with a series of topics for observation, the stars, moon, planets, &c., assuming that the readers are supplied only with an opera glass or small telescope. It is to be in no sense professional, "except to be accurate in statement of fact and principle without being technical in terms." The first number can be ready by September of this year if the subscribers are forthcoming.

OPTICAL TESTS FOR OBJECTIVES.—In a small pamphlet entitled "Optische Untersuchung von Objectiven," by Dr. Ludwig Mach of Prague, the contents of which have appeared in the *Photographischer Rundschau*, the writer describes a very simple means of obtaining photographs of objectives showing defects in the glass. After first referring shortly to the methods adopted by Schröder, Alvan Clark, &c., giving some excellent small photographs of some of the results obtained by these means, he decribes his method of making small optical errors visible. He casts, by means of an achromatic lens, an image of the sun on a screen in which is a small hole. Behind this screen, at some distance from it, he places the object glass to be tested, together with the camera at its focus, and it is found that in all places where the object glass is not perfect a system of interference marks or rings is formed. Experimenting with an object glass of 10·2 cm. aperture and 143 cm. focal length, by Sir Howard Grubb, the writer shows a photograph taken after this means.

PHOTOGRAPH OF A BOLID.—Although on fine nights many telescopes carrying with them photographic plates are turned towards the starry heavens for special objects, none, except a very few exceptions, have had the good luck to record the passage of a bright meteor. M. Lewis, at Ausonia (Connecticut) seems to have been very fortunate in this respect (*Bulletin Astronomique*, tome x., March), for on January 13 of this year, while photographing the comet Holmes, a very bright meteor crossed the field of view. An examination of the plate showed that the trail commenced at about 1h. 38m. R.A., and + 33° 40′ declination, terminating at 0h. 8m. R.A., and + 32° 12′ declination. Under the microscope he says that the centre of the trail is crossed by a very dark axis, clearly defined, while the other part is bounded by fringes of very irregular forms, indicating that fragments of matter had been detached from the meteorite: signs of rotary movement during its passage before the sensitised plate were also visible. For orbit determinations, photographs such as these, if they could be more often obtained, would be very valuable, for one could then fix the different points of the trajectory with far greater accuracy than is now done by the necessarily very approximate method of naked eye estimations.

GEOGRAPHICAL NOTES.

AN amusing instance of newspaper science occurred in a morning paper last week. A note on the salinity of the North Pacific, published in this column (vol. xlvii. p. 590), was reproduced without acknowledgment, but with annotations. After the quotation, "a tongue of considerably fresher water stretches nearly across the ocean about 10° N." came the interpolation, "caused no doubt by the dilution of the sea by the melting snow and ice of the northern regions," a far-fetched hypothesis, which ignores the rainy belt of calms. A worse error was to say that the curves of equal salinity "run through Behring Strait," when the original said Bering Sea. The use of a map would probably have prevented the blunders.

THE *Mouvement Géographique* publishes a useful *résumé* with route-maps and portraits of the officers of the various expeditions of the Katanga Company from May, 1890, to April, 1893. In July, 1890, the expedition of M. A. Delcommune left Europe for the Congo, went by the Lomami, discovered Lake Kassali, and reached Bunkeia, in Katanga on October 6, 1891. This expedition spent a year in exploring the upper Lualaba and the western side of Lake Tanganyika, then descended the Lukuga, crossed the Congo basin in a west-by-north direction to Lusambo, and arrived in Brussels on April 15, 1893. An expedition under Le Marinel left Lusambo on December 23, 1890, reached Bunkeia on April 18, 1891, and after taking possession of Katanga, returned to Lusambo in August of the same year. On July 4, 1891, Captain Stairs left the east coast, and travelling by Lake Tanganyika reached Bunkeia in December, but the leader died on the Zambesi on his way home on June 8, 1892. In September, 1891, Captain Bia's party left Stanley Pool, ascended the Sankuru, discovered Lakes Kabele and Kabire, near the Lualaba, and reached Bunkeia in January, 1892. Thence in June they reached Lake Bangweola, and after Captain Bia's death, Lieutenant Francqui led the expedition through the upper regions of the Lualaba, and in January, 1893, joined Delcommune at Lusambo, returning with him to Europe. The discoveries made by these four expeditions are of great importance; they fill in much of the detail of the Congo basin hitherto very lightly sketched on the maps.

