

approximation to the true theoretical shape of the arch is attained either by taking the "three-point circle," passing through the vertex A and the two points of equal curvature B_2 and B_2' , or by taking the "described circle," touching at B_1 and B_1' , or by taking the "inscribed circle," touching at A and again internally at B_3 and B_3' beyond the points of maximum curvature B_1 and B_1' , instead of taking, as customary, the circle of curvature at the vertex; and the authors show that if an elliptical arch is described, the proper approximation to its shape is obtained from an orthogonal projection of one of these circles.

The points B_1 and B_1' are called the *noes* of the transformed catenary, and give the name to the paper.

The transformed catenary, which may be taken as the line of thrust, is shown to lie below the "described circle," and above the "three-point circle," so that by taking these or similar circles for the boundaries of the ring of the arch, the proper stability of the arch is secured.

The mathematical treatment of the Catenary given by the authors would gain considerably in elegance by the employment of the hyperbolic functions, now no longer to be disregarded; thus, instead of writing—

$$y = r \frac{m}{2} \left(e^{\frac{x}{m}} + e^{-\frac{x}{m}} \right)$$

$$\tan \theta = \frac{dy}{dx} = \frac{r}{2} \left(e^{\frac{x}{m}} - e^{-\frac{x}{m}} \right),$$

the notation—

$$\frac{y}{m} = r \cosh \frac{x}{m}, \quad \tan \theta = r \sinh \frac{x}{m}$$

should be employed; and for purposes of numerical calculation of these hyperbolic functions, it is only necessary to notice that if $\tan \theta = \sinh u$, $\sec \theta = \cosh u$, then $u = \text{hyp. log} (\sec \theta + \tan \theta)$; so that the table of u , already calculated by Euler, used in conjunction with the tables of the ordinary circular functions, will give us the numerical values of the hyperbolic functions; for large values of u , when θ denotes an angle being nearly a right angle, the approximate equation—

$$\log \cosh u = \log \sinh u = u \log e - \log 2$$

being sufficient.

Tables of numerical results are given at the end of the paper, with practical illustrations, for the benefit of architects and engineers, and supplementary tables are added for the immediate designing of brick, sandstone, and granite arches, with circular soffits; so that these investigations should prove decidedly useful to those engaged in the design of similar structures. A. G. G.

SCIENTIFIC SERIALS.

American Journal of Mathematics, vol. xi. No. 3 (Baltimore, April 1889).—Were it not for the size of the pages, this number might be taken to be a number of the *Mathematische Annalen*, seeing that out of its ninety-eight pages sixty-eight are written in German. The first memoir, by Oskar Bolza (pp. 195-214), is entitled "On the Construction of Intransitive Groups," and touches on points discussed by Jordan ("Traité des Substitutions"), Capelli ("Sopra l'Isomorfismo dei Gruppi di Sostituzioni"), Netto ("Substitutionentheorie"), Cayley ("Theory of Groups"), and Dyck ("Gruppentheoretische Studien"). This is followed by a short note by Karl Heun (pp. 215-20), on "Die Herstellung einer lineären Differentialgleichung aus einem gegebenen Element der Integralfunction." Next we have an important memoir by Koenigsberger (pp. 221-82), "Ueber die Reduction von Integralen transcenderer Functionen." The closing note, by Dr. Franklin (pp. 283-92), "On the Double Periodicity of the Elliptic Functions," *inter alia*, proves a theorem of a bicircular quartic, enunciated by Clifford (see Crofton, L.M.Soc. Proc., vol. ii), and also results due to Siebeck and Greenhill, but all are established here from a different point of view.

Rivista Scientifico-Industriale, February 15.—Researches on the thermo-electric conductivity of magnetized iron (concluded), by Prof. Ercole Fossati. From these experiments the author concludes generally that the electric conductivity of iron either suffers no change under transverse magnetization or undergoes

some increase; this increase, however, is much less than the diminished conductivity of iron magnetized in the longitudinal direction. This inference agrees perfectly with the deductions he had already arrived at experimenting with iron conductors of varying dimensions.—Some experiments with a new battery, by Prof. Augusto Righi. Excellent results have been obtained from the battery here described, which consists of 108 condensers disposed in six groups of eighteen each, one above the other, in order to obtain high potentials. The outer armatures communicate with the conductors of a Holtz machine, the central with the ground, and a capacity is thus obtained equal to that of $\frac{18}{6} = 3$ jars, united in a single battery with armatures communicating directly with the two conductors of the machine. The capacity of the whole system is thus 18,810 electrostatic units, or about $\frac{1}{3}$ microfarad.

Rendiconti del Reale Istituto Lombardo, February 28.—Notice of the late Prof. Giuseppe Meneghini, by Prof. T. Taramelli. In this biographic memoir a summary account is given of the services rendered to geological studies by the eminent naturalist, who was born in Padua in 1811, and died in January of the present year. Meneghini is best known as joint translator, with Savi, of Murchison's work on the Alps, Apennines, and Carpathians, and by his systematic monograph on the fossils of the Upper Lias in Lombardy and Central Italy. His last publication was a paper on the Cambrian trilobites of the Igesiente district, and his name will always be associated rather with the palæontological than the stratigraphic or petrographic side of geology.—Meteorological observations made at the Royal Brera Observatory, Milan, during the month of February.

THE last issue of the *Memoirs of the St. Petersburg Society of Naturalists* (vol. xix., Botany) contains, besides several very interesting short papers in the Proceedings, a new contribution to the flora of Novgorod, by A. Antonoff, which raises the number of species of flowering plants discovered in the Government of Novgorod to 700; a note on the comparative anatomy of the tissues in the leaves of *Salicinea* as a basis for classification, by V. Dobrovlanski; and a suggestive description of the flora of the Shenkursk and Kholmogory districts of Archangelsk, by N. Kuznetsoff. Owing to the extension of a subsoil of limestone, which is much warmed by the sun during the summer, the flora of the region contains a number of species belonging to a more congenial climate, while several species characteristic of those latitudes are wanting. On the other hand, owing to its proximity to the Urals, and the existence on the west of such a barrier as Lake Onega, the flora contains a considerable number of Siberian species, while many West European species do not appear. M. Kuznetsoff's remarks on the extension of the *Abies sibirica*, which is slowly advancing towards the West, and the lime-tree, which seems to be, on the contrary, disappearing, are very interesting. Both the *Ulmus campestris* and the *U. effusa* were found as far north as the sixty-third degree of latitude. The presence of the *Lymnanthemum nymphoides*, which is found in Lithuania and South-Eastern Russia, but not in Central Russia, is especially remarkable, and M. Kuznetsoff explains the extension of this aquatic plant over Archangelsk by its having spread along the canal which connects the Volga with the Dvina. No fewer than twelve other South Russian plants, which must have migrated along the same canal, are named by the author. As to the *Mulgedium tataricum*, C. A. Mey., which is characteristic of the salt steppes of Astrakhan, it has been found on a shoal of the Dvina, under 64° N. lat. The paper is accompanied by a map, showing the western limits of extension of the *Abies sibirica*.

SOCIETIES AND ACADEMIES.

LONDON.

Royal Society, March 14.—"On the Organization of the Fossil Plants of the Coal-Measures. Part xvi." By W. C. Williamson, LL.D., F.R.S., Professor of Botany in the Owens College, Manchester.

In this memoir the author first calls attention to detached observations, made in his earlier memoirs, relating to the manner in which a medullary axis is developed in the interior of each of the primary vascular bundles of the Carboniferous Lycopodiaceæ. He then traces the changes undergone during the development of a small branch-bundle in *Lepidodendron Harcourtii*.