The general plan was to observe every star three times, and out of the total number of stars in the catalogue (3415) 289 stars were observed less than this number of times, while 1048, 491, and 194 stars were observed four, five, and six times respectively, and the rest seven times or more. The various differences of brightness were estimated by Argelander's method of step-estimations, each sequence comprising ten, five, or twenty stars according to the number of stars in the vicinity observed. Commencing in the year 1882, Mr. Sawyer says that nearly half of the whole work was done in that time, an opera glass being extensively used for fainter sequences, such as those in which the stars were of the 6th or fainter magnitude a field glass was employed. During the years 1883 and 1885 the observations, as he tells us, were wholly discontinued, "owing to the injury to the eyes from the trying nature of the work." In the method of reduction the magnitudes were deduced by plotting out the sequences, graphically using the *Uranometria Argentina* magnitudes as ordinates, and the observed differences of brightness, expressed in steps, as abscissas. The arrangement of the catalogue itself is as follows:—The columns give successively the catalogue current number of the star, U.A. catalogue number, constellation, Right Ascensions and Declinations for mean equinox 1875·0, number of observations, mean magnitude deduced, U.A. magnitude, and the three last the separate dates of the observations and magnitudes.

Comparing the average differences between the magnitudes here assigned and those given by Gould, it is found that ± 0·088m. about represents it, while the average error of a single determination, assuring equal degree of precision and including besides accidental errors, the effect of systematic difference is given as ± 0·059m.

While the work was in hand eight variables were discovered, which were as follows:—U Ophiuchi (1881), U Ceti (1885), U Aquilæ and Y Sagittarii (1886), R Canis Majoris (1887), Y Ophiuchi and W Hydræ (1888), and (?) Leporis (1891), and in addition several large discordances were noticed in many values obtained (the catalogue number of these are here given), rendering these stars worthy of special attention. The volume concludes with notes, in which several suspicious cases of variables, &c., are recorded.

A New Table of Standard Wave-lengths.—Under this title Prof. H. A. Rowland contributes to *Astronomy and Astrophysics* for April (No. 114) the new measurements of several metallic lines to be used as standards. The actual measures were made by Mr. L. E. Jewell, the probable error of one setting amounting to 1 part of 5,000,000 of the wave-length, and the reductions of the reading by Prof. Rowland himself. The measurements were obtained with a new machine, supplied with a screw specially made after Prof. Rowland's process. The standard wave-length of D used was the mean of the determinations of Angström, Müller and Kempf, Kurlbaum, Pierce, and Bell, and was 5896·156, different weights being given to these separate values. This value was utilised in six different ways, and the resulting table of wave-lengths from 2100 to 7700 was obtained, the accuracy of which might, as he says, be estimated as follows:—"Distribute less than $\frac{1}{100}$ division of Angström properly throughout the table as a correction, and it will be perfect within the limits 2400 and 7000."

Meteor Showers.—Among the principal meteor showers for the current year, a list of which is given in the *Companion to the Observatory*, the following two occur this week, the former of which is described by Denning as "one of the most briliant showers." The radiant points are:—

Date	Radiant α	Radiant δ	Meteors
April 20	270°	+ 33°	Swift
,, 25	272°	+ 21°	Swift; short

Wolsingham Observatory, Circular No. 35.—A plate taken with the Compton 8-inch photo-telescope, April 11, compared with a photo by Max Wolf, 1891, shows that the two stars

Es-Birm 545 18h. 28·9m. + 36° 55′ (1900)
,, 561 18h. 39·4m. 36° 52′ ,,

are var., the photo differences being approximately 9·1, 11·4; 8·8, 10·2.

GEOGRAPHICAL NOTES.

Letters dated March 9 have been received from the Antarctic whaling vessels confirming and extending the brief telegraphic information already published. The ships did not proceed farther south than 67° latitude, and discovered no signs of the existence of the Greenland whale, although whales of several other species were common, and there were great numbers of grampuses. In default of whaling, the energy of the crews was devoted to sealing, and the four vessels secured between them about 16,000 skins and a full cargo of oil. The seals were of several varieties, but until the return of the ships their species cannot be determined, nor their commercial value known. The weather throughout the whole stay in Antarctic waters was severe, and the formation of ice compelled the vessels to return at an earlier date than was at first intended. Flat icebergs of enormous size were seen, one being reported as fifty miles in length. The facilities afforded for scientific work were disappointing.

