LIGHT-WAVES AND THEIR APPLICATION TO METROLOGY.

EVERY accurate measurement of a physical quantity depends ultimately upon a measurement of length or of angle; and it will readily be admitted that no effort should be spared to make it possible to attain the utmost limit of precision in these fundamental quantities. At present, lengths are measured by the microscope, and angles by the telescope; and the extraordinary degree of accuracy already attained by the use of these instruments depends entirely on the properties of their optical parts in their relation to light-waves; so that, in fact, light-waves are now the most convenient and universally employed means we possess for making accurate measurements. It can readily be shown that this high degree of accuracy is especially due to the extreme minuteness of these waves.

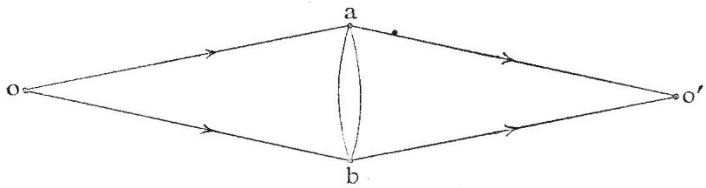

FIG. 1.

Thus it is well known that the image of a luminous point consists of a series of concentric coloured rings surrounding a bright central disc which is smaller the smaller the ratio of the wave-length of the light to the diameter of the objective employed. In fact, it can be shown that the radius of the bright central disc contains as many wave-lengths as the distance of the image from the objective contains the diameter of the objective. Thus in a telescope twenty diameters long, the diameter of the bright disc is forty wave-lengths or 0·02 mm. If the image be magnified by increasing its distance from the objective, or otherwise, these diffraction rings are magnified in the same proportion; so that nothing is gained thereby in *distinctness*, beyond the point where the rings are just large enough to be visible. But, were it not for the inevitable loss of light, it would be advantageous for *measurements of position* to increase the magnification much further.

This can be accomplished by an extremely useful instrument which has been misnamed the "interferential refractometer." It will be interesting to note that notwithstanding the apparent difference in form, this apparatus, when used as a measuring instrument, differs in no essential particular from the microscope or the telescope, or (what is perhaps a trifle unexpected) the spectroscope; and it is possible to change any one of these instruments into the other by unimportant modifications.

Thus, let *o*, Fig. 1, be a source of light, *a b* a lens which forms an image of *o* at *o′*. The operation of the lens, when used to distinguish minute objects, depends upon the accuracy with which all its parts contribute to make the elementary waves reach the focus in the same phase of vibration; but to determine the position of *o* with respect to *a b*, this is not at all necessary; and in fact, if we disregard the possible inconvenience due to the dissimilarity between the phenomenon observed and the object whose position is to be measured, it would be as well to entirely annul the central portions of the lens, leaving only an external annular ring, or better still, only two small portions at opposite ends of a diameter.

This involves no sacrifice of accuracy, but on the contrary a very considerable gain; for it is now possible to increase the size of the interference fringes up to any desired limit without diminishing the intensity of the light, the result being the same as could be obtained with a perfect microscope of unlimited magnifying power with a source of unlimited intensity.

For this purpose the two small portions to which the lens is reduced are replaced by plane mirrors or prisms, whose office is simply to bring the two interfering pencils into coincidence. Further, the pencils, instead of starting from a point or a line, may be separated by a plane transparent surface; and a second similar surface may be used to reunite the pencils after reflection. Thus the telescope or microscope will have been converted into a refractometer. The exact nature of the analogy will be apparent by a comparison of Figs. 1 and 2.

It may be assumed that under the most favourable circumstances the utmost attainable limit of accuracy of a setting of the cross-hair of a microscope on a fine ruled line is about $\frac{1}{20}$ of a micron. Now, it is usually admitted that the middle point of an interference fringe, if it be sufficiently broad and clear, can be determined within about $\frac{1}{30}$ of the width of a fringe. In the refractometer this would mean only $\frac{1}{60}$ of a light-wave, or about 0·01μ, from which it would follow that the refractometer is about five times as accurate as the microscope. But a number of trials with the form of refractometer shown in Fig. 8 gave as the mean error of a series of ten observations:

Fr.	Fr.	Fr.
Morley 0·0056 ...	Nicholson 0·0059 ...	X 0·0110

The third observer had no previous practice in this kind of measurement.

