THE SCIENTIFIC PRINCIPLES INVOLVED IN MAKING BIG GUNS.

I.

STEAMSHIPS are now called *boats*, and the largest cannon are called *guns*, according to a process in language which philologists have explained; but while steamships have increased in size and complication, the gun, however big, satisfies the Hibernian definition of a cylindrical hole with metal placed round it; and the most difficult problem of the gun-maker is to dispose the metal in the most efficient manner, hampered as he is by the limitations of the metallurgical art.

The difficulties increase with the size of the gun, according to the well-known law of Mechanical Similitude.

Geometrical Similitude is independent of scale; a geometrical theorem is true, however large the figure may be drawn; but the laws of Mechanical Similitude are complicated, when we notice the differences between a simple girder and the Forth Bridge, or between the anatomy of large and small animals.

As an example of mechanical similitude, consider what sort of a steamship would be required to reduce the voyage to America from six to five days. The present steamers crossing in six days have a speed of 20 knots, and displacement of 10,000 tons, and the indicated horse-power is close on 20,000. To cross in five days the speed would have to be increased 20 per cent., to 24 knots; and now if we apply Froude's law that, at corresponding speeds as the sixth root of the displacements, the resistances are as the displacements, we shall find that the steamer would have to be of 30,000 tons, and of 65,000 horse-power, thus exceeding even the *Great Eastern's* dimensions.

With given material, say steel, the strongest with which we are familiar, a limit of size is soon reached at which the structure falls to pieces almost of its own weight; and recent experience with the heaviest artillery seems to show that we are nearing this limit.

The larger the gun or structure, then, the greater the necessity for careful and scientific design and proportion. It is proposed to give here a sketch of the fundamental principles which guide the gun-maker, and which he applies to secure the safety of the gun under the greatest pressure it can ever be called upon to sustain.

While reaping almost all the glory of success, the gun-maker cannot risk the disgrace of a failure; on the other hand, the carriage-maker can work with a small margin of safety, as ample warning would be given of any failure, and breakage is easily repaired; but the failure of a gun may be so disastrous that it must be avoided at all cost, so that the gun-maker never allows himself to work very close to the limits which his theory allows.

At the present time the design and employment at sea or in forts of such monsters as our 110-ton or Krupp's 135-ton guns is severely criticized and condemned in certain circles; but it is a maxim in artillery that one big gun is worth much more than its equivalent weight in smaller guns; and for naval engagements a few line-of-battle ships armed with the heaviest artillery are invincible, if properly flanked and protected by the light cavalry of frigates.

So, too, with steamships; the largest and fastest always fill with passengers, and by making rapid passages, and therefore more in a given time, are found to be more profitable in spite of their great initial cost and expense of working.

The size of the gun is settled by the thickness of armour it is required to attack; the calibre increasing practically as the thickness to be pierced, but the weight of the gun mounting up as the cube of the calibre. Thus if an 8-inch gun weighing 13 tons can pierce 12 inches of armour, a 16-inch gun is required to pierce 24 inches, and the 16-inch gun will weigh 104 tons.

PART I.—THE STRESSES IN A GUN.

(1) The theory of gun-making begins with the investigation of the stresses set up in a thick metal cylinder, due to steady pressures, applied either at the interior, or exterior, or at both cylindrical surfaces.

So far, the dynamical phenomena which arise from the propagation and reflexion of radial vibrations are beyond our powers of useful analysis; so that we restrict ourselves to the investigation of the elastical problem of the thick cylinder of elastic material, subject to given internal and external pressures, applied steadily, as in the case of a tube tested under hydraulic pressure.

Fig. 1 is drawn representing the stresses set up in a cylinder or tube B, by an internal pressure p_i; we denote by r_i and r_o the inner and outer radii, the suffixes *i* and *o* denoting *inside* and *outside*; and then r can be used to denote any intermediate radius.

The stress at any point at a distance r from the axis will consist of a radial pressure, p, and a circumferential tension, t; the radial pressure p decreasing from p_i at the inner radius r_i to zero at the outer radius r_o, the atmospheric pressure not being taken into account; while the circumferential tension t at the same time diminishes from t_i to t_o.

The British units employed in practical measurements with guns are the inch and the ton; so that r being measured in inches, p and t are measured in tons per square inch.

(2) To determine the state of stress at any point of the cylinder, we suppose it divided by a diametral plane $r_o r_i \mathrm{O} r_i r_o$; and the equilibrium of an inch length of either half is considered.

The stresses p and t being represented graphically by the ordinates of the curves $p_i p p_o$, $t_i t t_o$, the equilibrium of either half of the cylinder requires that the area of the circumferential tension-curve $r_i t_i t_o r_o$ and its counterpart should be equal to the area of the rectangle $\mathrm{O}p_i$, and its counterpart, these latter representing the thrust due to the pressure p_i on the half cylinder.

Then, denoting the area $r_i t_i t_o r_o$ by Q, and calling it the resistance of the section $r_i r_o$,

$$Q = p_i r_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

If we divide the resistance Q by the thickness of the cylinder $r_o - r_i$, we obtain the *average* circumferential tension in the material; and when the cylinder is thin, the maximum circumferential tension t_i and the average tension $Q/(r_o - r_i)$ will not be appreciably different; so that a knowledge of the average circumferential tension will be sufficient for practical purposes in such cases as, for instance, of the cylindrical shell of a boiler; and we have thus the elementary formula ordinarily employed in the design of boilers.

But when, as in a gun or hydraulic press, the thickness has to be made considerable, we must have the means of determining the *maximum* tension t_i, and of contriving that t_i shall not exceed a certain proof limit suitable for the material.

