

potash destroys it gradually, forming potassium chloride, fluoride, and carbonate. It was obtained by treating carbon tetrachloride with a mixture of antimony trifluoride and bromine in equal molecular proportions. It is notable that the bromofluoride produced by the mixture acts not as a bromising but a fluorising agent.—On a simplification of some of Tesla's experiments, by H. Schoentjes. Like some recent workers in England, Prof. Schoentjes has found that most of the experiments can be produced, although with lesser intensity, without the bobbin immersed in oil, the discharge exciter, and the condenser, simply by the first Rhumkorff coil, whose dimensions need not exceed 7×17 cm.—On a process of sterilisation of albumin solutions at 100°C ., by Émile Marchal. Albumin can be easily sterilised at 100°C ., without coagulation, by first adding 0.05 gr. per litre of borax, or 0.005 of ferrous sulphate in a 2 to 5 per cent. solution, or 4 to 5 gr. nitrate of urea per litre of 10 per cent. solution. The "incoagulable albumin" thus obtained is perfectly suitable for cultivations.

SOCIETIES AND ACADEMIES.

LONDON.

Royal Society, November 24, 1892.—"Memoir on the Theory of the Compositions of Numbers," by P. A. MacMahon, Major R. A., F. R. S.

In the theory of the partitions of numbers the order of occurrence of the parts is immaterial. Compositions of numbers are merely partitions in which the order of the parts is essential. In the nomenclature I have followed H. J. S. Smith and J. W. L. Glaisher. What are called "unipartite" numbers are such as may be taken to enumerate undistinguished objects. "Multipartite" numbers enumerate objects which are distinguished from one another to any given extent; and the objects are appropriately enumerated by an ordered assemblage

of integers, each integer being a unipartite number which specifies the number of objects of a particular kind; and such assemblage constitutes a multipartite number. The 1st Section treats of the compositions of unipartite numbers both analytically and graphically. The subject is of great simplicity, and is only given as a suitable introduction to the more difficult theory, connected with multipartite numbers, which is developed in the succeeding sections.

The investigation arose in an interesting manner. In the theory of the partitions of integers, certain partitions came under view which may be defined as possessing the property of involving a partition of every lower integer in a unique manner. These have been termed "perfect partitions," and it was curious that their enumeration proved to be identical with that of certain expressions which were obviously "compositions" of multipartite numbers.

The generating function which enumerates the composition has the equivalent forms—

$$\frac{h_1 + h_2 + h_3 + \dots}{1 - h_1 - h_2 - h_3 - \dots},$$

$$\frac{a_1 - a_2 + a_3 - \dots}{1 - 2(a_1 - a_2 + a_3 - \dots)},$$

where h_s, a_s represent respectively the sum of the homogeneous products of order s and the sum of the products s together of quantities

$$a_1, a_2, a_3, \dots, a_n,$$

and the number of compositions of the multipartite

$$p_1 p_2 \dots p_n$$

is the coefficient of $a_1^{p_1} a_2^{p_2} \dots a_n^{p_n}$ in the development according to ascending powers.

It is established that

$$\frac{1}{2} \frac{1 - s_1(2a_1 + a_2 + \dots + a_n)}{1 - s_2(2a_1 + 2a_2 + \dots + a_n)} \dots \frac{1}{1 - s_n(2a_1 + 2a_2 + \dots + 2a_n)}$$

is also a generating function which enumerates the compositions; the coefficient of

$$s_1^{p_1} s_2^{p_2} \dots s_n^{p_n} a_1^{p_1} a_2^{p_2} \dots a_n^{p_n}$$

being the number of compositions possessed by the multipartite

$$p_1 p_2 \dots p_n.$$

The previous generating function may, by the addition of the fraction $\frac{1}{2}$ and the substitution of $s_1 a_1, s_2 a_2, \&c.$, for $a_1, a_2, \&c.$, be thrown into the form

$$\frac{1}{2} \frac{1}{1 - 2(\sum s_1 a_1 - \sum s_1 s_2 a_1 a_2 + \dots (-)^{n+1} s_1 s_2 \dots s_n a_1 a_2 \dots a_n)}$$

and hence these two fractions, in regard to the terms in their expansions which are products of powers of $s_1 a_1, s_2 a_2, \dots, s_n a_n$, must be identical. This fact is proved by means of the identity—

$$\frac{1}{2} \frac{1 - s_1(2a_1 + a_2 + \dots + a_n)}{1 - s_2(2a_1 + 2a_2 + \dots + a_n)} \dots \frac{1}{1 - s_n(2a_1 + 2a_2 + \dots + 2a_n)} = \frac{1}{2} \frac{1}{1 - 2(\sum s_1 a_1 - \sum s_1 s_2 a_1 a_2 + \dots (-)^{n+1} s_1 s_2 \dots s_n a_1 a_2 \dots a_n)}$$