The Delcommune expedition (p. 474) has returned to Europe, and M. Delcommune was received with great enthusiasm in Brussels. The expedition, together with the others sent out by the Katanga Company, has to a large extent completed the work of Livingstone and his successors in the Congo Basin, and in the main confirms the accepted geography of the region. One point of some interest which has been established is that the Lake Lanji, marked from Arab reports at the junction of the Lukuga and the Lualaba, has no existence.

The new number of *Petermann's Mitteilungen* contains a short paper by Prof. Krümmel on recent Russian oceanographical work in the north Pacific. This is accompanied by a map of the salinity of the surface water, which extends, and in a general way confirms, Mr. Buchanan's map founded on the *Challenger* work. The centre of maximum salinity lies between 20° and 30° N., and has its centre about 170° W. A tongue of considerably fresher water stretches nearly across the ocean, about 10° N. and sweeps round the coasts of America and Asia. The diminution of salinity northward is very interesting, the curves of equal salinity sweeping through Bering Sea without regard to the line of Aleutian Islands, thus showing that so far as regards surface water, Bering Sea is simply part of the Pacific ocean, standing in very marked contrast to the Sea of Okhotsk, a fact of some interest during the present international controversy.

Mr. T. H. Hatton-Richards read a paper on British New Guinea at the last meeting of the Royal Colonial Institute. While giving an account of the climate discouraging to would-be white settlers, Mr. Richards describes the native Papuans from personal experience as a fine race, possessing a keen sense of justice, and most laborious and successful as agriculturists.

RECENT INNOVATIONS IN VECTOR THEORY.[1]

Of late years there has arisen a clique of vector analysts who refuse to admit the quaternion to the glorious company of vectors. Their high priest is Prof. Willard Gibbs. His reasons for developing a vector analysis devoid of the quaternion are given with tolerable fullness in Nature, vol. xliii. p. 511. His own vector analysis is presented in a pamphlet, "Elements of Vector Analysis, arranged for the Use of Students in Physics, not Published" (1881–84). Mr. Oliver Heaviside, in a series of papers published recently in the *Electrician* and in an elaborate memoir in the *Philosophical Transactions*, supports some of Gibbs's contentions and cannot say hard enough things about the quaternion as a quantity which no physicist wants. Prof. Macfarlane, of Texas University, has added to the literature of the subject, and without altogether agreeing with Gibbs takes umbrage at a most fundamental principle of quaternions and developes a pseudo-quaternionic system of vector algebra which is non-associative in its products!

Between the years 1846–52, just at the time when Hamilton was developing the quaternion calculus, a series of papers was published by the Rev. M. O'Brien, Professor in King's College, London. The system developed by O'Brien is essentially that

[1] Abstract of a paper by Prof. C. G. Knott, read before the Royal Society of Edinburgh, on Monday, December 19, 1892.

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advocated by Gibbs and Heaviside. Two products of vectors are defined, which correspond to Hamilton's $V\alpha\beta$ and $-S\alpha\beta$; and applications are given of the linear and vector function and of the operator $\alpha\partial_1 + \beta\partial_2 + \gamma\partial_3$ which somewhat resembles the quaternion ∇.

The broad argument advanced by Gibbs in his letter to NATURE is that, in comparison with the quantities $V\alpha\beta$ and $S\gamma V\alpha\beta$, which symbolise an area and a volume which "are the very foundations of geometry," anything that can be urged in favour of the quaternion product or quotient as a "*fundamental* notion in vector analysis" is "trivial or artificial." These are brave words. Let us examine them by considering what is the purpose of a vector analysis. Clearly such a calculus is intended to show forth the properties of vectors in a form suitable for use.