(3) Now, just as the equilibrum of either half of the cylinder requires that the area $r_i t_i t_o r_o = p_i r_i$, so the equilibrium of either half of a part of the cylinder bounded internally by the radius r_i, and externally by any radius r, requires that the area $r_i t_i t r$ should equal the rectangle $\mathrm{O}p_i$ minus the rectangle $\mathrm{O}p$; or, in the notation of the Integral Calculus—

$$\int_{r_i}^{r} t dr = p_i r_i - pr \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

The first attempt at a solution of these equations (1) and (2) is due to Peter Barlow, when called upon to calculate the strength of the cylinder of the Bramah hydraulic press, in a paper read before the Society of Civil Engineers in February 1825, and published in the *Edinburgh Journal of Science*, and in the Trans. I.C.E., vol. i. 1836.

(4) Barlow assumed that under an internal pressure the metal is compressed radially as much as it is stretched circumferentially, so that the cubical compression of the metal is zero, and he is justified therefore in putting $p = t$ in the material of the cylinder.

Then equation (2) becomes

$$\int_{r_i}^{r} p dr = p_i r_i - pr;$$

so that, differentiating with respect to r,

$$p = -d(pr)/dr, \text{ or } dp/p + 2dr/r = 0;$$

and integrating again with respect to r,

$$\log p + \log r^2 = \text{constant},$$

or

$$pr^2 = a, \text{ a constant}; \; p = t = ar^{-2} \quad . \quad . \quad . \quad . \quad (3);$$

so that p and t, if equal, vary inversely as the square of the distance from the axis.

Thus, a cylindrical tube under internal and external pressures which are inversely as the squares of the internal and external radii respectively, will, according to Barlow's law, have at any point a radial pressure and an equal circumferential tension, also inversely as the square of the distance from the axis.

When the thickness of the cylinder is considerable, compared with the bore, this solution of Barlow will give a very fair indication of the true result.

(5) But Rankine showed ("Applied Mechanics," § 273) that, by superposing the state of hydrostatic stress produced by equal internal and external pressures, we obtain the algebraical solution of the most general case where the internal and external applied pressures are arbitrary.

For if we suppose the state of stress in the cylinder is a hydrostatic stress, composed of a radial pressure p, and an equal circumferential pressure $-t$, then equation (2) becomes—

$$\int_{r_i}^{r} p\,dr = pr - p_i r_i;$$

and differentiating with respect to r,

$$p = d(pr)/dr, \text{ or } dp/dr = 0;$$

so that

$$p = b, \text{ a constant; and then } t = -b \quad . \quad . \quad . \quad (4).$$

(6) The superposition of this state of stress on Barlow's state of stress gives—

$$p = ar^{-2} + b, \qquad t = ar^{-2} - b \quad . \quad . \quad . \quad . \quad (5),$$

or

$$(p + t)r^2 = 2a, \qquad p - t = 2b;$$

values which will be found to verify equation (2); and now the constants a and b are determined for arbitrarily applied internal and external pressures p_i and p_o by the equations

$$p_i = ar_i^{-2} + b, \qquad p_o = ar_o^{-2} + b;$$

so that

$$a = \frac{p_i - p_o}{r_i^{-2} - r_o^{-2}} = \frac{(p_i - p_o)\,r_i^2 r_o^2}{r_o^2 - r_i^2};$$

$$b = \frac{p_o r_o^2 - p_i r_i^2}{r_o^2 - r_i^2} = \frac{p_o r_i^{-2} - p_i r_o^{-2}}{r_i^{-2} - r_o^{-2}}.$$

These results were first obtained by Lamé and Hart (the late Sir Andrew Searle Hart, of Dublin), but in a much more complicated manner. Lamé's solution was given in his "Leçons sur la théorie mathématique de l'élasticité des corps solides"; while Hart's treatment of the question will be found in Note W to Robert Mallet's "Physical Conditions involved in the Construction of Artillery" (1856). An investigation of the same problem by Maxwell, when about eighteen years old, in the Trans. R. S. Edin., vol. xx. 1850, has been generally overlooked.

Rankine's treatment analyzes the mechanical signification of the separate terms of the solution, and obtains them by simple reasoning from the state of stress, without an appeal to the laws of elasticity and the consequent state of strain.

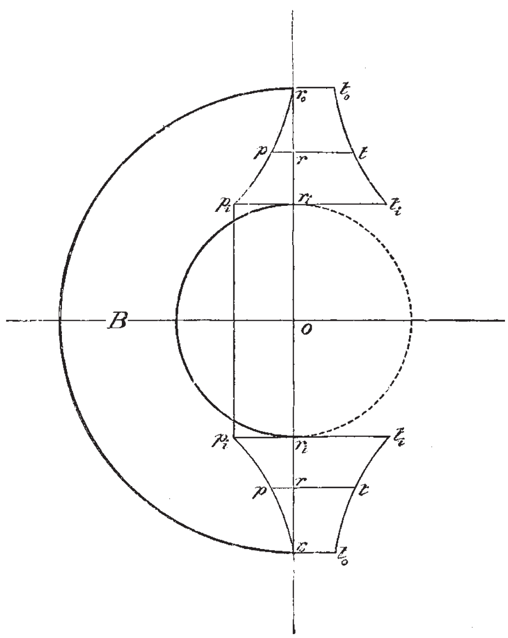

FIG. 1.

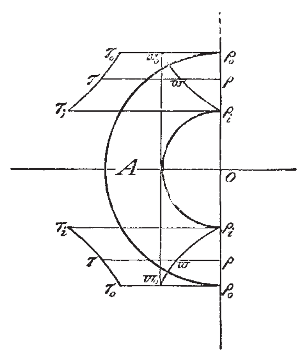

FIG. 2.