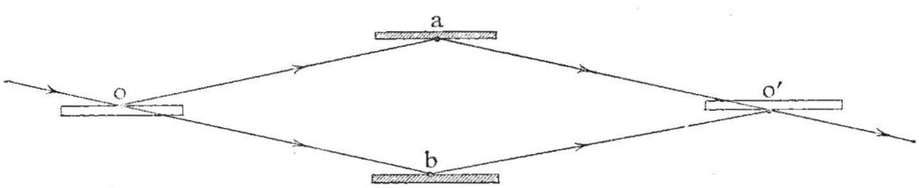

FIG. 2.

It is evident from these results that $\frac{1}{30}$ of a fringe is too large an estimate of the average error of a setting, and that it is, in fact, less than 0·01 of a fringe, corresponding to an error in distance of about 0.003μ.

For angular measurements the microscope is replaced by the telescope.

Fig. 3 represents a disposition sometimes adopted for observing minute angular displacements of the mirror *d c*; the light starts from *o*, is reflected by the plane parallel glass plate *p* to the objective *a b* of a telescope, whence the now parallel rays proceed to the mirror *c d*. Thence they retrace their path to the plate *p*, through which they are transmitted, forming an image of the source at *o′*, which is viewed through the eyepiece.

Fig. 4 is the exact analogue in the form of a refractometer; and Fig. 5, though slightly different in aspect, is still essentially the same instrument. The path of the rays is $o\,p\,a\,c\,a\,p\,o'$ for one of the pencils, and $o\,p\,b\,d\,b\,p\,o'$ for the other.

From considerations quite analogous to those employed in the former case, it can be shown that the

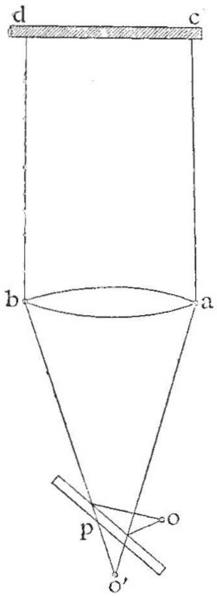

FIG. 3.

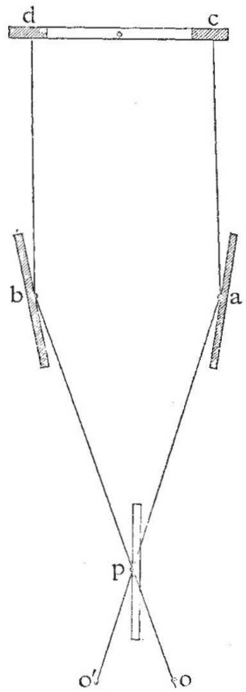

FIG. 4.

limit of accuracy attainable in the estimations of angles involves an error of about one-fifth of the angle subtended by a light wave at a distance equal to the diameter of the objective. This is halved by the fact that the angular motion of the beam is twice that of the mirror; so that

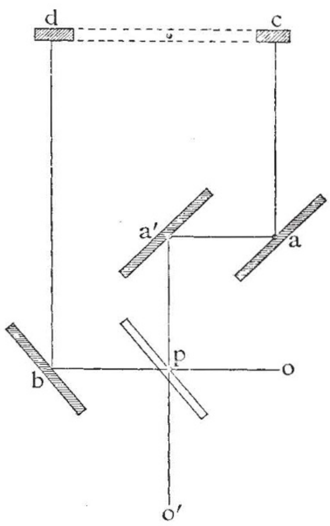

FIG 5.

with a telescope of 10 cm. aperture the limit of accuracy may be estimated at $\frac{1}{2000000}$, or say 0.1″. But taking 0.01 fr. as the smallest perceptible displacement of the mirrors $c\ d$, the corresponding angle of rotation of the line $c\ d$ (10 cm. long) would be only $\frac{1}{20000000}$, or say 0.01″.[1]

It is not at first evident that there is any relation between the refractometer and the spectroscope. A comparison of Fig. 6 and Fig. 7 shows, however, that there is a strict analogy. Fig. 6 represents a disposition sometimes adopted to observe the spectrum by means of a concave grating, and Fig. 7, with unimportant modifications, is the arrangement actually employed in the analysis of radiations by means of their "visibility curves," as will be explained below.