Having formed the conception of a vector, we have next to find what relations exist between any two vectors. We have to compare one with another, and this we may do by taking either their difference or their ratio. The geometry of displacements and velocities suggests the well-known addition theorem—

$$\alpha + \delta = \beta$$

in which by adding the vector δ we pass from the vector α to the vector β.

But this method is not more fundamental geometrically than the other method which gives us the quaternion. When we wish to compare two lengths a and b, we divide the one by the other. We form the quotient a/b, and this quotient is defined as the factor which changes b into a. Now a vector is a directed length. By an obvious generalisation, therefore, we compare two vectors by taking their quotient and by defining this quotient α/β as the factor which changes the vector β into the vector α. This is the germ out of which the whole of vector analysis naturally grows. A more fundamental conception it is hardly possible to make. Yet Gibbs calls it trivial and artificial! Far more fundamental, we are told, are the conceptions of a vector bounded area and a vector bounded volume, whose bounding vectors may have an infinity of values. Or take the more general case of a body strained homogeneously. The relative vector of any two of its points passes into its new position by a process which is a combination of stretching and turning. A simpler and more complete description cannot be imagined. It is perfectly symbolised by the quaternion with its tensor and versor factors. And *this* is trivial and artificial—as trivial, say, as the versor operation which every one performs when estimating the time that must be allowed to catch a train. . . .

Another argument advanced by Willard Gibbs is in the paragraph beginning: "How much more deeply rooted in the nature of things are the functions $S\alpha\beta$ and $V\alpha\beta$ than any which depend on the definition of a quaternion, will appear in a strong light if we try to extend our formulæ to space of four or more dimensions." To elucidate the "nature of things" by an appeal to the fourth dimension—to solve the Irish question by a discussion of social life in Mars—it is a grand conception, worthy of the scorner of the trivial and artificial quaternion of three dimensions. Further on we are told that there "must be vectors in such a space"; that is, space of four or more dimensions. True, and if there be vectors, must there not be operations for changing one vector into another? . . . .

"Vectors must be treated vectorially" is a high-sounding phrase uttered by Prof. Henrici and Mr. Heaviside. What does it mean? On the same sapient principle, I suppose, scalars must be treated scalarially, rotors rotorially, algebra algebraically, geometry geometrically. That is to say the remark is a very loose statement of a truism, or it is profound nonsense. Strictly speaking, to treat vectorially is to treat after the manner of vectors, or to treat *as vectors do.*

Now what does a vector do? Prof. Gibbs, the prince of vector purists, says on page 6 of his pamphlet that "the effect of the skew [or vector] multiplication by α [any unit vector] upon vectors in a plane perpendicular to α is simply to rotate them all 90° in that plane." Hence a vector *is* a versor. To which Mr. Heaviside in fierce denunciation: "In a given equation [in quaternion-vector analysis] one vector may be a vector and another a quaternion. Or the same vector in one and the same equation may be a vector in one place and a quaternion (versor or turner) in another. This amalgamation of the vectorial and quaternionic functions is very puzzling. You never know how things may turn out." Puzzling? Then must Heaviside find his own system as puzzling as any. For when he writes the vector product $ij=k$, he is simply acting on j by i or on i by j, and turning it through a right angle. It is impossible to get rid of this versorial effect of a vector. It stares you in the face from the very beginning.

A very sore grievance with Heaviside and Macfarlane—although Gibbs cautiously steers clear of the whole question—is that Hamilton puts i^2, j^2, k^2, each equal to negative unity, with the consequence that $S\alpha\beta$ is equal to $-ab\cos\theta$, where a and b are the lengths of α and β, and θ the angle between them. This putting the square of a vector equal to *minus* the square of its length vexes their souls mightily. It is so "unnatural," so troublesome.

Now Prof. Kelland, in Kelland and Tait's "Introduction to Quaternions," chap. iii., shows that if we assume, as do Heaviside and Macfarlane, the cyclic relations

$$ij=k=-ji \quad jk=i=-kj \quad ki=j=-ik,$$

and if in addition we desire an *associative* algebra, then *of necessity* we must have $i^2=j^2=k^2=-1$. If then, following these O'Brienites, we put what they consider to be so much simpler and more natural, namely, $i^2=j^2=k^2=+1$, we get a non-associative algebra of appalling complexity, which in the long run gives us no more than the associative quaternion algebra.