(7) Putting $p_i r_i^2 = p_o r_o^2$ makes $b = 0$, and gives the particular case considered first by Barlow; and putting $p_i = p_o$ makes $a = 0$, and gives the additional particular case of uniform hydrostatic stress invented by Rankine.

But, in the general case, a and b may have any values, positive or negative, according to the relations between p_i and p_o, r_i and r_o.

Thus, as in Fig. 1, with $p_o = 0$, we find—

$$a = \frac{p_i}{r_i^{-2} - r^{-2}}, \quad b = \frac{-p_i r_o^{-2}}{r_i^{-2} - r_o^{-2}};$$

and then

$$p = ar^{-2} + b = p_i \frac{r^{-2} - r_o^{-2}}{r_i^{-2} - r_o^{-2}}; \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$t = ar^{-2} - b = p_i \frac{r^{-2} + r_o^{-2}}{r_i^{-2} - r_o^{-2}}; \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$t = p_i \frac{r_i^{-2} + r_o^{-2}}{r_i^{-2} - r_o^{-2}} = p_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}; \quad . \quad . \quad . \quad . \quad (8)$$

$$t_o = p_i \frac{2r_o^{-2}}{r_i^{-2} - r_o^{-2}} = p_i \frac{2r_i^2}{r_o^2 - r_i^2} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

(8) Now using t to denote the *average* value of the circumferential tension, so that

$$t = p_i r_i/(r \quad r_i),$$

then

$$\frac{t_i}{t} = \frac{r_o^2 + r_i^2}{r_i(r_o + r_i)}, \quad \frac{t_i - t}{t} = \frac{r_o}{r_i}\frac{r_o - r_i}{r_o + r_i} \quad . \quad . \quad (10)$$

thus showing that the maximum tension t_i may exceed the average tension t by a considerable amount; and it is this *maximum* tension t_i which must be carefully watched and kept down below a certain working value; so that, with given t_i, the maximum allowable pressure in the tube is given by

$$p_i = t_i \frac{r_o^2 - r_i^2}{r_o^2 + r_i^2}.$$

This is the formula now used in the design of a hydraulic press, or of a thick tube, of bore $2r_i$, to stand an internal pressure p_i; t_i being fixed by the strength of the material, and then r_o being calculated.

We notice that p_i is always less than t_i, so that a tube, however thick, cannot stand, if unsupported, an internal pressure greater than the working tenacity of the material.

But, as the pressures in gunnery often exceed the tenacity of any known material, the requisite strength must be provided by an initial compression of the tube due to shrinking on one or more cylindrical jackets.

(9) Fig. 2 is drawn representing graphically the state of stress set up in a tube A by an external applied pressure ϖ_o, as in the tube or flue of a boiler by the external pressure of the water, or in the internal tube of a gun by the shrinkage pressure of the outside jacket.

Denote by ρ_i and ρ_o the inner and outer radii of the tube A, and by ρ any intermediate radius.

The stress at any point of the tube will now consist of a radial pressure ϖ, and of a circumferential *pressure* τ, represented by the ordinates of the curves $\varpi_o\varpi\varpi_i$, $\tau_o\tau\tau_i$; and dividing the tube by a diametral plane $\rho_o\rho_i O\rho_i\rho_o$, and considering the equilibrium of inch length of either half, we shall find as before that the area $\rho_i\tau_i\tau_o\rho_o$ = the rectangle $O\varpi_o = \varpi_o\rho_o$; while considering the equilibrium of any coaxial cylindrical portion, bounded by the radii ρ_o and ρ, then the area $\rho\tau\tau_o\rho_o$ = rectangle $O\varpi_o$ − rectangle $O\varpi$; or, in the notation of the Integral Calculus—

$$\int_\rho^{\rho_o} \tau d\rho = \varpi_o\rho_o - \varpi\rho\,; \quad \ldots \ldots \quad (11)$$

leading, by differentiation with respect to ρ, to

$$\tau = - d(\varpi\rho)/d\rho\,; \quad \ldots \ldots \quad (11^*)$$

the general solution of which can, as before, be exhibited in the form—

$$\varpi = \beta + \alpha\rho^{-2},\ \tau = \beta - \alpha\rho^{-2}, \quad \ldots \quad (12)$$

or

$$(\varpi - \tau)\rho^2 = 2\alpha,\ \varpi + \tau = 2\beta,$$

where α and β are arbitrary constants, determined from the values of the arbitrary pressures applied to the interior and exterior surfaces.

(10) Now with $\varpi_i = 0$,

$$0 = \beta + \alpha\rho_i^{-2},\ \varpi_o = \beta + \alpha\rho_o^{-2};$$

so that

$$\alpha = \frac{-\varpi_o}{\rho_i^{-2} - \rho_o^{-2}},\quad \beta = \frac{\varpi_o\rho_i^{-2}}{\rho_i^{-2} - \rho_o^{-2}};$$

and then

$$\varpi = \beta + \alpha\rho^{-2} = \varpi_o\frac{\rho_i^{-2} - \rho^{-2}}{\rho_i^{-2} - \rho_o^{-2}}, \quad \ldots \quad (13)$$

$$\tau = \beta - \alpha\rho^{-2} = \varpi_o\frac{\rho_i^{-2} + \rho^{-2}}{\rho_i^{-2} - \rho_o^{-2}}, \quad \ldots \quad (14)$$

$$\tau_o = \varpi_o\frac{\rho_i^{-2} + \rho_o^{-2}}{\rho_i^{-2} - \rho_o^{-2}} = \varpi_o\frac{\rho_o^2 + \rho_i^2}{\rho_o^2 - \rho_i^2}, \quad \ldots \quad (15)$$

$$\tau_i = \varpi_o\frac{2\rho_i^{-2}}{\rho_i^{-2} - \rho_o^{-2}} = \varpi_o\frac{2\rho_o^2}{\rho_o^2 - \rho_i^2}. \quad \ldots \quad (16)$$

Given, then, τ_i the maximum allowable crushing pressure of the material, then

$$\varpi_o = \tau_o(\rho_o^2 - \rho_i^2)/2\rho_o^2 = \tfrac{1}{2}\tau_o(1 - \rho_i^2/\rho_o^2) \quad \ldots \quad (17)$$

is the maximum allowable external pressure on the tube.

