on the South African gold-fields, which include much information on the present condition of the whole of South Africa as far north as the Zambesi. The observer points out that, while the Delagoa Bay and other lines of communication are much discussed, the fine artery of the perfectly navigable Limpopo is entirely neglected, notwithstanding Captain Chaddock's navigation of it a few years ago. The writer remarks that "this river flows mainly through regions under the influence or protectorate of England; the Transvaal people on the one side, and those of Matabeleland on the other, would certainly be glad to avail themselves of this outlet for their produce. As it traverses only a small tract of Portuguese territory about its estuary, I hope and believe that Portugal will not be allowed to treat the Limpopo as she is now attempting to treat the Zambesi. The subject is of such importance that it cannot fail soon to be brought before the British Parliament." Referring to the negotiations at present going on in connection with the Swaziland question, he observes, in the same spirit:—"The Swazi people must, sooner or later, yield either to the Transvaal or to England, and if to the former, it must be to the entire detriment of British interests. England, as the suzerain power in South Africa, should be the first in the field, both in her own interest and in that of her other colonies and subjects. If she does not assume the protectorate of Swaziland, besides losing the control of a vast and rich mineral district, she will deprive the colony of Natal of all further hope of expansion. If she ignores her responsibility in this matter, and allows the Transvaal Republic to absorb Swaziland, she will add another to the long list of blunders that threaten to destroy all prospect of consolidating a dominion as large as Canada, and may end disastrously for British interests in South Africa."

A FRENCH traveller has just achieved a feat of great interest. Captain Trivier, equipped by the newspaper *La Gironde*, started some eighteen months ago for the Congo State. He went up the river to Stanley Falls, and thence proceeded to Central Africa and the Lake region, accompanying caravans. He has just arrived at Mozambique.

Globus reports that during the past summer M. Thoroddsen, the well known student of Iceland, has carried out a journey in the waste region known as Fiskivötn, lying between Hecla and the Vatna Jökul, which has hitherto been unvisited for the most part by any inquirer. To the east and north of Hecla he discovered a new obsidian region. Crossing the Tunguaa, he went to the Fiskivötn group of lakes, all true crater lakes. The district between this and the Vatna Jökul has absolutely no plant-life whatever; it consists of lava-fields, and plains of volcanic sand. In it he found a lake, Thorisvatn, the second largest in the island. Thence, after a day's journey through an utterly desolate district, he reached the hitherto unknown source of the Tunguaa. To the south of this he discovered, between three ranges of hills, previously unknown, a new and very long lake.

MR. DAUVERGNE has, says the *Times of India*, completed an adventurous journey in the regions of North-West Cashmere. His course was from Leh northwards to the Kilian Pass, in Kashgaria, and then northwards across the Pamir to the Upper Oxus. He reached Sarhad in safety, and after six days' halt there, crossed the Hindu Kush by the Baroghil Pass, as he did not wish to visit Chitral. He then turned eastwards, and after a trying journey through the snow, crossed the Ishkaman Pass, north of Yasin. Thence he travelled southwards by the Karambar Valley, and eventually reached Gilgit, a short time after Captain Durand had started for Chitral. Mr. Dauvergne reports that the Russian explorer, Captain Grombchevsky, whose attempt to reach Kafiristan was noticed some time ago, was stopped at Kila Panjah on the Oxus, by the Afghan authorities.

THE ST. PETERSBURG PROBLEM.

THIS celebrated problem, which is first mentioned before 1708 in a letter from the younger Nicholas Bernoulli to Montmort, has been frequently discussed by Daniel Bernoulli (1730) and other eminent mathematicians. It may be briefly stated as follows:—

A tosses a coin, and undertakes to pay B a florin if head comes up at the first throw, two florins if it comes up at the second, four florins if it be deferred until the third throw, and so on. What is the value of B's expectation?

The chance of head appearing at the
1st, 2nd, 3rd, 4th . . . . nth throw is
$\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ . . . . $\frac{1}{2}^n$. A promises to pay for head
1, 2, 4, 8 . . . . 2^{n-1} florins, hence B's expectation is
$\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$, $\frac{8}{16}$ . . . . $2^{n-1}/2^n = \frac{1}{2}$ florin.

Hence the total value of B's expectation is an *infinite* series, each term of which is a shilling, or it is infinite.

This result of the theory of probability is apparently directly opposed to the dictates of common-sense, since it is supposed that no one would give even a large finite sum, such as £50, for the prospect above defined.

Almost all mathematical writers on probability have allowed the force of the objection, which they have endeavoured to evade by various ingenious artifices all more or less unsatisfactory.

The real difficulty of the problem seems to lie in the exact meaning of *infinite* and *value of the expectation.*

Since the infinite value of the result is *only* true if an infinite number of trials are paid for and made, all such considerations as want of time and the bankruptcy of A or B are precluded by the terms of the question.