Exactly as in the case of mirrors and lenses, we may here, too, sacrifice "resolution" and "definition" by using only the extreme portions of the surface, with an actual gain in "accuracy." To compare numbers, it appears that the average error in the comparison of

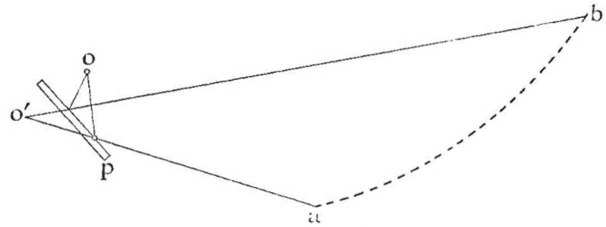

FIG. 6.

wave-lengths by a grating with 250,000 lines is about one part in half-a-million. With this number of waves in the difference of path of two interfering pencils, the corresponding error in the refractometer observations are of the order of one twenty-millionth.

The name "interferential refractometer" seems rather inappropriate to an instrument which has so many important applications beside the measurement of indices of refraction; but as it has been sanctioned by long usage it will be retained.

Among the many forms of the apparatus which have been rendered classic by the works of Arago, Fresnel, Fizeau, Jamin, and Mascart, and which are so admirably adapted to the work for which they were designed, there are none which are not open to serious objections when applied to the solution of such problems as the measure-

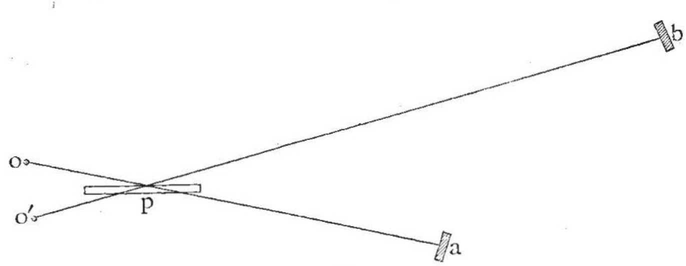

FIG. 7.

ment of lengths and angles, for the analysis of the constitution of the light of spectral lines, and especially for the determination of wave-lengths in absolute measure. For these, the form of instrument shown in Fig. 8 has many important advantages, among which the following may be mentioned:—It is simple in construction, and is easily adjusted; it may be used with a broad luminous

1 In the use of the revolving mirror as in galvanometers, gravity and torsion balances, &c., the accuracy can be increased by enlarging the surface of the mirror; but the moment of inertia is thereby increased, and in greater proportion. But in the refractometer the mirrors $c\ d$ may be made insignificantly small, and yet, with the same distance between the outer edges, the accuracy may be increased at least tenfold. It is important to note that any linear motion of the line joining the mirrors, or even a rotation about this line, has no effect on the fringes. It seems probable that this form of instrument may be of service in such problems as the measurement of the moon's attraction, constant of gravitation, variations of the vertical, &c.

surface as source of light; the pencils may be separated as far as desired; its range of difference of path between the interfering pencils is unlimited; and when properly adjusted the position of the interference fringes is perfectly definite, so that there is no uncertainty on account of parallax, and no difficulty in counting the number of fringes passing a given point. Finally, it may be added, that this is probably the only form of instrument which permits the use of white light (and consequently of the identification of the fringes) in the determination of the position or inclination of a surface without risk of disturbance due to contact or close approximation.

As shown in Fig. 8, the refractometer consists essentially of a plane parallel plate of optical glass G_1 and two plane mirrors M_1 M_2. The beam of light to be examined falls on the plate G_1 at an angle, usually 45°, part being reflected and part transmitted.[1] The reflected portion is returned by the mirror M_2, and passes back through the inclined plate. The transmitted portion is returned by the mirror M_1, and is reflected by the inclined plate, and from