A RUMOUR has been current that Dr. Nansen's polar expedition is likely to collapse at the last moment for lack of funds; but it is satisfactory to learn that this is not the case. The *Fram* is practically ready for sea, and the party will embark in the month of June, as originally intended.

THE recent advance in Arctic navigation is strikingly shown in the announcement by a Norwegian firm of a pleasure-trip to Spitzbergen, planned for this summer, with a vessel strengthened for ice-work and fitted with every comfort.

MM. FOUREAU AND MERY have during the past year carried out some important journeys in the Sahara. They have succeeded in reaching the country of the Tuaregs, which has not been visited by Europeans since the Flatters' mission was massacred in 1881, and they have induced the chiefs to acknowledge French protection. The French officials are diligently extending the cultivable area of the oases in the northern Sahara by sinking artesian wells and securing artificial irrigation.

THE USE OF HISTORY IN TEACHING MATHEMATICS.[1]

I HAVE ventured to make some suggestions to this Association as to the use of history in teaching mathematics, and the restrictions and limitations under which it may be advantageously employed. It will be perhaps the most convenient course to begin with the restrictions and limitations.

The three most important of these are:—

(1) The history of mathematics should be strictly auxiliary and subordinate to mathematical teaching.

(2) Only those portions should be dealt with which are of real assistance to the learner.

(3) It is not to be made a subject of examination.

Unless these conditions are observed, it is to be feared that the effect of the introduction of new matter for instruction would be injurious rather than beneficial. The ordinary schoolboy or schoolgirl now takes in hand quite as many subjects as he or she can satisfactorily study, and nobody wants the number to be increased.

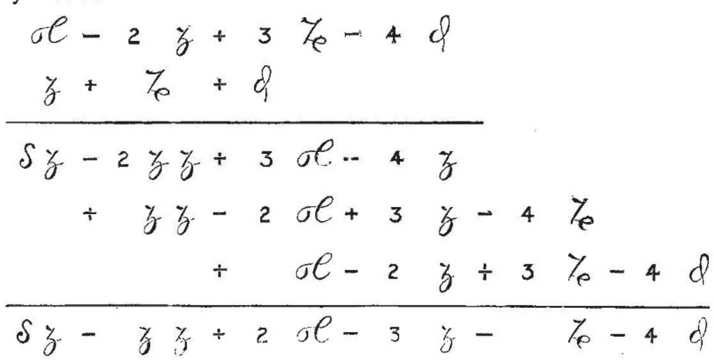
When men look back on their school days, they constantly feel some things they have always remembered and often applied came to them from their masters not as part of the regular course or as included in the work done for examination. It is just this outside illustrative position that I propose history should occupy in respect to mathematics. I want at the outset to free myself from any imputation of desiring to add one grain's weight to the heavy burden boys and girls have to bear in these days of competitive examination.

Coming now to the main question, which is in what ways history makes mathematical study easier, clearer, or more interesting, it may first of all be remarked that it gives us stereoscopic views instead of pictures and diagrams. A particular subject may be looked at from many sides, each aspect suggesting a different mode of treatment. Thus, although we do not want to go back to the method in Whewell's Mechanical Euclid, where the main truths of elementary statics were all derived from the fundamental axiom that a ruler would balance if its middle point were supported; it is yet a good thing for the pupil to know that such a method was successfully adopted. We do not want in arithmetic to go back to the old-fashioned rules of single and double false position, but the student is all the better for knowing what they were, and what could be effected by their means. Possibly some of us might really like to go back to the proof of Euclid I. 47 in the "Vija Ganita," depending only on the almost obvious truth that triangles of the same shape have their sides proportional, but at all events a student should know about this proof, even if he were to be warned of the objections to using it.

