not suited for testing uneducated persons. A similar instrument, introduced by Chibret and Meyer, of Paris, is to be found in ophthalmic hospitals.

I may further remark that I do not consider any test satisfactory unless made by an ophthalmic surgeon, as he alone is accustomed to deal with such people every day of the week, and can alone eliminate such errors as refraction-disease and stupidity. D. D. REDMOND.

14 Harcourt Street, Dublin, May 3.

The Green Flash at Sunset.

YOUR correspondents (vol. xli. pp. 495, 538) seem to imply that this phenomenon is only seen at sea, but I observed it on May 17 while walking from east to west, near Worms Heath (Warlingham, Surrey). It had been an exceptionally fine day, since the morning, and about 8 p.m. there was not a cloud in the sky, except to westward, where strips of cloud were rapidly forming, and covering up the glow of sunset; the sun had sunk behind a hill, when, suddenly, my companion and I both saw a flash of green light against the thickest cloud; it lasted 1 or 2 seconds, just long enough for there to be no doubt about it. We compared it to the glare thrown by "green fire," extending over an area whose diameter appeared about four times that of the moon.

At 12 p.m. the same night it was raining.

I think this observation definitely negatives the sea-wave theory, while the appearance was seen at least in association with the condensation of aqueous vapour. Perhaps the reason it was not *bluish*-green was that this vapour absorbed the blue rays? T. ARCHIBALD DUKES.

16 Wellesley Road, Croydon, June 2.

THE THEORY OF SCREWS.[1]

THE book before us, a large octavo volume of over 600 pages, gives in a connected form the results of Sir R. S. Ball's investigation in the theory of screws, as contained in his "Theory of Screws" and a series of publications in the Proceedings and Transactions of the Royal Irish Academy. But as its scheme is that of a text-book on theoretical mechanics, it begins with a chapter on the postulates and methods of mechanics; whilst chapter vii. is on the theory of moments of inertia; chapter viii. on impulsive forces capable of imparting to a rigid body a given state of velocity; and chapter x., on kinetic energy, contains a number of propositions from analytical dynamics. Here expressions for the kinetic energy, for its change in consequence of an impulse, Lagrange's equations of motion in generalized co-ordinates, Hamilton's principle of least action, and various other propositions, are developed in the usual form—that is to say, without the use of screws. The rest of the book relates to the theory of screws and its applications. This alone, as forming the characteristic feature of the book, concerns us here, and of it we shall try to give an outline.

In order not to be unintelligible to those who have no knowledge of Ball's creation, it will be necessary to begin with the very elements of the subject; and in order to form a just idea of the scope and importance of the new method, it will not be sufficient to give a sketch of the results obtained—it will be necessary to take a wider view of the subject. We shall then be able to form some idea of the inherent capabilities of the theory. These I believe to be very great—very great indeed. One of its peculiarities lies in this, that all the results obtained in modern algebra and geometry, as distinct from analysis, seem to be directly applicable to it.

Friends of synthetic geometry and of graphical methods, too, will find here a wide field for investigations. Grassmann's "Ausdehnungslehre" has already been pressed into its service, and the theory of vectors and quaternions is easily applicable. Clifford, in fact, has generalized the latter theory into that of biquaternions to embrace screws.

Mr. Cartesius, to make use of Sir Robert's personifications, has been dethroned, and Mr. Anharmonic together with Mr. One-to-one reign in his place.

Poinsot, whose investigations form the starting-point of the theory of screws, has proved that a rigid body can always be transferred from one position to any other by a rotation about a certain perfectly determined axis, together with a translation along this axis. These two motions combine to a motion identical with that of a nut on a screw. It is completely determined if the angle through which the rotation takes place, together with the ratio of the translation to the rotation, is given. This ratio—*the "pitch" of the screw*—characterizes the screw. As the motion does not at all depend upon the diameter of the screw, we may suppose this to become infinitely small, and then we have the notion of Sir R. Ball's *screw*.

A screw, therefore, is a line in space which has connected with it a certain pitch—*i.e.* a certain length, as the pitch is a linear magnitude. The compound motion considered is called a "twist" about a screw, and is known if the screw and the "amplitude" of the twist, *i.e.* the amount of rotation, is given. In the same way a system of forces can, according to Poinsot, always be reduced, and that in one way only, to a single resultant and a couple turning about the resultant; and these two dissimilar parts Ball combines to a "wrench on a screw," the line of action of the resultant force being the axis of the screw and the ratio of the moment of the couple to the force giving the "pitch," whilst the magnitude of the resultant force is called the "intensity" of the wrench.

We have thus a new entity—the screw—and its introduction forms the characteristic distinction of the theory. Connected with it is a kinematical and a kinetic entity—the twist about a screw, and the wrench on a screw.

If we now consider a rigid body under the action of any forces, then the latter combine at every moment to a wrench on some screw, whilst the motion itself is always a twist about some other screw. If the body is constrained in any manner, then the reactions due to the constraint will also at every moment combine to a wrench about some screw.

The problem first to be solved is that of the combination of twists and wrenches. Let any two screws, α and β, be given, then wrenches on them constitute together a system of forces, and therefore combine to a wrench on some other screw, γ, which has to be determined. If the ratio of the intensities of the two given screws be varied whilst the screws themselves remain unaltered, then the screw, γ, of the resultant wrench also varies, and its axis describes a surface called the cylindroid. This is a ruled cubic surface which can be described as follows:—Let through a fixed line, l, a plane be drawn, and in it a circle be taken. Let a point, P, move uniformly in the circumference, whilst the plane itself turns uniformly about l, completing half a revolution whilst P describes the whole circumference. The perpendicular from P to l will then generate the cylindroid, and the screw on any generator will have a pitch equal to the length of the perpendicular from P to l. The line l is a nodal line of the surface and perpendicular to all screws on it. All cylindroids are similar, and through any two screws one cylindroid can always be drawn. The projections of all generators on a plane, perpendicular to the nodal line, form a flat pencil in which each ray corresponds to one screw. Also to each point on the circle corresponds one screw. We may here mention that this generation of the cylindroid stands in a very close relation to the plane representation of the cylindroid which is given in chapter xx. For if A, B are the ends of the diameter of the above circle which is perpendicular to the nodal line l, then to A and B correspond two generators of the cylin-

[1] "Theoretische Mechanik starrer Systeme auf Grund der Methoden und Arbeiten, und mit einem Vorworte von Sir Robert Ball, Royal Astronomer of Ireland." Herausgegeben von Harry Gravelius. (Berlin: Georg Reimer, 1889.)