem, deinde Valentiae, deinceps Barcelonae munere Academicum functus, tot honorum spatio feliciter decurso, nunc denique in urbe, quod gentis totius caput est, histologiae scientiam praecelle proficit. Fere decem abhinc annos professoris munus Valentiae auspicatus, fore auguratus est, ut intra annos decem studiorum suorum in honorem etiam inter exteris gentes nomen suum notesceret. Non felicit augurium; etenim nuper etiam nostras ad oras a Societate Regia Londinensi honoris causa vocatus, muneri oratorio, virorum insignium nominibus iampridem ornato, in hunc annum destinatus est. Omitto opera eius maiora de histologia et de anatomia conscripta; praetereo etiam opuscula eiusdem quadraginta intra lustra duo in lucem missa; haec enim omnia ad ipsa scientiae penetralia pertinent. Quid vero dicam de artificio pulcherrimo quo primum auri, deinde argenti ope, in corpore humano fila quaesdam tenuissima sensibus motibusque ministrantia per ambages suas inextricabiles aliquatenus explorari poterant? In artificio illo argenti usum, inter Italos olim inventum, inter Hispanos ab hoc viro in melius mutatum et ad exitum feliciorum perductum esse constat. Si poeta quidam Romanus regione in eadem genitus, si Valerius Martialis, inquam, qui expertus didicit fere nihil in vita sine argento possi perfici, hodie ipse adisset, procul dubio popularem suum verbis suis paululum mutatis non sine superbia appellaret:—

‘Vir Celtiberis non tacende gentibus,
Nostraequa laus Hispaniae...’
Te nostri Hiberi ipsis gloriabitur,
Nec me tacebit Bilbilis.’

Martial, i. 49, 1-2; 61, 11-12.

Duco ad vos virum et in Hispania et inter exteris gentes laudem merito adeptum, histologiae professorem insignem, Santiago Ramon y Cajal.”

Dr. J. B. Bradbury, Physician to Addenbrooke's Hospital and Linacre Lecturer at St. John's College, has been elected to the Downing Professorship of Medicine, vacant by the resignation of Dr. Latham.

Mr. P. H. Cowell, Senior Wrangler in 1892, has been elected to the Isaac Newton Studentship in Physical Astronomy and Optics.

The Arnold Gerstenberg Studentship, worth about £90 a year, will be awarded next May, on the results of an examination in Psychology and Logic, commencing on May 21. Candidates must have obtained honours in one part of the Natural Sciences Tripos, and have commenced residence earlier than April 1888. The student elected must devote himself to moral or mental philosophy.

THE Queen has signified her approval of the appointment of the following Commissioners to consider what are the best methods of establishing a well-organised system of secondary education in England, taking into account existing deficiencies, and having regard to such local sources of revenue from endowment or otherwise as are available or may be made available for this purpose, and to make recommendations accordingly:—The Right Hon. J. Bryce, M.P.; the Right Hon. Sir J. T. Hibbert, M.P.; Mr. Henry Hobhouse, M.P.; Mr. H. Llewellyn Smith; Prof. R. C. Jebb, M.P.; Mrs. Henry Sidgwick; Mr. M. E. Sadler; the Rev. A. M. Fairbairn; the Hon. E. Lyttelton; Mrs. Bryant, D.Sc.; Dr. R. Wormell; the Very Rev. E. C. Maclure; Mr. George J. Cockburn; Mr. J. H. Yoxall; Sir Henry Roscoe, M.P., F.R.S.; Lady Frederick Cavendish; Mr. C. Fenwick, M.P.