multiplied by

$$1 + \sum \frac{2(A_{k1} + a_{k1}) \dots (A_{kt} + a_{kt}) - (A_{k1} + 2a_{k1}) \dots (A_{kt} + 2a_{kt})}{(1 - S_{k1}) \dots (1 - S_{kt})} s_{k1} s_{k2} \dots s_{kn},$$

where

$$S_k = s_k(2a_1 + \dots + 2a_k + a_{k+1} + \dots + a_n) = s_k(A_k + 2a_k),$$

and the summation is in regard to every selection of t integers from the series

$$1, 2, 3, \dots, n,$$

and t takes all values from 1 to $n - 1$.

This remarkable theorem leads to a crowd of results which are interesting in the theory of numbers.

The geometrical method of "trees" finds a place, and, lastly, there is the fundamental algebraic identity—

$$\frac{1}{k} \frac{1 - s_1(ka_1 + a_2 + \dots + a_n)}{1 - s_2(ka_1 + ka_2 + \dots + a_n)} \dots \frac{1}{1 - s_n(ka_1 + ka_2 + \dots + ka_n)} = \frac{1}{k} \frac{1 - k \sum s_1 a_1 + k(k-1) \sum s_1 s_2 a_1 a_2 - \dots + (-)^n k(k-1)^{n-1} s_1 s_2 \dots s_n a_1 a_2 \dots a_n}{1 - k \sum s_1 a_1 + k(k-1) \sum s_1 s_2 a_1 a_2 - \dots + (-)^n k(k-1)^{n-1} s_1 s_2 \dots s_n a_1 a_2 \dots a_n}$$

multiplied by

$$1 + \sum \frac{k(A_{t1} + a_{t1}) \dots (A_{tn} + a_{tn}) - (A_{t1} + ka_{t1}) \dots (A_{tn} + ka_{tn})}{(k-1)(1 - S_{t1})(1 - S_{t2}) \dots (1 - S_{tn})} s_{t1} s_{t2} \dots s_{tn},$$

which reduces to that formerly obtained when k is given the special value 2.

to move. The photographs show the successive phases of one entire motion of the fins, which consists of a wave-like motion beginning in front. Shortly after the anterior portion has been lifted it is depressed, the motion being meanwhile propagated to the lateral portions, and growing in amplitude as the fin grows in breadth. Just before the movement dies out near the tail the process recommences in front. The periodic time was 0.8 seconds. The photographs show a striking likeness to those obtained by chronophotography applied to the flight of birds. M. Marey intends to study the mechanical effect of the action of the fins upon the water, also by the aid of photography.—Microscopic researches on the contractility of the blood-vessels, by M. L. Ranvier. The pericesophagian membrane of the frog was placed on the disc of the slide-cell in one or two drops of peritoneal serum. It was kept extended by a platinum ring; electrodes of tinfoil were placed in connection, and a cover glass was fixed over the whole with paraffin. Thus mounted, the smooth muscular fibres and the internal elastic sheath are well seen. On connecting the induction coil with the electrodes, the muscular fibres contract as soon as the current is strong enough. At the same time, the folds of the internal sheath become more pronounced and finally touch, thus effacing the passage through the small artery. On breaking the current, the artery gradually regains its original diameter. If the current is not sufficiently strong for producing a regular contraction, some of the segments contract, while others are at rest. But the zone of contraction is never displaced, and, if interrupted, will reappear at the same place on reestablishing the current. Nothing corresponding to a peristaltic motion can be produced by direct electrical excitement. In none of the experiments, even with the strongest currents, was it possible to detect any signs of contraction in the capillaries.—On the sum of the logarithms of the first numbers not exceeding x , by M. Cahen.—On differential equations of a higher order, the integral of which only admits of a finite number of determinations, by M. Paul Painlevé.—On linear differential equations with rational coefficients, by M. Heuge von Koch.—Electric waves in wires; depression of the wave propagated in conductors, by M. Birkeland (see *Wiedemann's Annalen*, abstract).—On the minimum perceptible amount of light, by M. Charles Henry. This was estimated by Aubert at $\frac{1}{30000}$ th of the light of the full moon. This is about a thousand times too great, as proved by some measurements made with the zinc-sulphide (phosphorescence) photometer previously described. The corrected formula for the rate of loss of luminosity of the sulphide is $i^{0.5}(t - 18.5) = 1777.8$, which agrees even with the longest observations, and is theoretically justified by M. Henri Becquerel. The minimum perceptible amount of light was determined by noticing the time at which the eye, previously kept in the dark for one hour, could only just distinguish the light emitted by the phosphorescent substance, taking care to test for illusions by the successive interposition of ground-glass screens. The time thus found was four hours, giving an amount of light of 29×10^{-9} standard candles at 1 m. If the eye is previously kept in the dark during varying periods, the minimum varies inversely as the square of the time during which it is kept dark.—On phosphorescent sulphide of zinc, considered as a photometric standard, by the same. Careful tests showed that the light emitted by zinc sulphide at a given instant is independent of the distance of the illuminating magnesium ribbon, of the time of illumination, and of the thickness of the layer, and is also uniform in samples prepared under different conditions, thus exhibiting all the requisites of a secondary photometric standard.—On an acid platino-nitrite of potassium, by M. M. Vèzes.—Decomposition of chloroform in presence of iodine, by M. A. Besson.—On some ethers of homopyrocatechine, by M. H. Cousin.—On the determination of phosphorus in iron and steel, by M. Adolphe Carnot. The new method, based like most others on the employment of ammonium molybdate, differs from them in the mode of separation of the silicon, which is effected by sulphuric acid; in the process of destruction of the carbon compounds, brought about by chromic acid; and in the nature of the final compound, which is not magnesium pyrophosphate, but dry phosphomolybdate of ammonia, which only contains 1.628 per cent. of phosphorus, thus ensuring a greater accuracy in the quantitative estimation.—Losses of nitrogen in manure, by M. M. A. Muntz and A. Ch. Girard.—Researches on the localisation of the fatty oils in the germination of seeds, by M. Eugène Mesnard. It appears that, except in the grasses, the fatty oil