Heaviside apparently is unaware of the non-associative beauties of his system, which he believes "to represent what the physicist wants;" for he says, much to the credit of the *Philosophical Transactions*, that his system (which is demonstrably *not* quaternions) is "simply the elements of quaternions without the quaternions, with the notation simplified to the uttermost, and with the very inconvenient *minus* sign before scalar products done away with" (*Phil. Trans.*, vol. clxxxiii. 1892, p. 428).

We have seen how perfectly natural is the geometric conception of a quaternion as the quotient of two vectors; and the quaternion product is as simply conceived of as the operator $(\alpha\beta)$ which turns the vector β^{-1} into α. Space considerations quickly lead us to consider quaternions which rotate a given vector through a right angle. If we take two such right or quadrantal quaternions I I′ and operate severally on the vector α that is perpendicular to the axes of both, it is easy to show that

$$I\alpha + I'\alpha = (I + I')\,\alpha$$

gives a right quaternion $(I+I')$ bearing to I and I′ the same relation which would exist were I and I′ vectors. That is, right or quadrantal quaternions are added and subtracted according to the recognised rules for vector addition and subtraction, which so far, be it noted, are all we know about vectors. Hence in combinations other than addition and subtraction we may treat vectors as quadrantal quaternions, exactly as Gibbs, Heaviside, and Macfarlane do, although in a half-hearted fashion.

It remains now to consider wherein the systems advocated by these vector analysts improve upon Hamilton's. Do they give us anything of value not contained in quaternions?

Prof. Gibbs, having objected *in toto* to the quaternion product $\alpha\beta$, is for consistency's sake bound to object to Hamilton's selective principle of notation. His own notation is very similar in appearance to O'Brien's of old. He defines two products, the *direct* product $(\alpha . \beta)$ and the *skew* product $(\alpha \times \beta)$. The direct product is Grassmann's inner product or Hamilton's $-S\alpha\beta$; and the skew product is $V\alpha\beta$, so called probably because it has a value only when α and β are skew, or inclined to one another. Now there is a serious objection at the very outset to such a form as $\alpha \times \beta$ for the vector product of α and β. There corresponds to it no quotient amenable to symbolic treatment. The reason is, of course, that $\alpha \times \beta$ is not a complete product. Given the quaternion equation $\alpha\beta = q$, any one quantity is uniquely determined if the other two are given. But it is impossible, in spite of the suggestiveness of the form, to throw Prof. Gibbs's $\alpha \times \beta = \gamma$ into any such shape as $\alpha = \gamma \div \beta$. The point is that Hamilton's notation does not even suggest the possibility of such a transformation. It is certainly inexpedient, to say the least, to use a notation strongly resembling that for multiplication of ordinary algebraic quantities, but having no corresponding process by which either factor can be carried over as a generalised divisor to the other side of the equation.

One peculiar perspicuity of Hamilton's notation arises from the fact that S and V are thrown out in bold relief from amongst the small Greek letters used for vectors and the small

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Roman letters used for quaternions and scalars. A glance tells us what kind of quantity we have to deal with before we are called upon to inquire into its composition. There is no such eye-catching virtue in Gibbs's notation; and Heaviside largely destroys the contrast between the quantities and selective symbols by using capital letters for all. In print the vectors are made heavy and stand out prominently enough. But a vector analysis is a thing *to be used;* and with pencil or pen or chalk on a blackboard it is hopeless to prevent confusion between A and **A**. In suggesting a suffix notation for manuscript, Heaviside unconsciously condemns his own system. Two conditions for a good notation are (1) an *unmistakable* difference between *easily written* symbols for scalar and vector quantities; (2) the scalar and vector parts of products and quotients thrown out in clear relief. This second is quite as important as the first condition. So far, Hamilton's notation easily holds its own.

