water. The odd circumstance about the damage was that it occurred on a simple grass slope, about half way between a tall boat-house on the one side and a drinking fountain standing on more elevated ground on the other. Small trees also were in the neighbourhood, and there was no apparent cause why the flash should have selected this particular spot; though indeed it was not within any of the ordinarily accepted "areas of protection." A gentleman—Mr. Hewitt—proposed digging for the meteor, and although fairly convinced that it was nothing but an ordinary flash, we thought it just possible that an accidental meteorite might have fallen during the thunderstorm; in which event a flash down the rarefied air of its trail would be a natural consequence. It may be just possible that the popular belief in thunderbolts has some such foundation.

At any rate the excavation was made, with the result of proving that it was an ordinary flash and that the lightning made use of a surface drain-pipe, about four feet deep, to get at the water of the lake.

I enclose Mr. Hewitt's report.

OLIVER J. LODGE.

DURING a thunderstorm on the afternoon of Sunday, July 3, 1892, what is described as a "ball of fire" was seen by several persons to descend to the ground, near the south end of the lake in Sefton Park; and immediately afterwards a column of water, about sixty feet high, was shot up from the lake. On examining the spot where the ball of fire was seen to descend, several clean-cut holes were observed, and a sod was also found at a little distance from the spot.

A few days afterwards an excavation was carefully made. The sod being removed, the holes were traced down to a surface drain pipe four feet below the surface. At this drain the holes terminated, and the pipe was found shattered. The important holes were found to be six, the largest being seven inches in diameter, the others about two inches. No meteoric matter was found, but it seems curious that this effect was brought about by a flash of lightning only, in an open space of sloping grass, when there were trees and houses close by.

Aigburth, Liverpool.

GEORGE H. HEWITT.

Peripatus Re-discovered in Jamaica.

MRS. E. M. SWAINSON has been so fortunate as to find on Beacon Hill, near Bath, three specimens of *Peripatus*, which she has sent to the Institute of Jamaica. The species is doubtless identical with that found by Gosse many years ago at the other end of the island. Of the two specimens which we have studied, one has 36 pairs of legs, and is dark pinkish-brown, with the ends of the antennæ pure white, in striking contrast; the other is smaller and darker, without white ends to the antennæ, and with only 29 pairs of legs. The third example, which we have still alive, is larger, but dark in colour. Full details will be given elsewhere later on, and it may suffice for the present to state that the species is very closely allied to *P. Edwardsii* from Venezuela, as described by Sedgwick, but differs in the greater number of legs and the white-tipped antennæ of certain individuals (probably the females), in the only slightly curved (not hooked) claws, in the differentiation of the papillæ into two distinct kinds on the dorsal surface, and apparently in other minor matters. There is no dark dorsal line. The genital orifice is between the penultimate pair of legs; and the jaws are almost precisely as in *Edwardsii*. The Jamaican species being evidently new, it is proposed to call it *Peripatus jamaicensis*.

M. GRABHAM.

September 5.

T. D. A. COCKERELL.

Reflection on Valley Fog.

A LETTER from an observer at the Lick Observatory appeared in NATURE on August 25, reporting the reflection of mountains in a valley fog. I was therefore much interested to note the following in the *Yorkshire Herald* of September 7:—

"SIR,—Possibly it may interest your readers to hear of a natural phenomenon I noticed this morning before 6 a.m. Overlooking, from Leyburn, the valley of Wensleydale, it appeared as though more than half of the dale was filled with water, like a great lake with rising hills on either side, and these hill-sides, above the level of the (apparent) flood, were distinctly reflected in it. The sun was shining brightly at the time, but almost immediately the mist began to disperse, and the mirage faded away. What struck me as unusual was the

extraordinary distinctness of the reflection. Yours, AN EARLY RISER. September 5, 1892."

In both cases the reflecting film seems to have been near its vanishing point.

J. EDMUND CLARK.

Impure Water in Bread.

SOME accurate answers to the following questions would be desirable, in view of public health.

(1) What bacilli—if any—can survive in the amount and duration of the heat of baking in the interior of unfermented bread?