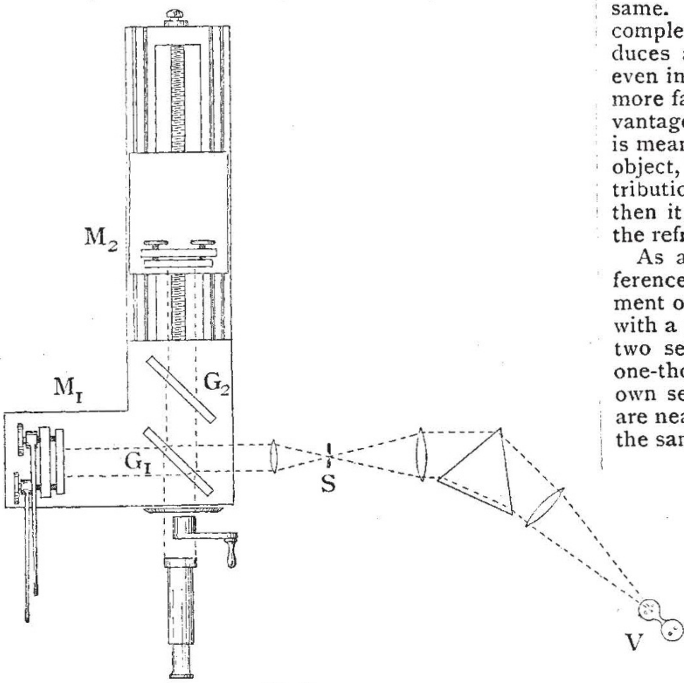

FIG. 8.

this point it coincides with the other beam, so that the two are in condition to produce interference fringes.[2]

A little consideration will show that this arrangement is in all respects equivalent to an air-film or plate between two plane surfaces. If the virtual distance between these surfaces is small, white light may be employed, and interference fringes may be observed similar in all respects to those between two plates of glass pressed nearly into contact.[3]

[1] The front surface of the plate G_1 is lightly coated with silver. The light which leaves the refractometer is a maximum where the thickness of the silver film is such that the intensities of the transmitted and reflected portions are equal. The silvering has another important advantage in diminishing the relative intensity of the light reflected from the other surface; and for this reason the thickness of the film may be advantageously increased, which permits also a more uniform surface. The ultimate ratio of intensities of the two pencils is not affected, for what is lost by transmission on entering the plate is made up by reflection on leaving it.

[2] One of the beams has to pass twice through the thickness of the glass plate G_1, and in order to equalise the two paths, a similar plate G_2 is introduced in the path of the other beam.

[3] If the plate G_1 be not silvered, the colours follow the same order as those of Newton's rings, but if the silvering be sufficiently heavy, the colours are complementary; this, if the plates G_1 and G_2 are exactly equal and parallel. Otherwise, the excess of path in glass of one of the pencils disturbs the order of colours by the effect of achromatism due to the dispersion of the glass, as was first pointed out by Cornu.

If, however, the distance exceeds a few wave-lengths, monochromatic light must be employed. In this case the fringes are in general invisible, unless they be viewed through a small aperture. If, however, the two surfaces are very accurately parallel, the fringes are always distinct, and it follows from the symmetry of the conditions that they are concentric rings. Their diameters increase as the square root of the order of the ring.

These rings are not formed at the surface of the mirrors (as is the case when the distance between them is small), but are perfectly distinct when the eye or the observing telescope is focussed for parallel rays.

In the preceding comparison between the refractometer and the telescope, microscope, or spectroscope, the "accuracy" has been increased at the expense of "definition." When, however, the object viewed is beyond the "limit of resolution" of the instrument, its form and distribution of light can no longer be inferred from that of the image. Thus, if the object be a disc, a triangle, or a double star, the appearance in the telescope is the same. Similarly in the spectroscope, a source of great complexity cannot be distinguished from one which produces a single spectral line. So that for such objects, even in the ordinary sense of the word "definition," the more familiar optical instruments cannot claim any advantage over the refractometer; but if by "definition" is meant not the actual resemblance of the image to the object, but the accuracy with which the form or the distribution of light in a minute source may be inferred, then it can be shown that all the advantage rests with the refractometer.

As an illustration of such an application of interference methods, let us consider the celebrated experiment of Fizeau, in which Newton's rings are observed with a sodium flame as source. The light, consisting of two separate systems of radiations differing by about one-thousandth in wave-length, each system produces its own series of interference fringes. When the surfaces are nearly in contact, the difference of path is very nearly the same for both systems, and the fringes coincide, and the clearness is a maximum. When, however, the difference of path reaches about 500 waves for one of the systems, it is a half wave more for the other; and the maxima of intensity of the one coincide with the minima of the other; hence at this point the fringes are faintest. But when the difference of path of the first system is about 1000 waves, it is a whole wave more for the second, and the fringes coinciding, there is again a maximum of distinctness. M. Fizeau has counted 52 such periods, corresponding roughly to a difference of path of 50,000 waves.