A very important symbol of operation is the Nabla, ∇, which may be defined in the form $\alpha\partial_1 + \beta\partial_2 + \gamma\partial_3$ where $\partial_1\partial_2\partial_3$ are space-differentiations along the mutually rectangular directions of the unit vectors $\alpha\beta\gamma$. Since Heaviside and Macfarlane make $\alpha^2\beta^2\gamma^2$ each equal to $+1$, they find that $\nabla^2 u$, where u is any scalar, is $d^2u/dx^2 + d^2u/dy^2 + d^2u/dz^2$. The real $\nabla^2 u$ is *minus* this quantity. When ∇^2 acts on a vector, Heaviside boldly defines $\nabla^2\omega$ as having the same significance; but Macfarlane, rejoicing in his non associative algebra, finds that $\nabla(\nabla\omega)$ is quite a different quantity from $(\nabla\nabla)\omega$. The net result attained by this tinkering of the signs is to get a pseudo-nabla non-associative with itself!

Gibbs moves more cannily. He defines separately the quantities ∇u, $\nabla \times \omega$, $\nabla . \omega$, and $\nabla . \nabla \omega$, which mean the same things as the quaternion quantities ∇u, $V\nabla\omega$, $-S\nabla\omega$, and $-\nabla^2\omega$. [In quaternions there is one definition of ∇, and everything else follows.] But even with these four definitions (all of which are properties somewhat distorted of the real Nabla) Gibbs finds his system lacking in flexibility. He has, so to speak, to lubricate its joints by pouring in the definitions of four other functions with as many new symbols. One of these is the Potential; the others are called the Newtonian, Laplacian, and Maxwellian. They are symbolised thus—Pot, New, Lap, Max. Their meanings will be evident when they are exhibited in quaternion form. Thus, as is well known,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \text{Pot } u = -4\pi u,$$

from which at once

$$\nabla^2 \text{ Pot } u = +4\pi u$$

or

$$\text{Pot } u = 4\pi\nabla^{-2}u.$$

Similarly, if ω is a vector quantity,

$$\text{Pot } \omega = 4\pi\nabla^{-2}\omega.$$

Then we have

$$\text{New } u = \nabla \text{ Pot } u = 4\pi\nabla^{-1}u$$

$$\text{Lap } \omega = V\nabla \text{ Pot } \omega = 4\pi V\nabla^{-1}\omega$$

$$-\text{ Max } \omega = S\nabla \text{ Pot } \omega = 4\pi S\nabla^{-1}\omega.$$

Now, Prof. Gibbs gives a good many equations connecting these functions and their various derivatives, equations which in quaternions are *identities* involving the *very simplest* transformations. But there is no such simplicity and flexibility in Gibbs's analysis. For example, he takes eight distinct steps to prove two equations, which are special cases of

$$\nabla^{-2}\nabla^2 u = u\ !$$

Another of his *theorems*, namely,

$$4\pi \text{ Pot } \omega = \text{Lap Lap } \omega - \text{New Max } \omega$$

is simply the quaternion *identity*

$$4\pi\nabla^{-2}\omega = 4\pi\nabla^{-1}\nabla^{-1}\omega$$
$$= 4\pi\nabla^{-1}V\nabla^{-1}\omega + 4\pi\nabla^{-1}S\nabla^{-1}\omega.$$

Similarly the equation

$$4\pi \text{ Pot } u = -\text{ Max New } u$$

is a travesty of

$$4\pi\nabla^{-2}u = 4\pi\nabla^{-1}\nabla^{-1}u!$$

These extremely simple quaternion transformations cannot be obtained with the operator used by Gibbs. Hence the necessity he is under to introduce his Pot, New, Lap, Max, which are merely inverse quaternion operators. . . .