(2) What is the further effect of the carbonic acid of fermentation?

(3) What is the effect of the water being highly carbonated without fermentation, as in aerated bread?

W. M. F. P.

The Comets of Brorsen (1846 VII.) and Brooks (1892 "d").

THE elements of Brooks's comet "1892 d," as computed by Berberich from four observations made between August 31 and September 5, bear a strong resemblance to those of Brorsen's comet of 1846, calculated by Oudemans, the figures being—

T	Comet Brorsen (1846 VII.)		Comet Brooks (1892 d)	
	1846 June 5	1847 June 5	1892 Dec. 19	1892 Dec. 19
ω	260	12 50	269	24 27
Ω	261	53 12	261	2 55
i	29	18 47	27	57 8
Log q	9.80188		9.84455	

Brorsen's comet of 1846 was visible to the naked eye on May 14 of that year. It was supposed to be revolving in an elliptical orbit, with a period of about 400 years.

W. F. DENNING.

Bristol, September 22.

NOTE ON THE PROGRESS OF THE DIOPTRIC LENS AS USED IN LIGHTHOUSE ILLUMINATION.

FRESNEL, in 1820, devised and constructed a lens for first order lights of 920 mm. radius. It was composed of a plano-convex lens, with five refracting prisms concentric with it, and four segments of rings in the corners all gradually decreasing in breadth as they receded from the centre. The separate pieces of which these lenses were made up were cemented together and mounted in metallic frames 30 inches square.

In 1835, the late Mr. Alan Stevenson introduced the French apparatus into Great Britain. In doing so he made several improvements, one of which was that he increased the height of the lens from 30 to 39 inches, at the same time diminishing the thickness of the glass. This refractor had eight prisms above and eight prisms below the central lens. From that time Alan Stevenson's lens was almost universally used until a comparatively recent date, when a revolution in the size of lenses took place.

A few years ago inventors were trying to obtain greater power by increasing the diameter and volume of the flames; but Messrs. Stevenson pointed out, in 1869, that after a certain point an increase of diameter of the luminary not accompanied by a corresponding increase of the radius of the apparatus was a mistake, as the light became ex-focal and divergent, and that the proper way to secure greater power was to enlarge the diameter of the apparatus. In 1885 they had a lens made to their design of 1330 mm. radius, and having a height of 5 feet. This lens, which was named "Hyper-radiant," was tried at the South Foreland against other lenses, and with a large 10-ring gas burner it was found to give a light from one and a half to twice as intense as the ordinary lenses which were pitted against it, with the same large burners in their foci, thus proving conclusively that to get the power out of large burners it was imperatively necessary

to increase the diameter of the apparatus. In 1883 Messrs. Stevenson got an offer from Messrs. Barbier for a lens of 1840 mm. focal distance.

All refracting lenses from the day of Alan Stevenson were cylindrical for fixed lights and plano-convex for revolving lights, and no alteration of any moment has been made in the mode of their construction until 1888, when, instead of making the lenses cylindrical or plano-convex, I proposed to give them a spherical form, that is to say, circular not only in the horizontal but also in the vertical section. This design was carried into practice in the apparatus for one of the Fair Isle Lighthouses. The introduction of the spherical refractor has made practicable the construction of very much larger and consequently more powerful apparatus, and occupying much less space both in the daylight size and diameter of lantern, and, hence, diameter of tower. It has rendered practicable the quadrilateral arrangement with hyper-radiant lenses which have already been erected at Fair Isle, the lenses being cut so as to give two flashes from each side of the quadrilateral. An experimental lens made for Mr. Wigham is to be tried in Ireland. It is 2m. focal distance, and the spherical refractor is 7 feet 6 inches diameter, and will give one flash from each side of the quadrilateral. M. Barbier says that in making this lens he was attempting to give the most powerful flash possible, and he adopted the spherical refractor. In this case, however, the spherical refractor has been carried, in my opinion, rather far, except in the view of economy in keeping the angles of the whole apparatus within reasonable limits, which is only possible (in an apparatus of 4m. diameter) by the use of the spherical refractor, and by its being made to subtend a great angle. When I proposed this form I pointed out that there was a loss of efficiency if it subtended more than 40° ; now M. Barbier has made the spherical portion subtend 64° , or 24° farther, and 8° farther than any spherical refractor yet made. The great amount of light which I experimentally found re-

by the greater divergence which takes place in the spherical refractor, and which would be a *small* source of loss in a revolving light, but would better illuminate the nearer sea in a fixed arc.

