refractive index of 1·53. The equiangular prisms cause less loss of light by absorption and reflection than either the spherical or Fresnel refractors, and also act on the light so that ex-focal light is better dealt with, thereby reducing the divergence.

CHARLES A. STEVENSON.

MODERN DYNAMICAL METHODS.

A DYNAMICAL system is said to possess a *given* number of degrees of freedom, when it is capable of assuming the *same* number of independent positions. The position of the system, in any possible configuration, is capable of being determined by a definite number of independent quantities, which are equal to the number of degrees of freedom of the system. These quantities are called the co-ordinates of the system.

When the system possesses *six* degrees of freedom, the motion may be completely determined by expressing in mathematical language a principle which may be conveniently termed the *principle of momentum*. This principle is specified by the following two propositions:—(i.) *The rate of change of the component of the linear momentum, parallel to an axis, of any dynamical system, is equal to the component, parallel to that axis, of the impressed forces which act upon the system;* (ii.) *the rate of change of the component of the angular momentum about any axis, is equal to the moment of the impressed forces about that axis.* Since the motion of the system may be referred to any set of fixed or moving rectangular axes, the above-mentioned dynamical principle furnishes six equations connecting the six co-ordinates, which, when integrated, will determine the latter in terms of the time and the initial circumstances of the motion.

The various ways of expressing this dynamical principle in mathematical language are explained in treatises on dynamics: and a variety of special forms and particular cases are obtained, by means of which the solution of numerous problems can be simplified. For example, Euler's equations, for determining the motion of rotation of a single rigid body about its centre of inertia, is a particular case of the second proposition; whilst Kirchhoff's equations, for determining the motion of a single solid in an infinite liquid, is a special form of both propositions.

When a conservative system possesses seven degrees of freedom, the motion may be completely determined by means of the principle of momentum combined with the principle of energy. The first principle, as we have already shown, furnishes six equations, whilst the second furnishes one; hence, we have a sufficient number of equations for determining the motion.

When a dynamical system possesses more than seven degrees of freedom, the principles of momentum and energy are insufficient to determine the motion; and under these circumstances, the most convenient method to adopt is to use Lagrange's equations; but inasmuch as these equations are double-edged tools, which are apt to cut the fingers of the unwary, their employment requires considerable care.

The kinetic energy of a dynamical system can be expressed in a variety of different forms, but it will only be necessary to mention the following three. In the first form, it is expressed as a homogeneous quadratic function of velocities, which are the time-variations of the co-ordinates of the system. This form, which will be denoted by T, is called the *Lagrangian form;* it is the only one which it is permissible to use when employing Lagrange's equations, and many mistakes have been made by persons who have attempted to use some other form.

In the second form, which is called the *Hamiltonian form,* the kinetic energy is expressed as a homogeneous quadratic function of the momenta of the system. If θ be any co-ordinate, and Θ the generalized momentum of type θ, it is known that

$$\frac{\partial T}{\partial \dot{\theta}} = \Theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

whence Θ is a linear function of the velocities. Hence, if the velocities be eliminated from the Lagrangian expression for the kinetic energy by means of (1), it follows that the latter will be expressible as a homogeneous quadratic function of the momenta Θ, which is the Hamiltonian form. We shall denote this form by $\mathfrak{T}$.

Lagrange's equations are

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} = -\frac{\partial V}{\partial \theta} \quad . \quad . \quad . \quad . \quad (2)$$

where V is the potential energy; and if the elimination be performed, we shall obtain

$$\frac{d\Theta}{dt} + \frac{\partial \mathfrak{T}}{\partial \theta} = -\frac{\partial V}{\partial \theta} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

we have also the reciprocal relation

$$\frac{\partial \mathfrak{T}}{\partial \Theta} = \dot{\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Equations (3) and (4) are Hamilton's equations of motion.