(11) If we make $\rho_o = r_i$ and $\varpi_o = p_i$, the tube A of Fig. 2 may be supposed to be gripped by the cylinder B of Fig. 1, of which only the upper halves need now be shown, as in Fig. 3; and now Fig. 3 will represent the cross-section of a tube, A, over which a jacket, B, has been shrunk, as at the breech end of an ordinary field-gun, and will represent graphically the stresses set up when the pressure, $\varpi_o = p_i$, at the common surface, is supposed known; these are called the *initial stresses*, or stresses of repose; the internal pressure at the radius ρ_i and the

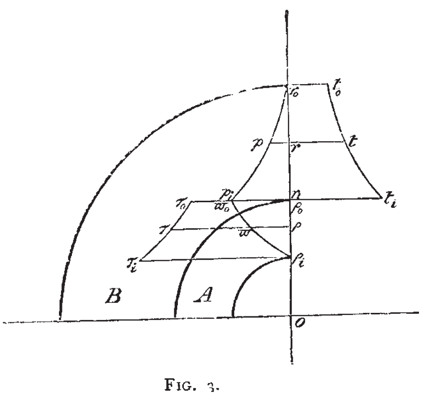

FIG. 3.

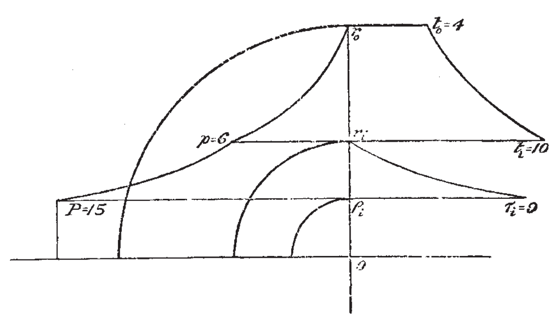

FIG. 4.

external pressure at the radius r_o being zero, as the atmospheric pressure is insensible in our calculations.

In Fig. 3 we notice that the total pull resistance across the section r_or_i, represented by the area $r_ot_ot_ir_i$, is equal to the total thrust resistance of the section $\rho_o\rho_i$, represented by the area $\rho_o\tau_o\tau_i\rho_i$, and each of these is equal to the resultant pressure thrust represented by the area of the rectangle Op_i.

(12) Now, suppose a pressure P (say 15 tons on the square inch) is applied at the interior of the tube, either by the steady pressure of water, as in a hydraulic press, or by the momentary pressure of gunpowder, as in the bore of a gun.

We suppose that the additional stresses due to this pressure, P, which we shall call the *powder stresses*, are the same as those which would be set up in a homogeneous cylinder of internal radius ρ_i, and external radius r_o, by a steady pressure, P; and these powder stresses will therefore, by what precedes, in equations (6), (7), (8) (Fig. 1), at a distance r from the axis, consist of a radial pressure—

$$P\frac{r^{-2} - r_o^{-2}}{\rho_i^{-2} - r_o^{-2}}, \quad \ldots \ldots \quad (18)$$

and a circumferential tension—

$$P\frac{r^{-2} + r_o^{-2}}{\rho_i^{-2} - r_o^{-2}}, \quad \ldots \ldots \quad (19)$$

having a maximum value at the bore of

$$T = P\frac{\rho_i^{-2} + r_o^{-2}}{\rho_i^{-2} - r_o^{-2}}.$$

We must superpose these powder stresses on the initial stresses of the compound cylinder to obtain the stresses when the cylinder is used as a gun (or hydraulic press); these are called the *firing stresses*, and they are exhibited graphically in Fig. 4.

(13) We now see the reason for setting up initial stresses in the gun by shrinking a jacket over the interior tube.

For the maximum circumferential tension at the bore on firing is reduced by the initial stresses from

$$T = P\frac{\rho_i^{-2} + r_o^{-2}}{\rho_i^{-2} - r_o^{-2}} \text{ to } P\frac{\rho_i^{-2} + r_o^{-2}}{\rho_i^{-2} - r_o^{-2}} - \varpi_o\frac{2\rho_i^{-2}}{\rho_i^{-2} - \rho_o^{-2}}, \quad (20)$$

while at the interior of the jacket the circumferential tension is altered from

$$P\frac{r_i^{-2} + r_o^{-2}}{\rho_i^{-2} - r_o^{-2}} \text{ to } P\frac{r_i^{-2} + r_o^{-2}}{\rho_i^{-2} - r_o^{-2}} + p_i\frac{r_i^{-2} + r_o^{-2}}{r_i^{-2} - r_o^{-2}}. \quad (21)$$

The maximum stresses in the gun are thereby equalized to a great extent, and material can be economized.

(14) Thus, with $\rho_i/\rho_o = r_i/r_o = \frac{1}{2}$, and $P = 15$, the powder stresses are given by circumferential tensions—

$$T = \tau_i = 17, \qquad t_i = 5;$$

so that, with a shrinkage pressure $\varpi_o = p_i = 3$, the principal firing stresses are given by circumferential tensions—

$$\tau_i = 17 - 8 = 9, \text{ a great reduction on } 17,$$
$$t_i = 5 + 5 = 10,$$

while

$$\tau_o = 5 - 5 = 0,$$
$$t_o = 2 + 2 = 4.$$

We need not consider the radial pressures for practical purposes.