The value of B's expectation is frequently confused with how much he can or ought to pay for it; thus Mr. Whitworth ("Choice and Chance," p. 234) finds that if B have 1024 florins, he may give very little more than 6 florins for the venture. This ingenious, solution seems to have no reference to the original problem, which has been modified by Mr. Whitworth's introduction of the word "advantageously" (p. 232).

B can pay for his expectation in three ways: (i.) a sum before each toss; (ii.) a sum before each series of tosses ending with head; (iii.) a sum for the total result of A's operations.

Mr. Whitworth apparently assumes the first method of payment, and shows that the larger B's funds are the more he may safely pay for each toss, since he can continue to play longer. Many mathematicians take the second method of payment. "However large a fee I pay for each of these sets, I shall be sure to make it up in time" ("Logic of Chance," p. 155).

It is easy to show in this case also that what may be safely paid before each series increases with the number of series.

Suppose a very large number of tosses made, about half would come up heads and half tails; each head would end a series, when a fresh payment must be made by B. Suppose the tosses limited to one series, if B pays one florin he cannot possibly lose, if he pay anything more he may lose by head coming up the first time, and the more he pays the greater will his chance of loss be, since the series of tails must be longer to cover it. But, however large a finite sum he pays, he is not *certain* to lose, *e.g.* head *may* not come up till the hundred and first toss, when he would receive

2^{100} = 1,267 650,600 228,229 401,496 703,205 376 florins.

If the sets are limited to one hundred, about

50 heads would probably come up the			1st toss.			B would receive for each series 50 florins.
25	,,	,,	,,	2nd	,,	
13	,,	,,	,,	3rd	,,	
6	,,	,,	,,	4th	,,	
3	,,	,,	,,	5th	,,	
2	,,	,,	,,	6th	,,	
1	,,	,,	,,	7th	,,	

Hence for the hundred sets, B would receive about 350 florins, or he could pay without loss seven shillings for each set.

If N be the number of sets, the total amount received by B will probably not be less than n terms of the series

$$\left\{\frac{N \times 2^0}{2^1} + \frac{N \times 2^1}{2^2} + \&c.\right\} = n\{\tfrac{1}{2}\}N,$$

but n is the number of times which N is successively divisible by 2, or $2^n = N$, or $n = \log N/\log 2$. But the amount x which B can afford to pay per set when multiplied by the number of sets is equal to the amount which he receives, or—

$$xN = \frac{\log N}{\log 2}\{\tfrac{1}{2}\}N,$$

hence $x = \log N/0{\cdot}6$ nearly.

This formula, though inexact for low, is very convenient for high, values of N.

$N = 1$	$x = 0$	$N = 10^6$	$x = 10$
$= 50$	$= 2{\cdot}7$	$= 10^9$	$= 15$
$= 100$	$= 3{\cdot}3$	$= 10^{12}$	$= 20$
$= 1000$	$= 5$	$= 10^{15}$	$= 25$

x increases with, though much more slowly than, N, and becomes infinite when N does. But to justify a payment of

£50 per set, we must expect a number of sets represented by 301 figures.

Lastly, what is the value of B's expectations if A's operations are continued indefinitely. With great deference to contrary opinions, I believe this to be the correct meaning of the problem in its original form. The theoretical result is in this case easily realized by the aid of the following illustration. Suppose the person A replaced by an automatic machine similar to that used for weighing sovereigns, which tosses continuously ten times per minute. On the average of a large number of tosses, B cannot receive less than one shilling a toss, £1 every two minutes, or £720 a day for ever. If the current rate of interest be 3 per cent., he may safely pay for this perpetual annuity £8,760,000. Suppose, instead of this comparatively slow rate, the machine increased the rapidity of its operations indefinitely, the sum to be paid for the result would also increase indefinitely, or the expectation would become infinite.

Sydney Lupton.

UNIVERSITY AND EDUCATIONAL INTELLIGENCE.

Cambridge.—The Newall Telescope Syndicate has drawn up a scheme for building a dome for the telescope on a site adjoining the present Observatory, with an observer's house; and they recommend that an observer be appointed, at a stipend of £250 per annum, with a house, to devote himself to research in stellar physics, under the general direction of the Director of the Observatory.

The results of this year's commercial examination, held by the School's Examinations Board, are satisfactory. Geography was still very imperfect. Elementary mechanics has now been added to the list of compulsory subjects.

An influential syndicate has been appointed to consider the question of the mechanical workshops, their management and utility.

SOCIETIES AND ACADEMIES.

London.

Royal Society, December 12.—"An Experimental Investigation into the Arrangement of the Excitable Fibres of the Internal Capsule of the Bonnet Monkey (*Macacus sinicus*)." By Charles E. Beevor, M.D., F.R.C.P., and Victor Horsley, B.S., F.R.S. (from the Laboratory of the Brown Institution).