Gibbs's system of dyadics, which Heaviside regards with such high admiration, differs from Hamilton's treatment of the linear and vector function simply by virtue of its notation. In his letter to NATURE he gives reasons why this notation is preferable to the recognised quaternion notation. As developed in the pamphlet, the theory of the dyadic goes over much the same ground as is traversed in the last chapter of Kelland and Tait's "Introduction to Quaternions." With the exception of a few of those lexicon products, for which Prof. Gibbs has such an affection,[1] there is nothing of real value added to our knowledge of the linear and vector function. As usual, the path is littered with definition after definition. Thus the *direct product*[2] of two dyads (indicated by a dot) is defined by the equation $\{\alpha\beta\}.\{\gamma\delta\} = \beta.\gamma\alpha\delta$. Quaternions gives at once

$$\phi\psi\rho = \alpha S\beta(\gamma S\delta\rho) + \&c. = \alpha S\delta\rho S\beta\gamma + \&c.$$

Then there follow the definitions of the *skew* products of ϕ and ρ, thus—

$$\phi \times \rho = \alpha\lambda \times \rho + \beta\mu \times \rho + \gamma\nu \times \rho$$
$$\rho \times \phi = \rho \times \alpha\lambda + \rho \times \beta\mu + \rho \times \gamma\nu.$$

These are not quantities but operators. To see what they mean let them operate on some vector σ. Then we find

$$\phi \times \rho\ .\ \sigma = \alpha S\lambda\rho\sigma + \ldots = \phi V\rho\sigma$$
$$\rho \times \phi\ .\ \sigma = V\rho\alpha S\lambda\sigma + \ldots = V\rho\phi\sigma.$$

The first is simply $\phi\varpi$, the old thing! The second is a well-known and important quantity in the theory of the linear and vector function. It is interesting to note, as bearing upon the *intelligibility* of the notation, that Heaviside, who dotes so on the dyadic, writes $\phi \times \rho$ in the form $V\phi\rho$, so that he makes

$$\phi V\rho\sigma = -V\sigma\phi\rho\ !\ !$$

As one example of our *gain* in following Gibbs's notation, take his dyadic identity—

$$\psi\ .\ \{\rho \times \phi\} = \{\psi \times \rho\}\ \ \phi,$$

on which the comment is that "the braces cannot be omitted without ambiguity." The quaternion expression is $\psi V\rho\phi\sigma$, where there is no chance of ambiguity, where everything is perfectly straightforward, and where there is much greater compactness in form. It seems to me that this last equation given by Gibbs condemns his whole principle of notation. It shows that one use of connecting symbols is to obscure the significance of a transformation. . . .

A beautiful example of virtually giving back with the left hand what he has taken away with the right is furnished on p. 42 of Gibbs's pamphlet. He writes: "On this account we may regard the dyad as the most general form of product of two vectors. We shall call it the indeterminate product." And then he shows how to obtain a vector and a scalar "from a dyadic by insertion of the sign of skew or direct multiplication."

This is exquisite. From the *operator* $\alpha\lambda + \beta\mu + \gamma\nu$, he forms —heedless of his high toned scorn for the quaternion product—the conception of the sum of three similar though more general products, but quiets his conscience by calling them *indeterminate!* This sum of products then becomes by simple insertion of dots and crosses the vector

$$\phi_\times = \alpha \times \lambda + \beta \times \mu + \gamma \times \nu,$$

and the scalar

$$\phi_s = \alpha\ .\ \lambda + \beta\ .\ \mu + \gamma\ .\ \nu.$$

Why, we naturally ask, is this *indeterminate* product welcomed where the *quaternion* product is spurned?

The truth is the quaternion, or something like it, has to come in; and in it most assuredly does come when Gibbs proceeds to treat the versor in dyadic form. The expression $\{2\beta\beta - I\}\ .\ \{2\alpha\alpha - I\}$ represents in Gibbs's notation the quaternion operator

$$\beta\alpha(\quad)\alpha\beta, \text{ or more simply } q(\quad)q^{-1}.$$

The I is called an *idemfactor* and is simply unity. . . .

There is something almost naïve in the way in which Heaviside introduces the dyadic as if nothing like it was to be found

[1] We are surprised that so much etymological erudition should accept such a monstrosity as parallelOpiped.

