

in the presentation of the subject. Of course, in some sense and to some extent it is and must be true. Whatever is special, accidental, and individual, will die, as it should; but that which is universal and essential should remain as an organic part of the whole intellectual acquisition. If that which is essential dies with the accidental, it must be because the accidental has been given the prominence which belongs to the essential. For myself, I should preach no such doctrine to those whom I wish to convert to the true faith.

In Italy, they say, all roads lead to Rome. In mechanics, kinematics, astronomy, physics, all study leads to the consideration of certain relations and operations. These are the capital notions; these should have the leading parts in any analysis suited to the subject.

If I wished to attract the student of any of these sciences to an algebra for vectors, I should tell him that the fundamental notions of this algebra were exactly those with which he was daily conversant. I should tell him that a vector algebra is so far from being any one man's production that half a century ago several were already working toward an algebra which should be primarily geometrical and not arithmetical, and that there is a remarkable similarity in the results to which these efforts led (see Proc. A. A. S. for 1886, pp. 37, ff.). I should call his attention to the fact that Lagrange and Gauss used the notation $(\alpha\beta\gamma)$ to denote precisely the same as Hamilton by his $S(\alpha\beta\gamma)$, except that Lagrange limited the expression to unit vectors, and Gauss to vectors of which the length is the secant of the latitude, and I should show him that we have only to give up these limitations, and the expression (in connection with the notion of geometrical addition) is endowed with an immense wealth of transformations. I should call his attention to the fact that the notation $[r_1 r_2]$, universal in the theory of orbits, is identical with Hamilton's $V(\rho_1 \rho_2)$, except that Hamilton takes the area as a vector, *i.e.* includes the notion of the direction of the normal to the plane of the triangle, and that with this simple modification (and with the notion of geometrical addition of surfaces as well as of lines) this expression becomes closely connected with the first-mentioned, and is not only endowed with a similar capability for transformation, but enriches the first with new capabilities. In fact, I should tell him that the notions which we use in vector analysis are those which he who reads between the lines will meet on every page of the great masters of analysis, or of those who have probed deepest the secrets of nature, the only difference being that the vector analyst, having regard to the weakness of the human intellect, does as the early painters who wrote beneath their pictures "This is a tree," "This is a horse."

I cannot attach quite so much importance as Mr. McAulay to uniformity of notation. That very uniformity, if it existed among those who use a vector analysis, would rather obscure than reveal their connection with the general course of modern thought in mathematics and physics. There are two ways in which we may measure the progress of any reform. The one consists in counting those who have adopted the *shibboleth* of the reformers; the other measure is the degree in which the community is imbued with the essential principles of the reform. I should apply the broader measure to the present case, and do not find it quite so bad as Mr. McAulay does.

Yet the question of notations, although not the vital question, is certainly important, and I assure Mr. McAulay that reluctance to make unnecessary innovations in notation has been a very powerful motive in restraining me from publication. Indeed my pamphlet on "Vector Analysis," which has excited the animadversion of quaternionists, was never formally published, although rather widely distributed, so long as I had copies to distribute, among those who I thought might be interested in the subject. I may say, however, since I am called upon to defend my position, that I have found the notations of that pamphlet more flexible than those generally used. Mr. McAulay, at least, will understand what I mean by this, if I say that some of the relations which he has thought of sufficient importance to express by means of special devices (see Proc. R. S. E., for 1890-91), may be expressed at least as briefly in the notations which I have used, and without special devices. But I should not have been satisfied for the purposes of my pamphlet with any notation which should suggest even to the careless reader any connection with the notion of the quaternion. For I confess that one of my objects was to show that a system of vector analysis does not require any support from the notion of the quaternion, or, I may add, of the imaginary in algebra.

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I should hardly dare to express myself with so much freedom, if I could not shelter myself behind an authority which will not be questioned.

I do not see that I have done anything very different from what the eminent mathematician upon whom Hamilton's mantle has fallen has been doing, it would seem, unconsciously. Contrast the system of quaternions, which he has described in his sketch of Hamilton's life and work in the *North British Review* for September, 1866, with the system which he urges upon the attention of physicists in the *Philosophical Magazine* in 1890. In 1866 we have a great deal about imaginaries, and nearly as much about the quaternion. In 1890 we have nothing about imaginaries, and little about the quaternion. Prof. Tait has spoken of the calculus of quaternions as throwing off in the course of years its early Cartesian trammels. I wonder that he does not see how well the progress in which he has led may be described as throwing off the yoke of the quaternion. A characteristic example is seen in the use of the symbol ∇ . Hamilton applies this to a vector to form a quaternion, Tait to form a linear vector function. But while breathing a new life into the formulæ of quaternions, Prof. Tait stands stoutly by the letter.

Now I appreciate and admire the generous loyalty toward one whom he regards as his master, which has always led Prof. Tait to minimise the originality of his own work in regard to quaternions, and write as if everything was contained in the ideas which flashed into the mind of Hamilton at the classic Brougham Bridge. But not to speak of other claims of historical justice, we owe duties to our scholars as well as to our teachers, and the world is too large, and the current of modern thought is too broad, to be confined by the *ipse dixit* even of a Hamilton.

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Glacial Drift of the Irish Channel.

It seems of interest to record that the eurite or microgranite containing blue amphibole (Riebeckite), the rock noticed by Mr. P. F. Kendall in the drifts of the Isle of Man and Caernarvonshire, occurs abundantly in the form of small pebbles on the shore at Killiney, co. Dublin, doubtless derived from the "glacial gravels" of the coast. I have also found a pebble in the raised beach at Greenore, co. Down.

Mr. Teall's description of the rock of Ailsa Craig (*Mineralogical Magazine*, vol. ix. p. 219) enabled the very characteristic pebbles collected by Mr. Kendall to be referred to that mass as a source, or to formerly existing bosses south of or adjacent to it. As far as I am aware, all the material is in the form of pebbles, often only an inch in diameter. This is hardly likely to be its original condition, if removed by ice from Ailsa Craig, and is only one of many points that indicate a redistribution of our so-called "glacial" beds by subsequent action of rivers or other waters.

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March 12.

THE SACRED NILE.

THAT Egypt is the gift of the Nile is a remark we owe to the father of history, who referred not only to the fertilising influence of the stream, but to the fact that the presence of the Nile and its phenomena are the conditions upon which the habitability of Egypt altogether depends. That that part of Egyptian archæology and myth which chiefly interests astronomers is also the gift of the Nile is equally true.

The heliacal rising of Sirius and other stars at the time of the commencement of the inundations each year; all the myths which grew out of the various symbols of the stars so used, are so many evidences of the large share the river, with its various water levels at different times, had in the national life. It was, in fact, the true and unique basis of the national life.

In this the Nile had a compeer, or even compeers. What the Nile was to Egypt the Euphrates and Tigris were to a large region of Western Asia, where also we find the annual flood to have been in ancient times a source of fertility over an enormous area which is now