Equiangular Refractor.

To obviate the loss of light at the outer face of the lenses, especially those remote from the focal plane—a

FIG. 1

turned from the inner face of the spherical refractor made for Fair Isle, however, shows that up to 20° , and perhaps farther, it is ample to make up for any loss of light caused

FIG. 2.

loss which stops the refractor being carried with due regard to efficiency farther than 30° to 40° in the cylindrical form, and 20° in the spherical form—it will be found that the most efficient form is a refractor which I proposed, with the inner face of each lens doing equal work with the outer face, or nearly so—and by a careful study of such a refractor it will be found that the locus of the centres of curvature of each refracting lens lies outside the refractor, and at points below the focal plane, and more and more remote from the lens, as the lenses are more and more remote from the focal plane, and that the inner face of the refractor ought, in fact, to be a parabolic curve (Fig. 1). This can very closely be approximated to by a circular curve with a suitably chosen centre on the focal line produced, but the centre is so far distant that when the spherical form and the equiangular form are used in combination (Fig. 2), the inner face of the equiangular prisms above and below it may, with sufficient accuracy, be made straight and leaning outwards in place of being vertical as in Fresnel's form. By the combination of the spherical refractor and the equiangular, or a refractor of the equiangular form alone, the defect in Fresnel's refractor, namely the loss of light at emergence from the lenses, especially those remote from the focal plane, is avoided, and the refractor may thus be made to subtend an angle which has hitherto been considered inexpedient, with glass of the ordinary

refractive index of 1.53. The equiangular prisms cause less loss of light by absorption and reflection than either the spherical or Fresnel refractors, and also act on the light so that ex-focal light is better dealt with, thereby reducing the divergence.

CHARLES A. STEVENSON.

MODERN DYNAMICAL METHODS.

A DYNAMICAL system is said to possess a *given* number of degrees of freedom, when it is capable of assuming the *same* number of independent positions. The position of the system, in any possible configuration, is capable of being determined by a definite number of independent quantities, which are equal to the number of degrees of freedom of the system. These quantities are called the co-ordinates of the system.

When the system possesses *six* degrees of freedom, the motion may be completely determined by expressing in mathematical language a principle which may be conveniently termed the *principle of momentum*. This principle is specified by the following two propositions:— (i.) *The rate of change of the component of the linear momentum, parallel to an axis, of any dynamical system, is equal to the component, parallel to that axis, of the impressed forces which act upon the system*; (ii.) *the rate of change of the component of the angular momentum about any axis, is equal to the moment of the impressed forces about that axis*. Since the motion of the system may be referred to any set of fixed or moving rectangular axes, the above-mentioned dynamical principle furnishes six equations connecting the six co-ordinates, which, when integrated, will determine the latter in terms of the time and the initial circumstances of the motion.

The various ways of expressing this dynamical principle in mathematical language are explained in treatises on dynamics: and a variety of special forms and particular cases are obtained, by means of which the solution of numerous problems can be simplified. For example, Euler's equations, for determining the motion of rotation of a single rigid body about its centre of inertia, is a particular case of the second proposition; whilst Kirchhoff's equations, for determining the motion of a single solid in an infinite liquid, is a special form of both propositions.

When a conservative system possesses seven degrees of freedom, the motion may be completely determined by means of the principle of momentum combined with the principle of energy. The first principle, as we have already shown, furnishes six equations, whilst the second furnishes one; hence, we have a sufficient number of equations for determining the motion.

When a dynamical system possesses more than seven degrees of freedom, the principles of momentum and energy are insufficient to determine the motion; and under these circumstances, the most convenient method to adopt is to use Lagrange's equations; but inasmuch as these equations are double-edged tools, which are apt to cut the fingers of the unwary, their employment requires considerable care.