[2] Gibbs calls the quantity $\phi.\sigma$ (which is simply Hamilton's $\phi\sigma$) the direct product of the dyadic ϕ and the vector σ. The direct product of two vectors is $\alpha.\beta$, and this Heaviside calls the scalar product. Similarly translating the Gibbsian dialect, he speaks of $\phi\sigma$ as being the "scalar product of the dyadic and the vector"—and gets a scalar product equal to a vector! This "is most tolerable and not to be endured."

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in either Hamilton or Tait. The truth is it is all there. Hamilton showed long ago that if

$$\phi\rho = \alpha S\lambda\rho + \beta S\mu\rho + \gamma S\nu\rho,$$

then

$$\phi^{-1}\rho = \lambda_1 S\alpha_1\rho + \mu_1 S\beta_1\rho + \nu_1 S\gamma_1\rho,$$

where

$$\alpha_1 S\alpha\beta\gamma = V\beta\gamma, \&c., \lambda_1 S\lambda\mu\nu = V\mu\nu, \&c.$$

Now Heaviside fusses greatly over this method of inverting ϕ, and any reader of § 172 ("Electromagnetic Theory," in the *Electrician*), would infer that the invention of the name dyadic suggestion this demonstration which Hamilton and Tait had somehow missed in their development of "the very clumsy way" of expressing $\phi^{-1}\rho$ in terms of ρ, $\phi\rho$, and $\phi^2\rho$. But the whole thing is given in Hamilton's "Elements" (p. 438, equation xxvii.), and in Tait's "Quaternions" (p. 89, second edition; p. 123, third edition). I would also refer to § 174 of Tait's third edition (§ 162 of the second), a comparison of which with Heaviside's tall talk in the *Electrician* of November 18, 1892 (§ 171), will show that, on the most lenient hypothesis available, our self-appointed critic of Tait's methods has never really read Tait's "Quaternions." . . .

All through his system Prof. Gibbs has refused to consider the complete product of two vectors. He has used the form $\alpha\beta$ to mean a "dyad" or operator of the form $\alpha S\beta$ or $\beta S\alpha$. What, then, can he mean us to understand by the equations—

$$\iint d\sigma\omega = \iiint dv\nabla\omega \quad ((2) \text{ of § } 164),$$

and

$$\int d\rho\omega = \iint d\sigma \times \nabla\omega \quad ((2) \text{ of § } 165).$$

In quaternion notation the last would be written

$$\int d\rho\omega = \iint V(d\sigma\nabla)\omega.$$

Both equations are quite correct if and only if $d\sigma\omega$, $d\rho\omega$, and $\nabla\omega$ are taken in their quaternion meaning of *quantities*. But Gibbs has wilfully cut himself adrift from this interpretation. How, then, does he interpret these equations?

The chief arguments of the paper may be briefly summarised thus:—

(1) It is maintained that the quaternion is as fundamental a geometrical conception as any that Prof. Gibbs has named.

(2) In every vector analysis so far developed, the versorial character of vectors in product combinations is implied if not explicitly stated.

(3) This being so, it follows as a *natural* consequence that the square of a unit vector is equal to negative unity.

(4) The *assumption* that the square of a unit vector is positive unity leads to an algebra whose characteristic quantities are non-associative, and whose ∇ is not the real efficient *Nabla* of quaternions.

(5) The invention of new names and new notations has added practically nothing of importance to what we have already learned from quaternions.

EXPERIMENTAL MEDICINE.[1]

THIS volume is the fourth number of this remarkable publication, and will prove of surpassing interest to the bacteriologist, physiologist, and physician, chiefly on account of the first paper which it contains.[2]

In 1877 Dr. N. V. Eck invented an operation by which it was possible to alter the circulation in such a manner that the blood flowed from the portal vein into the inferior vena cava without passing through the liver. He succeeded in establishing an artificial opening between these veins in several dogs, and then tied the portal vein near the liver; unfortunately, only one dog lived for any length of time (two and a half months), and, owing to an accident, Dr. Eck was unable to control the result by post-mortem examination. The operation has now been repeated at the St. Petersburg Institute, and it has been found that in successful cases the blood passed entirely from the portal vein into the inferior vena cava.

