Dr. Lodge gives the formula $T = \rho v^2$, and at the equator, $v = 1526$ f.s., and now our practical man looks out the density of steel in a table, and finds it given as about 8.

With $\rho = 8$, $v = 1526$, he finds T is very nearly 20 million, so there is a misconception somewhere; however, the result is given in tons, so, dividing by 2240, he finds T is now about 8000, still very far from the result. He next tries dividing by 144, as the result is given in tons per square inch, and now T is about 60—only half the true result.

By this time he remembers that he ought to have taken $\rho = 8 \times 62{\cdot}4$, and T is now about 3700; and as this looks like 32 times the true result, he now thinks of dividing by g, so that, finally, the formula which gives the true result is, not the simple $T = \rho v^2$, but $T = \frac{62{\cdot}4}{2240 \times 144} \frac{\rho v^2}{g}$.

What does the formula $T = \rho v^2$ then mean? With C.G.S. units we may put $\rho = 8$, and $v = 4 \times 10^9 \div (24 \times 60 \times 60)$, and now $T = 10^{10} \times 1{\cdot}7$ *barads;* and as a stress of one pound per square inch is about 70,000 barads, we divide by 70,000 and 2240, and find $T = 110$ tons per square inch.

But with the ordinary British legal units of the foot and the pound weight, the formula $T = \rho v^2$ is meaningless; and a practical man has a just complaint against our vague system of theoretical teaching in dynamics, when he comes across a formula such as $T = \rho v^2$, where it is merely stated that T is the tension, ρ the density, and v the velocity.

It is in the interest of those to whom dynamics is a reality, and not a mere combinational analysis, that I am encouraged to write this criticism; but now returning to the telegraph cable, which, as a girdle round the equator, ought to rise out of the ocean bed, and stand like an arch under a tension of 120 tons per square inch, in consequence of the whirling effect of the earth's rotation.

But, on taking into account the gravity due to the earth's attraction, which is about 289 times the whirling effect, our cable, if it still stood as an arch, would have a pressure of about 32,000 tons per square inch; and now we are confronted with the old Ostrogradsky paradox. A. G. GREENHILL.

March 16.

PROF. LODGE has invited me to follow up his letter on "The Flying to Pieces of a Whirling Ring" (NATURE, March 12, p. 439), by sending to NATURE a note about the strains and stresses in a whirling disk—a matter which has lately been the subject of some correspondence between us. Before speaking of the disk, however, let me (as an old cable hand) confess that I do not follow the reasoning which leads Prof. Lodge to say that a submerged cable of the average density of sea-water, if lying parallel to the equator, would be subject to a stress of 30 tons per square inch, or more (in latitude less than 60°), in consequence of the earth's rotation. This is in startling disagreement with one's recollection of the behaviour of say a "caya" rope (which satisfies the condition as to density). But surely a cable that is wholly supported by water is as much protected by gravity from flying to pieces as a cable that is partly supported by the mud or rocks of the bottom.

The strains in a revolving disk have been discussed by Mr. Chree (*Quart. Journal of Pure Math.*, No. 89, 1888; see also Proc. Camb. Phil. Soc., vols. xiv. and xv.) and by the late Prof. Grossmann (*Verhandlungen des Vereins zur Beförderung des Gewerbfleisses*, Berlin, 1889, p. 216). Prof. Grossmann—for the reference to whose paper I am indebted to Mr. J. T. Nicolson—treats the case of a disk with a central hole in it, and points out that the hoop tension is greatest just round the hole.

There is, however, an important difference between the values of the hoop tension when there is and when there is not a hole. The difference in question appears to have escaped notice, and my object in writing this note is to point it out.

The case supposed is that of a thin disk, homogeneous, isotropic, and uniformly thick. We have to consider two principal stresses—namely, the radial tension p_1, and the hoop tension p_2.

Let ω be the angular velocity,

ρ the density,

E Young's modulus for the material,

μ Poisson's ratio of lateral to longitudinal strain in a simple stress.

u the radial displacement at any point where the stress is considered, and to which the radius is r.