The kinetic energy of a dynamical system can be expressed in a variety of different forms, but it will only be necessary to mention the following three. In the first form, it is expressed as a homogeneous quadratic function of velocities, which are the time-variations of the co-ordinates of the system. This form, which will be denoted by T , is called the *Lagrangian form*; it is the only one which it is permissible to use when employing Lagrange's equations, and many mistakes have been made by persons who have attempted to use some other form.

In the second form, which is called the *Hamiltonian form*, the kinetic energy is expressed as a homogeneous

quadratic function of the momenta of the system. If θ be any co-ordinate, and Θ the generalized momentum of type θ , it is known that

$$\frac{\partial T}{\partial \theta} = \Theta \quad \dots \quad (1)$$

whence Θ is a linear function of the velocities. Hence, if the velocities be eliminated from the Lagrangian expression for the kinetic energy by means of (1), it follows that the latter will be expressible as a homogeneous quadratic function of the momenta Θ , which is the Hamiltonian form. We shall denote this form by $\mathfrak{T}$.

Lagrange's equations are

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\theta}} \right) - \frac{\partial T}{\partial \theta} = - \frac{\partial V}{\partial \theta} \quad \dots \quad (2)$$

where V is the potential energy; and if the elimination be performed, we shall obtain

$$\frac{d\Theta}{dt} + \frac{\partial \mathfrak{T}}{\partial \theta} = - \frac{\partial V}{\partial \theta} \quad \dots \quad (3)$$

we have also the reciprocal relation

$$\frac{\partial \mathfrak{T}}{\partial \Theta} = \dot{\theta} \quad \dots \quad (4)$$

Equations (3) and (4) are Hamilton's equations of motion.

The third form of the expression for the kinetic energy is of special importance in hydrodynamics and other branches of physics. It sometimes happens that a quantity occurs which can be recognized as a momentum, or as a quantity in the nature of a momentum, whilst the velocity corresponding to this momentum is either unknown or would be inconvenient to introduce. This occurs in problems relating to the motion of perforated solids in a liquid, when there is circulation, and is a particular case of Dr. Routh's theory of the "Ignorance of Velocities."¹ We therefore require a form of Lagrange's equations in which certain velocities are eliminated, and are replaced by the corresponding momenta.

Let the co-ordinates of the system be divided into two groups, θ and χ ; and let κ denote the generalized momentum corresponding to χ . Then

$$\frac{\partial T}{\partial \dot{\chi}} = \kappa \quad \dots \quad (5)$$

By means of (5) all the velocities $\dot{\chi}$ can be eliminated from the expression for the kinetic energy; and it is remarkable, that the result of the elimination does not contain any products of the form $\kappa \dot{\theta}$. The expression for T may accordingly be written

$$T = \mathfrak{T} + \mathfrak{R} \quad \dots \quad (6)$$

where $\mathfrak{T}$ is a homogeneous quadratic function of the velocities $\dot{\theta}$, and $\mathfrak{R}$ is a similar function of the momenta κ .

Equation (6) is therefore a mixed form, which is partly Lagrangian and partly Hamiltonian. We now require the corresponding form of the equations of motion in which all the $\dot{\chi}$'s have been eliminated from Lagrange's equations.

From (1) it follows that the generalized momentum Θ is a linear function of the velocities $\dot{\theta}$, $\dot{\chi}$; and if the latter velocities be eliminated by means of (5), it follows that Θ is expressible as a linear function of $\dot{\theta}$, κ . Let the portion which is a linear function of the κ 's be denoted by $\bar{\Theta}$; then it can be shown, that if

$$L = \mathfrak{T} + \mathfrak{Z}(\bar{\Theta}) - \mathfrak{R} - V \quad \dots \quad (7)$$

the equation of type θ is

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = 0 \quad \dots \quad (8)$$

¹ Having regard to the object of the theory, I think the phrase "Ignorance of Velocities" is better than "Ignorance of Co-ordinates."