To equalize these maximum tensions, $\tau_i = 9$ and $t_i = 10$, the tube might be made slightly thicker and the jacket thinner, or else the shrinkage pressure ϖ_o or p_i slightly diminished, keeping to the same bore and external diameter.

(15) We have thus shown how the initial stress, the powder stress, and the firing stress at any point of a gun composed of a tube and a single jacket is found, and exhibited graphically in Figs. 3 and 4.

The curves in the figure are seen to be all similar to a curve whose equation is of the form $y = ax^{-2}$, now called the Barlow curve.

When the gun is built up of three or more concentric cylinders, the method of procedure is the same; the initial pressure between the cylinders may be supposed known from the amount of shrinkage given in the manufacture; and now, taking any intermediate cylinder of the gun under initial pressures p_i and p_o at the internal and external surfaces, of radii r_i and r_o, we erect ordinates to represent p_i and p_o, and draw the Barlow curve joining their ends.

The Barlow curve representing the circumferential tension or pressure will always appear as an equal reflection of the pressure curve, the position being assigned so as to make the area of the circumferential *tension* curve equal to $p_i r_i - p_o r_o$; and it may happen that this area may vanish or become negative, showing that some or all of the initial circumferential tensions are really pressures.

(16) But practically the gun-maker reverses this procedure; with him it is the maximum circumferential firing tension t_i of a tube or hoop which is limited by the strength of the metal; so that, starting with these t_i's, as given, he calculates the pressures between the successive coils of the gun, proceeding inwards, and finally determines the maximum allowable powder pressure in the interior of the bore.

Afterwards he subtracts the powder stresses from these firing stresses, and thus obtains the initial stresses in the gun; and then from these initial stresses he calculates the amount of shrinkage to be given to the coils or hoops to obtain the requisite state of initial stress. But we shall show subsequently that the requisite amount of shrinkage is given just as simply from the firing stresses as from the initial stresses; so that henceforth we need only determine the firing stresses.

(17) Then Fig. 5 represents the maximum allowable firing stress over the powder-chamber of the American 8-inch gun, shown in cross-section, as composed of an inner tube, A, over which a jacket, B, and two hoops, C and D, have been shrunk on.

In practice, the maximum allowable tension in the jacket and hoops is restricted to 18 tons per square inch, but in the inner tube to 15 tons per square inch, so as to allow for erosion of the bore, the weakening due to the rifling grooves, and the possible failure of the tube.

(18) In the notation of the "Text-book of Gunnery," 1887, by Major Mackinlay, R.A., supposing there are n cylinders in the cross-section of the gun, the successive radii of the cylindrical

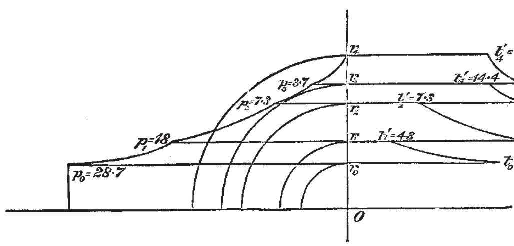

FIG. 5.

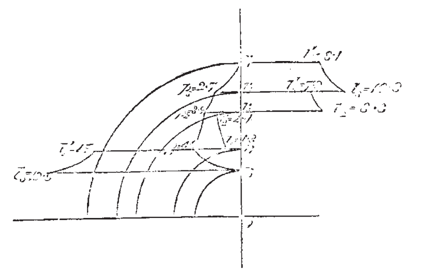

FIG. 6.

surfaces, beginning from the outside, are denoted in inches by $r_n, r_{n-1}, \ldots, r_2, r_1, r_0$; and the firing pressures at the surfaces of separation are denoted by $p_{n-1}, \ldots, p_2, p_1$; and p_0 finally denotes the powder pressure at the radius r_0 of the bore.

We notice that there is no sudden change in the value of the radial pressure; but that the circumferential tension, t, changes suddenly from one cylinder to the next.

As we are concerned principally with the *maximum* tensions, which occur practically at the inner surface of a cylinder, we denote them, proceeding inwards, by $t_{n-1}, t_{n-2}, \ldots t_2, t_1, t_0$; and we shall suppose them to change suddenly to $t'_{n-1}, t'_{n-2}, \ldots t'_2, t'_1$, in proceeding inwards to the next cylinder.

(19) Starting from the outside cylinder, the stress formulas give, since $p_n = 0$,

$$p_{n-1} = ar_{n-1}^{-2} + b, \qquad 0 = ar_n^{-2} + b,$$

while

$$t_{n-1} = ar_{n-1}^{2} - b, \qquad t'_n = ar_n^{-2} - b;$$

so that

$$p_{n-1} = a\left(r_{n-1}^{-2} - r_n^{-2}\right),$$

$$t_{n-1} = a\left(r_{n-1}^{-2} + r_n^{-2}\right),$$

or

$$p_{n-1} = \frac{r_{n-1}^{-2} - r_n^{-2}}{r_{n-1}^{-2} + r_n^{-2}} t_{n-1}, \quad \ldots \ldots (22)$$

giving p_{n-1} when t_{n-1} is assigned by its maximum allowable value.

Also

$$t_{n-1} - t'_n = p_{n-1}, \quad \ldots \ldots (23)$$

giving t'_n, if required for the diagram.