The third form of the expression for the kinetic energy is of special importance in hydrodynamics and other branches of physics. It sometimes happens that a quantity occurs which can be recognized as a momentum, or as a quantity in the nature of a momentum, whilst the velocity corresponding to this momentum is either unknown or would be inconvenient to introduce. This occurs in problems relating to the motion of perforated solids in a liquid, when there is circulation, and is a particular case of Dr. Routh's theory of the "Ignoration of Velocities."[1] We therefore require a form of Lagrange's equations in which certain velocities are eliminated, and are replaced by the corresponding momenta.

Let the co-ordinates of the system be divided into two groups, θ and χ; and let κ denote the generalized momentum corresponding to χ. Then

$$\frac{\partial T}{\partial \dot{\chi}} = \kappa \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

By means of (5) all the velocities $\dot{\chi}$ can be eliminated from the expression for the kinetic energy; and it is remarkable, that the result of the elimination does not contain any products of the form $\kappa\dot{\theta}$. The expression for T may accordingly be written

$$T = \mathfrak{X} + \mathfrak{K} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where $\mathfrak{X}$ is a homogeneous quadratic function of the velocities $\dot{\theta}$, and $\mathfrak{K}$ is a similar function of the momenta κ.

Equation (6) is therefore a mixed form, which is partly Lagrangian and partly Hamiltonian. We now require the corresponding form of the equations of motion in which all the $\dot{\chi}$'s have been eliminated from Lagrange's equations.

From (1) it follows that the generalized momentum Θ is a linear function of the velocities $\dot{\theta}$, $\dot{\chi}$; and if the latter velocities be eliminated by means of (5), it follows that Θ is expressible as a linear function of $\dot{\theta}$, κ. Let the portion which is a linear function of the κ's be denoted by $\overline{\Theta}$; then it can be shown, that if

$$L = \mathfrak{X} + \Sigma(\overline{\Theta}\dot{\theta}) - \mathfrak{K} - V \quad . \quad . \quad . \quad . \quad (7)$$

the equation of type θ is

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

[1] Having regard to the object of the theory, I think the phrase "Ignoration of Velocities" is better than "Ignoration of Co-ordinates."

whilst that of type χ is

$$\frac{d\kappa}{dt} - \frac{\partial \mathrm{L}}{\partial \chi} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

We have also the additional equations

$$\Theta = \frac{\partial \mathfrak{T}}{\partial \theta} + \Theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$\dot{\chi} = \frac{\partial \mathfrak{K}}{\partial \kappa} - \Sigma\left(\dot{\theta}\frac{\partial \Theta}{\partial \kappa}\right) \quad . \quad . \quad . \quad . \quad (11)$$

Equations (7) to (11) were first given by myself in a paper published in the Proc. Camb. Phil. Soc. for 1887; and it will be observed that they include the equations of Lagrange and Hamilton. A form of the modified Lagrangian function, which is equivalent to (7), was given by Dr. Routh a few years previously; but it is not of much practical use, owing to the fact that the elimination of the velocities $\dot{\chi}$ has not been performed.

It sometimes happens that the *co-ordinates* of the type χ do not enter into the expression for the energy of the system, in which case they are called *ignored co-ordinates*;[1] under these circumstances it follows from (9), that all the momenta κ are absolute currents. A top spinning about its point under the action of gravity is one of the most familiar examples of ignored co-ordinates, and one which illustrates several important dynamical theorems.

When there are ignored co-ordinates, the steady motion of the system, and the stability of the steady motion, can very easily be investigated. For if we suppose that all the co-ordinates θ have constant values, (8) reduces to

$$\frac{\partial \mathfrak{K}}{\partial \theta} + \frac{\partial \mathrm{V}}{\partial \theta} = 0.$$

There are as many equations of this type as there are co-ordinates θ, and an examination of this system of equations will show whether steady motion is possible, and if so, will determine the necessary conditions which the co-ordinates θ and the constant momenta κ must satisfy.

It can also be shown that the steady motion will always be stable when $\mathfrak{K} + \mathrm{V}$ is a minimum (see Proc. Camb. Phil. Soc. May 1892).