is not specially localised. It is in all cases independent of the starch and the glucose, but it appears superposed upon the albuminoid materials in the reserves of ripe seeds.

BERLIN.

Physical Society, December 16, 1892.—Prof. Kundt, President, in the chair.—Dr. Lummer spoke on the principles involved in the use of half-shade polarimeters. He showed that the difference in brightness of the two halves of the field of the instrument depends first on the angle between the two polarising prisms, the less this is the greater being the difference produced by a minimal rotation of the analyzer, and secondly on the power of perceiving minute differences of brightness. In connection with the latter he had made some changes in the Lippich instrument which presented some distinct advantages.—Prof. Goldstein gave an account of some experiments made many years ago, but not yet published. He first dealt with the light which appears at the anode, and which, as compared with that of the kathode, has as yet been but little investigated. As is well known, a kathode consisting of two metals emits rays of different brightness from its two parts, thus for instance the aluminium emits brighter rays than does the silver. When this electrode is used as an anode, the reverse holds good, inasmuch as the anodic light of silver is brighter than that of aluminium. The difference is, however, only observed in rarefied oxygen, and does not exist in a hydrogen tube, and is hence due to oxidation of the silver. The second set of experiments dealt with Crookes' supposed reciprocity deflection of cathodic rays of similar direction. The speaker had shown, by shielding one of the electrodes, that the deflection is apparent, not real. The change in the path of the cathodic radiation is due entirely to the effect of the second electrode upon the rays emitted by the first.

CONTENTS.

	PAGE
Modern Advanced Analysis. By P. A. M.	289
The Darwinian Theory	290
Ferns of South Africa. By J. G. Baker, F.R.S.	291
Our Book Shelf:—	
Vogel: "Newcomb-Engelmann's Populäre Astronomie, Zweite vermehrte Auflage."—A. T.	291
Saunders: "The Hemiptera Heteroptera of the British Islands."—W. L. D.	292
Treves: "Physical Education"	292
Letters to the Editor:—	
The Geology of the North-West Highlands.—Sir Archibald Geikie, LL.D., F.R.S.	292
The Identity of Energy.—Prof. Oliver Lodge, F.R.S.	293
A Proposed Handbook of the British Marine Fauna. Prof. W. A. Herdman, F.R.S.; W. Garstang	293
Fossil Plants as Tests of Climate.—Chas. E. De Rance	294
Racial Dwarfs in the Pyrenees.—R. G. Haliburton; Wm. McPherson	294
British Earthworms.—Frank J. Cole	295
Dante's "Quæstio de Aqua et Terra." (With Diagrams.) By Edmund G. Gardner	295
Morocco	298
The Rate of Explosion in Gases. Prof. Harold B. Dixon	299
Notes	300
Our Astronomical Column:—	
Comet Holmes	303
Comet Brooks (November 19, 1892)	304
Photographic Absorption of our Atmosphere	304
Harvard College Observatory	304
Solar Observations at Rome	304
The Total Solar Eclipse, April 15-16, 1893	304
Geographical Notes	304
The Approaching Eclipse of the Sun, April 16, 1893. M. De la Baume Pluviney	304
Memorial of Sir Richard Owen	307
Scientific Serials	309
Societies and Academies	310