Proceeding inwards to the next cylinder, we have (with different values of a and b)—

$$p_{n-2} = ar_{n-2}^{-2} + b, \qquad p_{n-1} = ar_{n-1}^{-2} + b,$$

$$t_{n-2} = ar_{n-2}^{-2} - b, \qquad t'_{n-1} = ar_{n-1}^{-2} - b;$$

so that, eliminating the a and b, first b and then a,

$$p_{n-2} - p_{n-1} = a\left(r_{n-2}^{-2} - r_{n-1}^{-2}\right),$$

$$t_{n-2} + p_{n-1} = a\left(r_{n-2}^{-2} + r_{n-1}^{-2}\right),$$

and therefore, by division,

$$p_{n-2} - p_{n-1} = \frac{r_{n-2}^{-2} - r_{n-1}^{-2}}{r_{n-2}^{-2} + r_{n-1}^{-2}} (t_{n-2} + p_{n-1}),$$

or

$$p_{n-2} = \frac{r_{n-1}^{2} - r_{n-2}^{2}}{r_{n-1}^{2} + r_{n-2}^{2}} (t_{n-2} + p_{n-1}) + p_{n-1}. \quad (24)$$

giving p_{n-2} when p_{n-1} is known, and when t_{n-2} is assigned by its maximum allowable value in practice.

Also

$$t_{n-2} - t'_{n-1} = p_{n-2} - p_{n-1} \quad \ldots \quad (25)$$

thus determining t'_{n-1}; a knowledge of t'_{n-1} is required when we come to the determination of the amount of shrinkage necessary to produce p_{n-1}.

Proceeding successively in this manner, we finally obtain—

. . . .

$$p_2 = \frac{r_3^2 - r_2^2}{r_3^2 + r_2^2}(t_2 + p_3) + p_3 \quad . . . . \text{(iii.)}$$

$$p_1 = \frac{r_2^2 - r_1^2}{r_2^2 + r_1^2}(t_1 + p_2) + p_2 \quad . . . . \text{(ii.)}$$

$$p_0 = \frac{r_1^2 - r_0^2}{r_1^2 + r_0^2}(t_0 + p_1) + p_1 \quad . . . . \text{(i.)}$$

thus determining p_0, the maximum allowable powder pressure in the gun, for maximum working values of $t_0, t_1, t_2 . . . t_{n-1}$; and these are the fundamental equations employed in gun-making.

(20) With no shrinkage, or a homogeneous gun, the maximum allowable powder pressure would be reduced to

$$t_0 \frac{r_n^2 - r_0^2}{r_n^2 + r_0^2},$$

so that we perceive the advantage of the shrinkage in strengthening the gun.

(21) In Fig. 5 the dimensions are taken from the American "Notes on the Construction of Ordnance," No. 31, by Lieut. Rogers Birnie, slightly altered to round numbers; the diameter of the powder-chamber of the 8-inch gun is supposed to be 10 inches; so that $r_0 = 5$; and we put $r_1 = 7$, $r_2 = 11$, $r_3 = 13$, $r_4 = 16$; instead of 4·75, 7, 11, 13·15, and 15·75, as given in the Note 31.

Now, solving equations (25), (iv.), (iii.), (ii.), (i.) with $p_4 = 0$, $t_3 = t_2 = t_1 = 18$, $t_0 = 15$, we shall find—

$$p_3 = \frac{16^2 - 13^2}{16^2 + 13^2} \times 18 = 3{\cdot}7, \qquad t'_4 = 14{\cdot}3;$$

$$p_2 = \frac{13^2 - 11^2}{13^2 + 11^2}(18 + 3{\cdot}7) + 3{\cdot}7 = 7{\cdot}3, \; t'_3 = 14{\cdot}4;$$

$$p_1 = 18, \qquad t'_2 = 7{\cdot}3;$$

$$p_0 = 28{\cdot}7, \qquad t'_1 = 4{\cdot}3.$$

Thus, the maximum allowable powder pressure in the chamber of this gun is nearly 29 tons per square inch; so that if the pressure is limited to 17, the gun has a factor of safety $29 \div 17 = 1{\cdot}7$.

Joining the tops of the ordinates for p_0 and p_1, t_0 and t'_1, &c., by Barlow's curves, we have the graphical representation of the maximum allowable firing stresses of this gun; in which it must be noticed that the area of the rectangle, $p_0 r_0 = 143{\cdot}5$, is equal to the area of all the circumferential tension curves bounded by the jagged edge $t_0 t'_1 t_1 t'_2$. . . .

(22) With a powder pressure $p_0 = 28{\cdot}7$ (tons on the square inch) the powder stresses will be given by

$$t_0 = \frac{16^2 + 5^2}{16^2 - 5^2} \times 28{\cdot}7 = 34{\cdot}9,$$

$$p_1 = p_0 \frac{7^{-2} - 16^{-2}}{5^{-2} - 16^{-2}} = 13{\cdot}1,$$

$$t_1 = t_0 - p_0 + p_1 = 19{\cdot}3 = t'_1;$$

$$p_2 = 3{\cdot}5, \quad t_2 = 9{\cdot}7 = t'_2;$$

$$p_3 = 1{\cdot}0, \quad t_3 = 7{\cdot}2 = t'_3;$$

and

$$p_4 = 0, \quad t_4 = 6{\cdot}2.$$

Subtracting these powder stresses from the firing stresses, we are left with the initial stresses in the gun in a state of repose, represented in Fig. 6, and given by

$p_0 = 0$,	$t_0 = -19{\cdot}9$,	
$p_1 = 4{\cdot}9$,	$t'_1 = -15$,	$t_1 = -1{\cdot}3$;
$p_2 = 3{\cdot}8$,	$t'_2 = -2{\cdot}4$,	$t_2 = -8{\cdot}3$;
$p_3 = 2{\cdot}7$,	$t'_3 = 7{\cdot}2$,	$t_3 = 10{\cdot}8$.
$p_4 = 0$,	$t'_4 = 8{\cdot}1$,	

(23) The data to which the gun-maker works are, first, the calibre of the gun; and secondly, the maximum powder pressure to be expected at any point of the bore; from these data, and the quality of the steel at his command, and also from the capacity of his machinery in producing and shaping the various pieces, the gun-maker proceeds to calculate the requisite thickness and number of the coils, arranged so that the maximum working tension shall not exceed certain practical limits laid down (18 tons per square inch in the coils, and 15 in the tube).