In some instances there is a further direct advantage in recalling old methods that are now superseded. Though the change

[1] Abstract of a paper by Mr. G. Heppel, read before the Association for the Improvement of Geometrical Teaching.

has been wisely made, yet it may happen that some important particulars have become comparatively obscured under the new treatment, that were in full light when the older plan was in vogue. Since Harriot introduced into England the grand and powerful improvement of making letters of the alphabet stand for unknown quantities, school boys have been for the most part regularly trained to look on algebra as a game of hide and seek, where x is concealed under conditions, and has to be dragged out into the light. The idea of some undetermined radix of a scale of notation, which was the very essence of the algebra of Stifel and Stevin, has not been brought prominently before them. It may be of interest to give four successive stages by which a process of multiplication in algebra has arrived at its present form. The first, originated by Stifel and adopted by Recorde, made use of very strange signs with very odd names. In the product, beginning from the right, the first term was called the *absolute*, the second the *root*, the third the *square*, the fourth the *cube*, the fifth the *zenzizenzike*, and the sixth the *sursolide*. In the second stage, Stevin's notation, adopted by Briggs, is self-explanatory. The third system is Vieta's, adopted by Harriot.



$$
\begin{array}{l}
\textcircled{3} - 2\,\textcircled{2} + 3\,\textcircled{1} - 4 \\
\textcircled{2} + \textcircled{1} + 1 \\
\hline
\textcircled{5} - 2\,\textcircled{4} + 3\,\textcircled{3} - 4\,\textcircled{2} \\
\quad + \textcircled{4} - 2\,\textcircled{3} + 3\,\textcircled{2} - 4\,\textcircled{1} \\
\qquad + \textcircled{3} - 2\,\textcircled{2} + 3\,\textcircled{1} - 4 \\
\hline
\textcircled{5} - \textcircled{4} + 2\,\textcircled{3} - 3\,\textcircled{2} - \textcircled{1} - 4
\end{array}
$$

$$
\begin{array}{l}
aaa - 2aa + 3a - 4 \\
aa + a + 1 \\
\hline
aaaaa - 2aaaa + 3aaa - 4aa \\
\quad + aaaa - 2aaa + 3aa - 4a \\
\qquad aaa - 2aa + 3a - 4 \\
\hline
aaaaa - aaaa + 2aaa - 3aa - a - 4
\end{array}
$$

$$
\begin{array}{l}
a^3 - 2a^2 + 3a - 4 \\
a^2 + a + 1 \\
\hline
a^5 - 2a^4 + 3a^3 - 4a^2 \\
\quad a^4 - 2a^3 + 3a^2 - 4a \\
\qquad a^3 - 2a^2 + 3a - 4 \\
\hline
a^5 - a^4 + 2a^3 - 3a^2 - a - 4
\end{array}
$$

As another example, a boy can use logarithms and understand what they are, directly he has mastered the law of indices, but in order to calculate them he imagines that he must know the Binomial and Exponential Theorems. Surely it would aid him to comprehend the relations of logarithms to numbers, if he knew that they were originally calculated when the Binomial and Exponential Theorems were unknown, and if he were given some slight sketch of the means by which they were then determined.

In the *Daily News* of December 16, 1892, a verse was quoted as being often found written in a schoolboy's Euclid or Algebra:—

> "If there should be another flood,
> Hither for refuge fly,
> Were the whole world to be submerged,
> This book would still be dry."

The schoolboy's charge of dryness must be met by showing him how the progress of the arithmetic, geometry, algebra, and trigonometry that he is learning has gone on in answer to the needs that men have felt, and the desires they have formed.

There have been periods in which men, under the influence of some widely-spread motive, have called for the aid of the theorists to help them on their course, and the endeavour to supply the great want of the time has brought about a great advance in theoretical knowledge. As we look at the course of these great movements, we find that it is the practical men that supply the stimulus to exertion, that set the few thinking for the advantage of the many. Three instances of these great wants of life—one of them now dead, the other two in ever-increasing life and vigour, stand out prominently beyond the rest—astrology, commerce, and navigation. The influence of astrology extended over such a vast period of time that we cannot trace its progress step by step from the ancient Chaldeans to the Doctor Dee of the reign of Elizabeth, who was the last eminent English mathematician of the astrological sort, and at the same time one of the great promoters of mathematics in its more modern applications. We can see, however, what has been left to us as the result of the attention that was paid to astrology. The works of Bhascara, himself an astrologer, show the extent to which the Indian arithmetic and algebra had gone, and what stock was in hand to be turned to the new purpose of facilitating European commerce. We had also from these ancient scholars the elements of trigonometry and tables of sines and cosines.