We have therefore the following simple rule for determining the steady motion of a dynamical system when there are ignored co-ordinates. Eliminate all the velocities corresponding to these co-ordinates from the expression for the kinetic energy of the system, so that the latter is expressed in terms of the velocities $\dot{\theta}$ and the momenta κ. Let $\mathfrak{K}$ and V be that portion of the *total* energy which does not depend upon the $\dot{\theta}$'s; then the conditions of steady motion are, that $\mathfrak{K} + \mathrm{V}$ should be stationary, and the steady motion will be stable provided this quantity is a minimum.

The preceding theorem also enables us to deduce by a very concise method all the results connected with the steady motion of a liquid ellipsoid, which is rotating about a principal axis under the influence of its own attraction. It also enables us to examine the stability of these different cases of steady motion, for disturbances which produce an ellipsoidal displacement.

A. B. BASSET.

THE PASSAGE OF GRANITE ROCK INTO FERTILE SOIL.

HAVING for the last three or four years paid particular attention to the natural formation of soil, I venture to believe that a concise account, or rather summing up, of the results of my researches, and of the mass of my observations—in one typical direction—may be of interest to the readers of NATURE. As indicated in the heading, the making of soil from granite is the only section of a very large subject which will be briefly considered in this paper.

The agents concerned with the turning of granite (or any other rock) into a fertile soil may be shortly classified as mechanical, chemical, and vital. The first-named produce the largest results in bulk, and the principal mechanical agent with which we have to deal is frost. The second and third classes of forces are extremely important, as it is by their actions that the raw material of plant-food is prepared, though unfortunately poisons also are brought into being through their activity. These last-named classes, however, likewise materially aid the action of frost (or, in tropical countries, of varying temperatures) in the mechanical separation of rocky matter. To render my descriptions as little confusing as possible I will endeavour, without regard to classification, order, or divisions, to trace the history of a granite soil as I have observed it in many localities in Scotland, from the practically unbroken rock into the condition in which it has been made by nature fit to bear the most luxuriant crops. But first of all I must remind my readers of two or three geological facts about granite. It is a holocrystalline (*i.e.* wholly crystalline) igneous rock, composed essentially of orthoclase, quartz, and mica. In its most typical condition the last-named mineral is always of the biotite or magnesia-mica species. Besides these essentials we always find (in Scotch granites at least) plagioclase, other species of mica than the essential, apatite as an endomorph, *i.e.* locked up in the mass of other minerals, and magnetite, and almost invariably, if not always, a little pyrites, and more or less hornblende, &c.

A rough mineralogical analysis of Kemnay granite taken from the lowest working of the well-known quarry in Aberdeenshire gave the following percentages:—

Orthoclase-felspar	42·00
Quartz...	22·00
Biotite-mica	20·00
Plagioclase-felspar	9·00
Hornblende	3·25
Muscovite-mica	2·00
Magnetite (and Ilmenite)	1·00
Pyrites	0·50
Apatite	0·25
Total...	100·00

The first change which comes over granite is the peroxidation of some of the iron always present in its mass. This sets in, and increases to the greatest extent, of course, where air and water can most readily enter. The surface of the rock becomes browned with the hydrated ferric oxide formed, and brown skins, of a deeper colour than the surface generally, coat the walls of the original rock joints. But in the mass of the rock, away from these primary fissures, there are areas which are more permeable than others from the surface, and through these, streaks of ferric oxide—anhydrous first, afterwards hydrated—are produced. Those lines of rust are the beginnings of a new set of joints, which have not yet been properly recognized in geological literature, and which I will here call weather joints to distinguish them from the primary joints of consolidation and rock movements. The first oxidation streaks of the coming weather fissures are invisible to the eye, but can be determined under the microscope. They gradually increase in width above as they extend their lines beneath, and they afford passages through which water can more readily percolate than in the surrounding fresher areas, and as a consequence planes along which frost can more powerfully act. By

[1] I must confess that I do not like the phrase *speed co-ordinates*, introduced by Prof. J. J. Thomson, for it conveys absolutely no meaning to my mind. I have no sympathy with the attempts, which have occasionally been made, to introduce short words of Teutonic origin into scientific nomenclature, as the words in question appear to me to be singularly deficient in point.