Thus, suppose he is called upon to design the cross-section of a gun over the powder-chamber, 10 inches in diameter, to stand a pressure of 20 tons per square inch.

He will generally take a factor of safety, say 2, and allow for double the pressure, so that he puts $p_0 = 40$, and then $t_0 = 15$.

He has r_0 given as 5 inches, and now r_1 is settled by the manufacture of the solid steel block or log, which is bored out to form the inner tube A; and now he can calculate p_1 and t'_1.

Practical considerations of manufacture decide the thickness and external radius r_2 to be given to the jacket B; and now, knowing r_1, r_2, p_1, and $t_1 = 18$, he can calculate p_2 and t'_2.

Similar practical metallurgical and manufacturing considerations decide the most suitable thickness for the hoops C, D, . . . ; and when he finds the radial pressure has become zero (or negative) the gun-maker knows that he has given his gun sufficient thickness and strength.

(24) A rule, suggested by Colonel Gadolin, was originally found convenient, by which the radii of the coils were made to increase in geometrical progression; this rule, though useful when guns were formed of a steel tube strengthened with wrought-iron hoops, is obsolete now that steel is used throughout; it was, however, formerly employed as a first approximation in the tentative solution of the problem.

The Longitudinal Stress in the Gun.

(25) So far we have not yet taken into account the distribution of longitudinal tension in the gun; and it must be confessed that no satisfactory rigorous theory exists at present for the determination.

Practically it is usual to take the longitudinal tension as uniform across a cross-section, and as due to the powder-pressure in the bore, treated as a closed vessel, closed at one end by the breech-piece, and at the other by the projectile.

Thus, with r_0 and r_2 as the internal and external radii, and p_0 as the powder-pressure, the longitudinal tension will have its average value

$$\pi r_0^2 p_0 / \pi(r_2^2 - r_0^2) = p_0/(r_2^2/r_0^2 - 1) \quad . . . (26)$$

tons per square inch.

The average circumferential tension being

$$p_0 r_0/(r_2 - r_0),$$

this longitudinal tension will be

$$\frac{r_0^2}{r_2^2 - r_0^2} \cdot \frac{r_2 - r_0}{r_0} = \frac{r_0}{r_2 + r_0}$$

of the average circumferential tension, reducing to one-half in a thin cylinder, in which we may put $r_2 = r_0$.

For this reason it was formerly considered safe to leave the longitudinal strength to take care of itself; but some alarming failures, in which the gun on firing drew out like a telescope, have shown the necessity of carefully hooking the coils together, to provide the requisite longitudinal strength.

The larger the gun, the greater the number of separate parts requisite in its construction, and the greater the difficulty of providing for longitudinal strength.

(26) By taking a simple cylindrical tube under given internal and external pressures, and supposing it closed by hemispherical ends, a certain theory of distribution of longitudinal tension can be constructed.

For while the cylindrical part has the same transverse stresses as previously investigated, the stresses in the hemispherical ends may be considered the same as would be produced if they were joined up into a complete spherical vessel, under the same applied pressures.

A similar procedure to that already given for the cylinder is shown by Rankine ("Applied Mechanics," § 275) to lead to radial pressure $p = ar^{-3} + b$, and tension $t = \frac{1}{2}ar^{-3} - b$, in all directions perpendicular to the radius r.

For equation (2) for the cylinder becomes modified in the sphere to

$$\int_{ri}^{r} 2\pi r t dr = \pi r_i^2 p_i - \pi r^2 p \; ; \quad . . . . . (27)$$

or, differentiating with respect to r,

$$2rt = -d(r^2p)/dr$$
$$= -2rp - r^2dp/dr,$$

or

$$t = -p - \tfrac{1}{2}rdp/dr. \quad \ldots \ldots \ldots (28)$$

(27) The first assumption of Barlow, that there is no cubical compression, gives $t = \frac{1}{2}p$; and therefore

$$dp/p + 3dr/r = 0,$$

or

$$pr^3 = a, \text{ a constant,}$$
$$p = 2t = ar^{-3}.$$

Rankine's second assumption of uniform hydrostatic stress gives $t = -p$; and therefore

$$dp/dr = 0, \; p = b, \text{ a constant.}$$

Hence, in the general case,

$$p = ar^{-3} + b, \; t = \tfrac{1}{2}ar^{-3} - b; \ldots \ldots (29)$$

where a and b are determined from the given values p_i and p_o of the internal and external applied pressures; so that

$$p_i = ar_i^{-3} + b, \quad p_o = ar_o^{-3} + b,$$

$$a = \frac{p_i - p_o}{r_i^{-3} - r_o^{-3}}, \quad b = \frac{p_o r_o^3 - p_i r_i^3}{r_o^3 - r_i^3} \quad \ldots (30)$$

(28) We may now take $\frac{1}{2}ar^{-3} - b$ to represent the longitudinal tension at radius r in the cylindrical part of the closed vessel.

Unfortunately for the strict mathematical accuracy of this method, we must suppose the circumferential tension to change suddenly from its value given from the formula $ar^{-2} - b$ to one given by a formula of the form $\frac{1}{2}a'r^{-3} - b'$, in passing from the cylindrical part to the hemispherical end.