Suppose, now, that this double line were so close that it could not be resolved by the spectroscope; then from the evidence furnished by the variations in distinctness of the interference fringes as the difference of path increases, the duplicity of the line could be readily detected. But beside this, it can be shown that the relative intensities of the components, their distance apart, and even the distribution of intensities within the component lines can be inferred.

Thus it has been shown (*Philosophical Magazine* for September, 1892) that among some twenty radiations which were examined (though all give simple lines in the spectrum) the great majority are shown to be highly complex. Thus, the red hydrogen line is a double whose components have the intensity ratio 7 : 10, and whose distance is about a fiftieth of the interval between the sodium lines. Each component of the yellow sodium lines is itself a double whose components are in the ratio 7 : 10, and whose distance is about one-hundredth of that between the principal components. Thallium gives a double line whose components are in the ratio 1 : 2, at a distance of about a fiftieth of that of the sodium lines, while each component has a small companion whose intensity is about a fifth of that of the principal lines, at a distance of about one three-hundredth of that of the sodium lines. The green mercury line is made up of a group of five or six lines, the strongest of which is itself double (or perhaps triple) the distance of the components, being less than a five-hundredth part of that between the sodium lines.

These distances, small as they are, can be measured within about a twentieth part, so that by this means it is possible to detect a change of wave-length corresponding to the ten-thousandth part of that between the two sodium lines.

The red line of cadmium is the simplest of all the radiations thus far examined, consisting of a single narrow line whose intensity falls off symmetrically according to an exponential law, its width (at the points where its intensity is reduced to half its maximum value) being only 0·002 (D_1-D_2). The green and the blue cadmium lines are also comparatively simple, and all three of these lines give interference fringes clearly visible at a difference of path of 100 mm., and under appropriate conditions they all satisfy the requisites for a definite and inalterable standard of length.

The most important of these conditions is that the radiating vapour be so rare that the molecules may vibrate freely; in other words, that the time occupied in the collisions between the molecules be so short relatively to that of the free path, that its influence in disturbing the free vibration may be neglected. Experience shows that in general this limit corresponds to a pressure of one or two thousandths of an atmosphere.

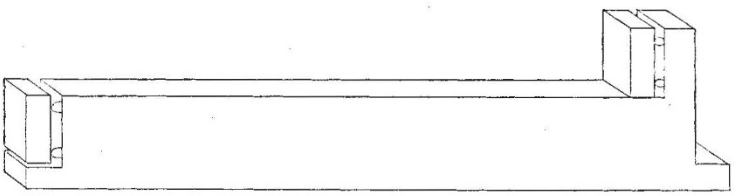

FIG. 9.

It may be noted that at atmospheric pressure—even when the radiating substance is introduced in quantity barely sufficient to colour a Bunsen flame—the greatest difference of path attainable is only one or two centimetres, whereas with mercury vapour in a vacuum tube interference fringes have been observed with a difference of path of 47 centimetres, or about 850,000 waves.

In order to make any practical use of these minute quantities for standards of length, it is necessary to employ an intermediate standard, such as that shown in Fig. 9, consisting of a bronze bar carrying two plane-parallel glasses, silvered in front, the distance between which can be compared on the one hand with the fundamental standard in actual use—the metre or the yard—and on the other with the length of a light-wave.

The former process is accomplished by moving the standard (whose length it is convenient to take at 10 centimetres) ten times through its own length, the coincidence and the parallelism of the surfaces being controlled at every step by the interference fringes in white light formed between these surfaces and that of the *reference plane* (the virtual image of the mirror MM in G_1, Fig. 8). The position of a fiducial mark on this standard is compared by means of two micrometer microscopes with the lines defining the standard metre at the first and last steps.

In the second process the only difficulty encountered is due to the very great disproportion between the length of a wave and that of the 10 centimetre standard, and the consequent difficulty in keeping the correct count of the very large number of waves which pass as the reference plane is moved from one surface to the other.