The animals which successfully resisted this severe operation showed no alteration in the appetite, though after a period of ten days or so their temper underwent marked changes. Although perfectly docile before the operation, they now became bad-tempered, bit everything that came in their way, and showed undue excitement on trifling provocation. The animals became weak, and their gait ataxic, whilst the sensory apparatus was also greatly disturbed, as they often became blind, and appeared to lose all sensation of pain. In a further stage convulsions and coma supervened; though the animals occasionally recovered perfectly after a time, many of them died when the first attack of excitement and convulsions occurred, or succumbed to subsequent attacks, although, on the whole, the latter rarely proved fatal. The temperature showed no changes attributable to the veinous fistula, but the weight generally diminished until death supervened, although, in animals which recovered it reached, or even exceeded, the original weight. The appetite was good, though capricious; but a distinct relation was found to exist between the state of the alimentary canal and the attacks of excitement before mentioned. The animals which absolutely refused to eat meat remained free from the attacks, while the "crises" invariably occurred in the dogs that ate meat voraciously. It is a remarkable fact that many of them learnt by experience that meat was bad for them, and declined to take it.

Some dogs recovered perfectly, and at the postmortem it was found that a collateral circulation had been set up, so that the portal blood again circulated through the liver.

It would appear from further observations that these symptoms are due to the toxic action of the products of the transformation of nitrogenous food, the liver being unable to convert them into urea and uric acid. Carbamic acid was found in the urine of these animals, and carbamate of sodium or calcium, when introduced into a healthy animal's stomach, produced exactly the same symptoms as the fistula above described. On the other hand, it was found impossible to poison healthy dogs with the same salt, provided the setting free of carbamic acid was prevented by the simultaneous introduction of carbonate of soda into the stomach, while the introduction of both salts gave rise to all the symptoms of carbamic acid poisoning, when the circulation through the liver had been interrupted. The authors conclude, therefore, that the carbamates formed during digestion in passing through the liver are transformed into a harmless substance, and that this substance is most probably urea.

In some cases the experimenters removed the entire liver; but the animals never lived more than six hours, and fell at once into a comatose state, followed by convulsions, tetanus, and death through arrest of the respiration. Similar results were obtained by establishing a veinous fistula in the first place and tying the hepatic artery afterwards.

According to Messrs. Hahn and Nencki, who performed the chemical part of these observations, the reaction of the urine remained normal until one of the attacks of excitement set in, when it became alkaline. If the hepatic artery were tied at the same time, the urine contained a little albumin and hæmaglobin, together with small quantities of urobilin and biliary pigment, provided the gall-bladder had not been emptied before the operation. The quantity of urea was always greatly lessened if the hepatic artery were also tied, or the greater part of the liver removed. The relation of the nitrogen in urea to the total quantity of nitrogen excreted was much smaller than normal, being only 77 per cent. instead of 89 per cent. On the other hand, the uric acid in the urine ultimately increased in quantity, even when the hepatic artery was not tied, although the total quantity of nitrogen excreted was not greater than normal, the increase in the uric acid corresponding to the setting in of the convulsions. With regard to the ammonia contained in urine, the authors have come to the following conclusions:—(1) Eck's operation, combined with the ligature of the hepatic artery, causes in dogs an increase in the excretion of ammonia. In some cases this increase is relative only with regard to the nitrogen of urea or the total nitrogen, whereas in other cases it is absolute, and this absolute increase takes place when the animals survive the operation for twenty hours at least; (2) the secretion of ammonia increases rapidly in animals which have been subjected to Eck's operation as soon as the first symptoms set in.

In a further series of researches the authors showed that car-

[1] "Archives des Sciences biologiques publiées par l'institut impérial de médecine experimentale à St. Pétersbourg," vol. i. no. 4.

[2] "La fistule d'Eck de la veine cave inférieure et de la veine porte, et ses conséquences pour l'organisme, par MM. les Drs. M. Hahn, V. Massen, M. Nencki, et J. Pawlow."

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