The equilibrium of any small element of mass under the two radial tensions at its inner and outer surfaces, the two hoop tensions on its front and back faces, and the "centrifugal force," requires that

$$p_1 = \frac{d(p_2 r)}{dr} + \omega^2 \rho r^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The strain in the direction of the hoop stress p_1 is $\frac{u}{r}$. The radial strain is $\frac{du}{dr}$. Hence

$$\frac{Eu}{r} = p_1 - \mu p_2 \,; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$E\frac{du}{ar} = p_2 - \mu p_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From (2) and (3),

$$p_1 = \frac{E}{1 - \mu^2}\left(\frac{u}{r} + \mu\frac{du}{dr}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$p_2 = \frac{E}{1 - \mu^2}\left(\frac{\mu u}{r} + \frac{du}{dr}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

And, substituting in (1),

$$\frac{rd^2u}{dr^2} + \frac{du}{dr} - \frac{u}{r} + \frac{(1 - \mu^2)\,\omega^2\rho r^2}{E} = 0 \quad . \quad . \quad . \quad (6)$$

From which,

$$\frac{u}{r} = \frac{C}{r^2} + C_1 - \frac{(1 - \mu^2)\,\omega^2\rho r}{E} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In this I have simply followed Grossmann, who goes on to find the stresses in a disk *with a hole* by applying the boundary conditions that $p_2 = 0$ when $r = a_1$, the radius of the disk, and also when $r = a_2$, the radius of the hole. The result is—

$$p_1 = \frac{\omega^2\rho}{8}\left\{(3 + \mu)\left(a_1^2 + a_2^2 + \frac{a_1^2a_2^2}{r^2}\right) - (1 + 3\mu)r^2\right\};$$

$$p_2 = \frac{\omega^2\rho}{8}\left\{(3 + \mu)\left(a_1^2 + a_2^2 - \frac{a_1^2a_2^2}{r^2} - r^2\right)\right\}.$$

From this it is clear that the maximum hoop tension occurs close to the hole, with the value

$$\text{Max. } p_1 = \frac{\omega^2\rho}{4}\left\{(3 + \mu)a_1^2 + (1 - \mu)a_2^2\right\};$$

and the maximum radial tension occurs when $r = \sqrt{a_1 a_2}$, with the value

$$\text{Max. } p_2 = \frac{\omega^2\rho}{4}(3 + \mu)(a_1 - a_2)^2.$$

In the special case when the hole is very small

$$\text{Max. } p_1 = \text{Max. } p_2 = \frac{\omega^2\rho a_1^2(3 + \mu)}{4}.$$

With a given material, the stresses depend simply upon the peripheral velocity.

Next take the case of a disk which has no hole. The boundary conditions are $p_2 = 0$ when $r = a$ (the radius of the disk), and $u = 0$ when $r = 0$. Hence, by equation (7), $C = 0$, and then, by equation (5), $C_1 = \frac{(1 - \mu)(3 + \mu)\omega^2\rho a^2}{8E}$.

The stresses in the disk without a hole are therefore,

$$p_1 = \frac{\omega^2\rho}{8}\left\{(3 + \mu)a^2 - (1 + 3\mu)r^2\right\};$$

$$p_2 = \frac{\omega^2\rho}{8}(3 + \mu)(a^2 - r^2).$$

Each of these is a maximum at the centre, namely,

$$\text{Max. } p_1 = \text{Max. } p_2 = \frac{\omega^2\rho a^2(3 + \mu)}{8};$$

and this is just half the value which the intensity of stress reaches when there is a very small hole.

If we take 15 tons per square inch as the greatest safe stress in steel, a thin disk of uniform thickness, with a hole at the centre, may be whirled with a peripheral velocity of about 620 feet per second. If there were no hole, the peripheral speed might be about 870 feet per second.

A plate intended to whirl at a high speed should evidently have its thickness increased in the neighbourhood of the nave.

Cambridge, March 14. J. A. EWING