

Satisfactory reasons have still to be given for deserting the quaternion highway. The asserted weakness of Hamilton's calculus, as contrasted with the implied strength of its rivals, has still to be disclosed.

With a view to bring us all to one mind, Prof. Alfred Lodge suggests (NATURE, June 29) that the quaternion be regarded as the difference of its vector and scalar parts, so that the square of a vector becomes *minus* the scalar product of a vector into itself. It is not easy to see what ultimate advantage this change of sign would bring. The most obvious disadvantage would be that it would to a large extent render Hamilton's and Tait's classical treatises of little service to the student. Moreover, it would bring in the quaternion in a very artificial manner, as a kind of after-thought, so to speak; it would, I think, confuse the beginner by forbidding him to make use of powers of vectors in the way generally familiar in analysis; it would accentuate the importance of the product at the expense of the quotient of vectors; and it would tend to obscure the significance of the versor. I am afraid it is too much to ask of any who have got accustomed to the quaternion method to introduce confusion by such a change of sign. Up to a certain point, and along certain lines, Gibbs's and Heaviside's systems lead to results identical with those obtained by quaternions. It has not been shown that they lead to these results more simply or more directly, or that they are more easily mastered by the student than is the calculus of Hamilton. And the same may be predicted of the modified quaternionic system suggested by Prof. Lodge.

Musselburgh, September 4.

C. G. KNOTT.

Grassmann's "Ausdehnungslehre."

SIR ROBERT BALL asks why no one has translated the "Ausdehnungslehre" into English. The answer is as regrettable as simple—it would not pay. The number of mathematicians who, after the severe courses of the universities, desire to extend their reading is very small. It is something that a respectable few seek to apply what they have already learnt. The first duty of those who direct the studies of the universities is to provide that students may leave in possession of all the best means of future investigation. That fifty years after publication the principles of the "Ausdehnungslehre" should find no place in English mathematical education is indeed astonishing. Half the time given to such a wearisome subject as Lunar Theory would place a student in possession of many of the delightful surprises of Grassmann's work, and set him thinking for himself. The "Ausdehnungslehre" has won the admiration of too many distinguished mathematicians to remain longer ignored. Clifford said of it: "I may, perhaps, be permitted to express my profound admiration of that extraordinary work, and my conviction that its principles will exercise a vast influence upon the future of mathematical science." Useful or not, the work is "a thing of beauty," and no mathematician of taste should pass it by. It is possible, nay, even likely, that its principles may be taught more simply; but the work should be preserved as a classic.

I should be glad to subscribe £10 towards the expenses of translation. If others will join, perhaps some publisher will take the matter up. Is there no machinery by which the universities could be induced to subscribe?

A good book on the subject, entitled "The Directional Calculus," by Prof. E. W. Hyde, is published by Ginn and Co., Boston; and a valuable and very clever elementary exposition, on a geometrical basis, of important parts of the Calculus, by M. Carvallo, appeared in the *Nouvelles Annales de Mathématiques* of January, 1892. The latter will, in one day, enable a student to comprehend the power and elegance of Grassmann's methods.

R. W. GENESE.

Astronomical Photography.

THE nature of chromatic correction adopted for visual telescopes is uniform enough to make it possible to state what kind of photographic plate is desired for use with such telescopes.

A plate which is sensitive to light between C and F in the solar spectrum, with a marked maximum between D and  $\delta$ , and insensitive to other light, would be suitable for nearly all visual telescopes, which might in other respects (e.g. aperture, focal length, position as affected by climate) be available for taking special photographic records. With existing plates, so far as I have been able to acquaint myself with them, the sensitiveness in the blue and violet is the difficulty.

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But whilst such a special plate as I describe would be warmly welcomed, we must not forget that the proved goodness of the photographic star-images of what may be called violet refractors, *i.e.* refractors corrected so that the minimum focus is for violet light, is in great measure to be attributed to the fact that light of short wave length is used. The increase in the diameter of star-images with increased exposures or great brightness of the star, may be, as Scheiner has lately suggested, due to defects in the mode of support of the object-glass or mirror, but doubtless the *goodness* of the images with proper exposures must be connected with the smallness of the scale of the diffraction pattern, and with the concentration of light to the centre of the pattern, which may be got at smaller expense with a violet refractor than with a visual.

Probably few astronomers would have been bold enough, if no photographic plates had been available except plates sensitive only to yellow and green, to urge the preparation of plates sensitive in the violet, on the ground that a violet refractor would give much better results, because short wave lengths were used. And yet a comparison of the results obtained with violet refractors and with reflectors would lead one to the view above expressed, and, I believe, generally accepted.

The increased *range* of sensitiveness of modern photographic plates, with respect not only to the colour, but also to the intensity of the light affecting them, is all in favour of the reflector. A greater and more desirable advance than even the preparation of plates to suit visual telescopes would, I think, be made if the difficulties of supporting, adjusting, and maintaining a mirror were overcome; so that the measurement of star-images may be regarded with as much confidence in the case of plates exposed in reflectors as in refractors.

H. F. NEWALL.  
Maddingley Rise, Cambridge, September 25.

Hering's Theory of Colour Vision.

I AM very much surprised to see that Prof. Ebbinghaus, in the last number of the *Zeitschrift für Psychologie*, announces as new a discovery which has a critical bearing upon Hering's theory of colour-vision—the fact, namely, that two grays composed the one of blue and yellow, and the other of red and green, and made equally bright at one illumination (by admixture of black with whichever of them turns out to be the brighter), do not continue to be equally bright at a different illumination. If two complementary colours were purely antagonistic—that is, if the colour-processes simply destroyed each other, as processes of assimilation and dissimulation must do, and if the resulting white was solely due to the residual white which accompanies every colour and gives it its brightness, then the relative brightness of two grays composed out of different parts of the spectrum could not change with change of illumination. The fact that they do change is therefore completely subversive of the theory of Hering, or of any other theory in which the complementary colour-processes are of a nature to annihilate each other. This consequence of the fact, as well as the fact itself, I stated at the Congress of Psychologists in London in August, 1892, and it was printed in the abstract of my paper, which was distributed at the time, and also in the Proceedings of the Congress.

Prof. Ebbinghaus' discovery is apparently independent of mine, for he supposes that the phenomenon cannot be exhibited upon the colour-wheel. This is not the case; with fittingly-chosen papers (that is, with a red and green which need no addition of blue or yellow to make a pure gray, and with a corresponding blue and yellow) it is perfectly evident upon the colour-wheel. The same paper circles which I used to demonstrate it in Prof. König's laboratory in Berlin are, at the request of Prof. Jastrow, now on exhibition at the World's Fair at Chicago. While Prof. Ebbinghaus' discovery of the fact is therefore doubtless independent of mine, I allow myself to point out that mine is prior to his in point of time.

Baltimore.

CHRISTINE LADEL FRANKLIN.

"Megamicros."

IN NATURE of August 24 the following extract from the *Bulletin de l'Académie de Belgique*, No. 6 (1893), is given, viz. :—

"According to Laplace, if the dimensions of all the bodies of the universe, their mutual distances and velocities were to increase or diminish in a constant proportion, these bodies