The old astrologers were maintained and were enabled to carry on their researches by the wealth of princes: Alphonso, King of Castile; Frederick II., Emperor of Germany; Matthias Corvinus, King of Hungary, are instances of monarchs who had astrologers in their train, filling recognised positions in their courts. Some of these were men of real learning; others, like Galeotti, introduced with the romance writer's licence as to place and time in Scott's "Quentin Durward," and Lilly, who successfully deluded the Parliamentary leaders in the Civil War, were not much better than quacks.

When we leave the astrological age and proceed to the commercial, the history is much more complete and more interesting. The whole story of the introduction of Indian arithmetic into Europe by means of the Arabians, first as the result of the Moorish conquests in Spain, and then, after a long interval, as a result of the commercial enterprise of Italy, is full of romantic interest. It is curious to notice how strongly the commercial element comes out in the algebra of Mahommed ben Musa. It is all about questions of money, partnerships, and legacies. When the practical objects for which mathematics were studied became different, there was a corresponding alteration in the mode by which such researches were encouraged and maintained. There still remained the patronage of great princes and nobles, but a new class of promoters arose among the great merchants and trading communities. A great wave of public enthusiasm seems to have borne along with it all classes of society, engaging them in the advancement of the new learning. Benedetti held the office of mathematician to the Duke of Savoy, with a good salary; Torricelli was mathematician to the Duke of Tuscany; Harriot received £300 a year regularly from the Earl of Northumberland, and while his noble patron was for fifteen years in prison for complicity with some of the ambitious plots of his friend Sir Walter Raleigh, Harriot, Hues, and Warner bore him company, and were generally spoken of as the Earl's three magi. As showing the interest taken by the traders of great cities, it may be noticed that some of the most important treatises of the time were written at the instigation of the merchants of Florence, and published at their expense. In our own country, the first English translation of Euclid was made by a citizen of London. Recorde dedicates the first English algebra to the company of Merchant Adventurers trading to Muscovia.

Important advances in mathematics were made by the professors at the college in London, founded by Sir Thomas Gresham. This feeling among the trading classes produced results in Italy which Libri tells us were unparalleled in any previous time. We all know of the Floral Games of Toulouse, and the athletic contests of the Greeks at Olympia and Corinth. But Libri tells us that just this interest, just this popular excitement was felt in Italy when Ferrari or Bombelli had made a step in advance in the solution of cubic and biquadratic equations. There were public challenges to contests of skill, proclamations by heralds, wagers to be decided. There is a collection of answers given by Tartaglia to questions submitted to him for solution by men from all ranks in society, princes, monks, doctors, ambassadors, professors, architects, and merchants, and a large proportion of them had to do with cubic and biquadratic equations. It may seem rather strange that this particular portion of Algebra should have excited so much interest, but it must be remembered that it is not possible to determine beforehand what researches into abstract truth will afterwards lead to the greatest practical benefits. There was a widespread belief that the new powers of calculation would bring about material advantage.

I trust that I may be pardoned for thus bringing forward matters which are no doubt very familiar to most of the members of this Association; but the object has been to give a sample of the kind of facts that would be likely to appeal to the minds of young learners, and to attach some human interest to the abstract subjects they are studying. This human interest is to be found in the history of navigation not less than in that of commerce. The relation between the commercial impulse and the navigation impulse was not exactly one of succession. The former was the earlier, then the two for a time went on together, and afterwards the latter was supreme as a ruling motive for promoting mathematics.

The two great problems in navigation were first, if you knew where you were, to find how you could best get somewhere else; and secondly, if you did not know where you were, to find this out by astronomical observation. The solution of the first was mainly dependent on maps and charts, and consequently for a long time men were hard at work making these for the use of sailors. The first great promoter of this work in modern times was Prince Henry of Portugal, called the Navigator, and after his death in 1460 to the close of the century, Portugal, eagerly engaged in the exploration of the coast of Africa, continued to be the great chart-producing country. Later on it was to the Netherlands that we were principally indebted for improvements in this direction, and in the long list of those thus engaged a prominent place is taken by Stevin. Mercator's projection is so called from Kauffman, who invented it in 1566, but did not clearly show the principles on which it is founded, a task that was afterwards accomplished by an Englishman, Edward Wright, whose great services to science have been but scantily recognised.