After an historical introduction, the authors proceed to describe the method of investigation, which was conducted as follows. The animal being narcotized with ether, the internal capsule was exposed by a horizontal section through the hemisphere. By means of compasses the outline of the basal ganglia and capsule were accurately transferred to paper ruled with squares of one millimetre side, so that a projection of the capsule was thus obtained, divided into bundles of one millimetre square area. Each of these squares of fibres was then excited by a minimal stimulus, the same being an induced or secondary interrupted current. The movements were recorded and the capsule photographed.

In all forty-five experiments were performed, and they are arranged in eight groups, representing eight successive levels (*i.e.* from the centrum ovale to the crus) at which the capsule was investigated.

Before the results are described in detail a full account is given of previous investigations, experimental, clinical, and anatomical, on the arrangement of the internal capsule.

The anatomy of the part and the relation of the fibres to the basal ganglia are then discussed, and a full description given of each of the groups examined.

The general results are next given at length, of which the following is a *résumé*.

Firstly, the rare occurrence of bilateral movement is discussed, and the meaning of the phenomenon defined. Secondly, the lateral arrangement and juxtaposition of the fibres are considered. Thirdly, the antero-posterior order in which the fibres for the movements of the different segments are placed is described, and that order found to be practically identical with that observed on the cortex, viz. from before back:—

Movements of eyes.
,, head.
,, tongue.
,, mouth.
,, upper limb (shoulder preceding thumb).
,, trunk.
,, lower limb (hip preceding toes).

The character or nature of these movements is set out in a table giving the average localization of each segment. Speaking generally, it may be said that the movements are arranged in the same way as has already been shown by the authors to exist in the cortex (*vide* previous papers in Phil. Trans., 1887, 1888), viz. that the representation of extension is situated in front of flexion for the segments of the upper limb, while for the toes flexion is obtained, as in the cortex, in front of extension.

Numerous tables and diagrams are appended, showing the extent of appropriation of fibres for each movement.

Physical Society, November 15.—Prof. Reinold, F.R.S., President, in the chair.—Mr. Enright resumed the reading of his paper on the electrification due to contact of gases with liquids. Repeating his experiments with zinc and hydrochloric acid, the author, by passing the gas into an insulated metallic vessel connected with the electrometer, proved that it was always charged with electricity of the opposite kind to that of the solution. The electrical phenomena of many other reactions have been investigated, with the result that the gas, whether H, CO_2, SO_3, SH_2, or Cl, is always electrified positively when escaping from acids, and negatively when leaving a solution of the salt. In some cases distinct reversal is not obtainable, but all these seem explicable by considering the solubility and power of diffusion of the resulting salts. Various other results given in the paper tend to confirm this hypothesis. Seeking for an explanation of the observed phenomena, the author could arrive at no satisfactory one excepting "contact" between gases and liquids, and if this be the true explanation he hoped to prove it directly by passing hydrogen through acid. In this, however, he was unsuccessful, owing, he believes, to the impossibility of bringing the gas into actual contact with the liquid. True contact only seems possible when the gas is in the nascent state. Some difficulty was experienced in obtaining non-electrified gas, for the charge is retained several hours after its production, even if the gas be kept in metallic vessels connected to earth. Such vessels, when recently filled, form condensers in which the electricity pervades an inclosed space, and whose charge is available on allowing the gas to escape. Soap bubbles blown with newly generated hydrogen were also found to act as condensers, the liquid of which, when broken, exhibited a negative charge. This fact, the author suggested, may explain the so-called "fire-balls," sometimes seen during thunderstorms; for if, by any abnormal distribution of heat, a quantity of electrified air becomes inclosed by a film of moisture, its movements and behaviour would closely resemble those of fire-balls. A similar explanation was proposed for the phenomenon mentioned in a recent number of Nature, where part of a thundercloud was seen to separate from the mass, descend to the earth, and rise again. The latter part of the paper describes methods of measuring the contact potential differences between gases and liquids, the most satisfactory of which is a "water dropper," and by its means the P.D. between hydrogen and hydrochloric acid was estimated to be about 42 volts. Prof. Rücker asked if the experiment with zinc and hydrochloric acid could be started in the second stage by having the acid partly saturated with salt. Dr. C. V. Burton thought it probable that contact could be made between a gas and a liquid by shaking them up together in a bottle. In reply, Mr. Enright said the experiment could be started at any stage, and reversal effected as often as desired by adding either acid or a solution of salt to the generating vessel.—Mr. Herbert Tomlinson, F.R.S., read a paper on the effect of repeated heating and cooling on the electrical resistance and temperature coefficient of annealed iron. In a paper recently presented to the Royal Society, the author has brought forward an instance of an iron wire, which when subjected to magnetic cycles of minute range alternately at 17° and 100° C., had its molecular friction and magnetic permeability reduced respectively to about one-quarter and one-half their original values. The present experiments were undertaken to see whether by