A. G. GREENHILL.

(*To be continued.*)

STUDIES IN BIOLOGY FOR NEW ZEALAND STUDENTS.[1]

IT is now generally recognized that of all recent works dealing with elementary natural science, none have more thoroughly revolutionized our methods of teaching than those of Huxley, well known; and the years 1875–77 will be for all time memorable to English-speaking students, as those which marked their publication. The principles therein laid down are now so well known and generally adopted, that explanation of them would be here superfluous. In his work on "Physiography" the author points out (preface, p. viii.) that any intelligent teacher will have no difficulty in making use of the resources of his surroundings, in the manner and to the end laid down by himself; and this, in the long run, is the refrain of the method by which he has effected the revolution alluded to. So far as external evidences go, this wise counsel appears to have been nowhere more readily acted upon than in New Zealand.

Prof. Hutton, now of Christchurch, New Zealand, early took the hint; and, in so doing, produced the first of the series of pamphlets now under consideration. He chose for his purpose the Shepherd's Purse (cf. NATURE, vol. xxiv. p. 188), and Prof. Parker, who succeeded him, has, in turn, prepared notes serial with those of his predecessor—upon "The Bean Plant" (1881), and now upon "The Skeleton of the New Zealand Crayfishes." During the interval between the publication of Prof. Parker's pamphlets there appeared the third of the series, entitled "The Anatomy of the Common Mussels (*Mytilus latus, edulis,* and *magellanicus*)." This, the work of Alex. Purdie, and the least didactic of the series, was originally presented as a thesis for the degree of M.A. in the University of New Zealand.

The pamphlets alluded to are illustrated—in the case of that before us, by six clear woodcuts; and those of Parker, with which we need now alone be concerned, chiefly depart from the precedent laid down by Huxley in their less rigid adherence to the single type organism chosen for study. Wherever parts of this are, by adaptive change, so modified as to be non-typical in structure, Parker has introduced supplementary descriptions of corresponding parts of less modified allies. The necessity for this mode of procedure is now generally recognized; and the only danger to be averted in the future is that of unconscious reversion to the old condition of the "*omnium gatherum* of scraps." Let the type organism be always adhered to as closely as possible. Prof. Parker has exercised, in the matter, a wise discretion; and, having availed himself of the researches of Boas, has given to the world of carcinologists a laboratory guide which cannot fail to be of great service to them. The arthropods of the genus *Palinurus* happen to have furnished him, a few years ago, with material for original observation; the results of his inquiry are brought to bear upon the needs of the beginner in the pamphlet before us, and the value of the latter is thereby enhanced.

In dealing with the morphology of the eye-stalk (and of the pre-oral region generally), Prof. Parker states the alternative views, and gives the names of their leading upholders. Although he adopts the belief that the ophthalmic and antennulary regions of the arthropod body do not form the first and second metameres, and introduces, in accordance therewith, a revised nomenclature, his remarks, when dealing with the real point at issue, are so framed as to leave the mind of the student unbiassed. And moreover, he has so arranged his book that consideration of this vexed question in morphology is deferred until the concluding paragraph. This is as it should be. He naïvely summarizes the position in the words:—

"The fact of the eye-stalk bearing a flagellum seems to prove conclusively that it and the antennule are homologous. The question then resolves itself into this: Are the eye-stalks and antennules appendages in the ordinary sense, *i.e.* lateral offshoots of the first two metameres, or are they to be looked upon as prostomial appendages comparable with the tentacles of Chætopods and the antennæ of insects?"

Mindful of comments upon the general question raised in the above, which have already appeared in this journal (NATURE, vol. xxxv. p. 506), we are of opinion that equally good arguments are still to be adduced on both sides. The extraordinary facts of development of the invertebrate nervous system which are now accumulating, render it doubtful if we are justified in regarding the prostomium as something so very different from the rest of the body as we are wont; and we are led to ask whether it may not merely represent a precociously differentiated portion of the common perisoma? If there is any truth in the belief that the symmetry of the bilateralia is a laterally compressed radial one, the probability that the prostomium may represent that which we suggest becomes vastly increased; and it is worthy of remark that that lobe in some Chætopods (*Nemodrilus, Phreoryctes*) so far conforms to the characters of a body segment as to become externally subdivided. Nor must it be forgotten that the *Catometopa* bear (especially the *Ocypodidæ*), an optic style which would appear to present us, in its variations, with a series of conditions transitional between that of the eye-stalk of Milne-Edwards's *Palinurus* (to which Parker appeals in seeking to show that that appendage and the antennules are homologous) and that of the ordinary podophthalmatous forms.

We congratulate the students of the University of Otago upon the good use which, in their interests, their Professor has made of the advice of his distinguished master. We cannot, however, allow to pass unnoticed the statement (p. 7) that the seventh abdominal somite (by which term Prof. Parker designates the telson) bears appendages only in Scyllarus. This is not the case, as has been previously pointed out in these pages (NATURE, vol. xxxii. p. 570). The supposed appendages, did they exist, would be at least peri-proctous in position; and, as there is reason to believe the antennules (which Parker, be it remembered, admits to be serially homologous with the ophthalmites) to have been originally peri-stomial, if not meta-stomial, the supposed peri-proctous appendages might, with equal reason, be denied homology with the other abdominal members.

Finally: the altered position of the sterna in the anterior cephalic region and the consequent displacement of their appendages is said to be "a result of the *cephalic flexure*, by which, in the embryo, the anterior cephalic sterna become bent strongly upwards." Allowance has not yet been made, in dealing with this question, for the fact that, in the Decapods, these changes are greatly exaggerated by the general enlargement of the cephalo-thoracic region, consequent upon the aggregation therein of the more important and specialized viscera, and upon specialization of the thoracic appendages for ambulation.

G. B. H.

[1] "Studies in Biology for New Zealand Students." No. 4. "The Skeleton of the New Zealand Crayfishes (*Palinurus* and *Paranephrops*)." By T. J. Parker, B.Sc., F.R.S., of the University of Otago. (Wellington: Colonial Museum and Geological Survey Department. London: Trübner and Co.)