The second great problem—to find out where you are by astronomical observation—was a pressing question in the sixteenth and seventeenth centuries. The chief instrument the Elizabethan mariner had at his command was the astrolabe. This was made in very various forms. For use at sea, of course the simplest form was chosen. There is a plate in Hutton's Mathematical Dictionary of one, consisting of a graduated circle held up by a ring, and so keeping a vertical position by its own weight, furnished with an arm and two sights, by which the altitude of the sun, moon, or stars could be estimated. The astrolabes in use on land were fitted up with much greater refinement.

An instrument perhaps more frequently used, easier to work with than the astrolabe, but less accurate, was called the cross-staff or fore-staff. It was composed of a graduated wooden rod, about three feet long, with cross pieces sliding along it of different heights, and the angle was observed in the same way that a volunteer uses the sights on his rifle. This fore-staff could be applied to roughly determine the distance between two stars.

To determine with any accuracy a ship's place at sea, three things are requisite. First, a theory that is true and workable as far as it goes; secondly, means of observation; thirdly, means of calculation. A defect in any one of these requisites renders comparative excellence in the other two of small use. Now, the mariners of Drake's time had scanty theoretical knowledge, poor instruments, and very deficient means of calculation. They could, in a rough fashion, find out in about what latitude they were; the longitude remained a mystery.

It was at the beginning of the seventeenth century that the first great improvement took place. The invention of logarithms, by Napier, placed the calculating power at one bound far in advance of either the theoretical knowledge or the means of observation. His system, further developed by Briggs, the Gresham professor, so completely supplied the want previously existing, that any improvements made between then and the present time are mere matters of detail.

The improvements in theory and in instruments went on gradually and together. Tycho Brahe did much to advance the efficiency of instruments, and every step in this direction gave the means of correcting or developing previous defective theory, and each theoretical advance suggested or rendered possible some new instrument of observation. It is no proper part of my subject to trace the steps of this progress. It is sufficient to say that now the shipmaster, often a man of no great scientific attainments, generally accustomed to work by rules, the reasons for which he does not know, has in his cabin a chronometer and a book of navigation tables, which represent in a material form the genius and the toil of the master minds that have arisen during the centuries of the past.

In the application of pure mathematics to navigation, as well as to many other purposes, it is curious to notice the changes in the relations between graphic methods and calculation methods. At first the former greatly predominated. The quantities of straight lines and curves engraved on Drake's astrolabe, the profusion of scales on old sun dials, that but few thoroughly understand, were originally intended and were accepted as the most simple means of determining practical problems. They gradually gave place to numerical calculation, but not very quickly. Fifty years ago a boy's training in the elements of navigation was conducted far more on the lines of geometrical construction than it is at present. In quite recent times there has been a revival of graphic methods in a somewhat different aspect. Besides the value they have always had for illustration and explanation, it has been seen that there is a special field for them in cases where calculation would be long and troublesome, and this special field is being clearly marked off.

The correspondence between the practical aims of men and the progress of theoretical knowledge and of means of calculation does not stop with navigation. In recent times the need for more powerful or more exact machinery, the employment of steam and electricity, our increased knowledge of what is meant by heat and light have had the effect of demanding fresh advances in mathematical methods; or, perhaps, more exactly of selecting from the mass of abstract truth acquired for its own sake the particular portion suited to the special purpose. These influences have had, however, nothing to do with the schoolboy's elementary programme, and are, therefore, outside the immediate subject of this paper.

In conclusion, I would urge that if there is any sound foundation for the views that have been expressed, we ought not in England to be without some elementary primer of the History of Mathematics.

FOGS AND HORTICULTURE.

PROF F. W. OLIVER'S second report on the effects of urban fog upon cultivated plants has been presented to the scientific committee of the Royal Horticultural Society, and is now printed in the Society's Journal. The following is the passage in which he deals with possible remedial measures:—

There is very little of what I can say likely to be consoling to the horticulturist. We must recollect that in the employment of measures directed towards mitigating the injuries incident to fog, two factors—the presence of poisons in the atmosphere and the reduction of light—have to be considered. To counteract these the urban cultivator is asked to construct air-tight houses, with definite openings where the admitted air can be filtered; whilst to compensate for the loss of light due to the absorption which the rays undergo in traversing a stratum of dense fog, he must provide a generous installation of electric light. Without doubt, the entire preservation of vegetation in foggy weather is only a matter of *£ s. d.* But it is for the cultivator to sit down