This problem has been solved in the following manner. Nine standards were constructed similar in all respects to that of ten centimetres, save that each succeeding one was half as long as the preceding. The last of the series is thus approximately 0·39 mm. long, corresponding to a difference of path of 0·78 mm. The number of waves in this distance in red cadmium light is 1212 plus a fraction, which is corrected by direct observation of the difference of phase of the circular fringes on the upper and the lower (front and rear) surfaces of the standard. This verification is also made with the green and the blue radiations.

It is important to note that the measurement of these fractions alone is sufficient to fix the whole number, even if there be an uncertainty of several waves. Thus, the relative wave-length of the three radiations being known, the number of green and of blue waves corresponding to the observed number of red waves can be readily calculated, as is shown in the following table:—

Wave-length.	Number of Waves.	
	Observed.	Calculated.
μ		
0·64389	1212·34	1212·34
0·50863	1534·76	1534·76
0·48000	1626·16	1626·13

If the whole number assumed as the basis of this calculation were in error by one or more waves, there would be no correspondence between the observed and the calculated fractions. The length of this standard and the succeeding one are now compared as follows:—The two standards being placed side by side in the refractometer II on a fixed support, and I on a movable carriage, the reference plane (R, Fig. 10) is moved until it coincides with A, the lower (or front) surface of II, and the interference fringes in white light are adjusted to the proper distance and inclination by adjusting the inclination of the reference plane. Next, C, the lower surface of I is brought to coincidence with the reference plane, and similarly adjusted, and then all the adjusting pieces are released from the carriages, so that these rest undisturbed on the ways. This completes the *first stage* of the comparison.

Second Stage.—The reference plane R′ is now moved back till it coincides with D, the upper surface of I and

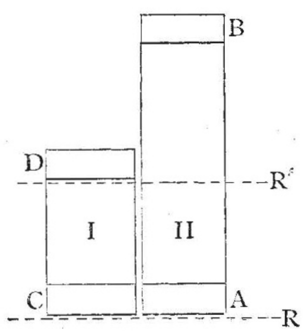

FIG. 10.

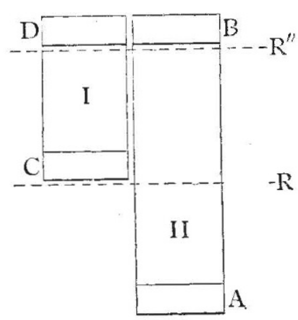

FIG. 11.

the adjustment of the interference fringes carried out as before.

Third Stage.—The standard, I, is moved back till its lower surface (C, Fig. 11) once more coincides with the reference plane, R′, and its inclination is again adjusted by the interference fringes.

Fourth Stage.—The reference plane is finally moved back till it coincides with D, the upper surface of I, and its inclination is again adjusted. If now the standard II is just twice as long as I, the fringes will appear simultaneously on *both* upper surfaces, D and B.

The adjustment of the length of the standards is usually made to within a few waves, and the outstanding difference is measured by a compensating device.

This is furnished by the rotation of the compensating plate, G_2, Fig. 8. The plate is held in a metal frame which is supported at one end by a short thick rod firmly fixed to the bed. At the other end a delicate spiral spring is attached; the tension of the spring *twists* the rod through a minute angle, and thus alters the thickness of glass traversed by one of the interfering pencils. The other end of the spring is attached to a flexible cord passing over a pulley which is connected with a graduated circle. The angular motion is thus reduced about 100,000 times, and yet the proportionality is preserved.

Suppose the outstanding difference is ϵ a fraction of a wave-length known to within one or two tenths, then

$$II = 2I + \epsilon$$

and consequently the number of red waves should be $2 \times 1212{\cdot}34 + \epsilon$. This fraction is corrected by direct observation, as in the case of standard I, and the same control is furnished by the concordance of the results for the three colours; so that an error in the whole number of waves is well-nigh impossible.

The process of comparison and correction is repeated in the same way with the other standards, until we finally arrive at the whole number of waves and approximate fraction in the 10 centimetre standard. Up to this point the question of temperature and pressure is of minor importance, for the comparisons and corrections are made while both standards are under the same conditions; and being all made of the same material, it is sufficient to know that the temperature is the same for both. In the measurement of the fractions on the 10 centimetre standard, however, it is necessary to know the temperature and pressure with all possible accuracy, and it is also important that the comparison of this standard with the metre should be made, as nearly as may be, under the same conditions as that of the determination of the standard in light-waves.

The author having been honoured by an invitation from the International Bureau of Weights and Measures to undertake a series of experiments upon the lines here briefly indicated, the necessary apparatus was constructed in America, and shortly afterward installed in the Bureau International des Poids et Mesures at Sèvres.

Two complete and entirely independent determinations were made. These have not yet been completely reduced, but an approximate calculation gives for the number of waves of red light in one metre of air at 15° C. and 76 mm.

1st series	1553163·6
2nd series	1553164·6

The difference from the mean is half a wave, or about one fourth of a micron.[1]

From these results it follows that we have at hand a means of comparing the fundamental standard of length with a natural unit—the length of a light-wave—with about the same order of accuracy as is at present possible in the comparison of two metre bars.

This unit depends only on the properties of the vibrating atoms of the radiating substance, and of the luminiferous ether, and is probably one of the least changeable quantities in the material universe.

If, therefore, the metre and all its copies were lost or destroyed, they could be replaced by new ones, which would not differ from the originals more than do these among themselves. While such a simultaneous destruction is practically impossible, it is by no means sure that, notwithstanding all the elaborate precautions which have been taken to insure permanency, there may not be slow molecular changes going on in all the standards; changes which it would be impossible to detect except by some such method as that which is here presented

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1 The error in the determination of the *relative* wave-lengths of the three radiations is very much smaller, probably less than one twenty-millionth.

FURTHER NOTES AND OBSERVATIONS UPON THE INSTINCTS OF SOME COMMON ENGLISH SPIDERS.

MANY of what would otherwise be most interesting anecdotes respecting the habits of spiders have been related by persons who, being unacquainted with the immense number of "kinds" of this group that there are in England, not to mention the rest of the world, have apparently considered that all needful information in the way of the animal's identity has been supplied by the simple statement that it is a spider.

Such anecdotes have of course a certain value, inasmuch as they furnish some general information respecting the instincts of the class as a whole. But to those who are anxious to compare together the instincts of individuals of the same or different species, genera, and families, who are anxious to acquire in short some little knowledge of the comparative psychology of the group, they are distressingly incomplete.

To remedy in part these deficiencies, to verify the experiments of others, and to make fresh observations upon some points that are open to dispute, I took the opportunity, during a recent visit to North Cornwall, of compiling a set of notes upon the habits of some of the commonest spiders in the neighbourhood.

In the following paper, which is based upon these notes, I have added some brief accounts of the webs, habitats, or general appearance of the spiders, so that those persons who are not acquainted with the animal by name, may yet, with but little trouble, ascertain what the species are that are under discussion.

Agalena labyrinthica.—This spider may be looked upon as the country cousin of the common house spider, *Tegenaria atrica*, which being essentially a lover of bricks and mortar, is found in lofts, disused rooms, &c., where it spins in corners and other angles a horizontal, triangular sheet of web, a familiar structure which must be associated in all minds with the word cobweb.

The snares of *Agalena* are essentially like those of *Tegenaria*, consisting of a short silken tube or funnel, one end of which is buried in the bush that the spider has chosen to build in, while the other opens upon, and is continuous with, a widely extended horizontal sheet composed of fine closely woven silken threads. During the daytime the spider, if cautiously approached, may usually be seen squatting at the entrance of her funnel. She is, however, remarkably wary, and this, coupled with her equally remarkable agility, makes the task of capturing her by no means an easy one. For, by means of the further open extremity of the tube, she can make her escape into the bush beyond. Wherever I have had an opportunity of observing this spider, I have noticed that it appears to have a special liking for furze bushes; and it seems reasonable to suppose that this selection of so prickly a site for the building saves the young and also the nest from destruction at the hands, or rather the noses and legs, of cattle.

Upon examining the *débris* of prey, with which the orifice of the funnel was usually strewn, I was surprised to find that it consisted more often of the remains of bees than of flies—generally, indeed, the limbs, wings, &c., were those of some species of Bumble-bees (*Bombus*). Being curious to see how the spider would manœuvre to overcome so redoubtable an adversary, I captured upon one occasion a small specimen of a bumble-bee and