A closer consideration of the figures shows that the direction of the force changes from instant to instant for such points as lie either in the axis of z or in the plane xy. If we represent the force at a point, therefore, in the customary way by a line, the end point of this line oscillates, not indeed in a straight line, but in an ellipse. In order to see whether there are points for which this ellipse approximates to a circle, in which, therefore, the forces go through all the directions of a windrose without important change of magnitude, let us superpose two of the representations expressing times which differ by $\frac{1}{4}$T ; for instance, Figs. 1 and 3, or 2 and 4.

For the points we seek, the lines of the one set must plainly cut those of the other system orthogonally, and the distances of the lines of the one figure must be equal to those of the other. The small quadrangles formed by the superposition of the two systems must therefore be squares for the sought points.

There may be now remarked, in actual fact, a region of the kind sought : it is represented in Figs. 1 and 2 by circular arrows, whose directions give at once the direction of the rotation of the force. The dotted lines are inserted for convenience ; they belong to the line system of Figs. 3 and 4.

One finds, moreover, that the force exhibits the behaviour here described, not only at the specified points, but also in the whole strip-formed region which, spreading out from those points, forms the neighbourhood of the z axis. Nevertheless, the magnitude of the force decreases so quickly in these directions, that only in the points above-mentioned can its singular behaviour be important.

The system of forces now described and required by theory can be quite well recognized in an incomplete observation, not hitherto indicated by theory, which I formerly described (*Wied. Ann.* xxxiv. p. 155, 1888). One cannot, indeed, explain everything about those experiments, but one can get the main points correctly.

By both experiment and theory the distribution of force in the neighbourhood of the oscillator is chiefly an electrostatic distribution. By both experiment and theory the force spreads out chiefly in the equatorial plane and decreases in that plane at first quickly, afterwards slowly, without being zero at a mean distance. By both theory and experiment the force, in the equatorial plane, in the axis, and at great distances, is of constant direction and varying magnitude, while at intermediate points it changes its magnitude but little and its direction much. The correspondence between theory and those experiments only breaks down in this, that at great distances, according to theory, the force remains always normal to the straight line through the source, while by experiment it appears to be parallel to the oscillator. For the neighbourhood of the equatorial plane where the forces are strongest this follows from the equations too, but not for directions which lie between the equatorial plane and the axis. I believe that the error is on the side of experiment. In these experiments the direction of the oscillator was parallel to both the main walls of the laboratory, and the component of the force which was parallel to the oscillator might be thereby strengthened in proportion to the normal components.

I have therefore repeated the experiment with a different arrangement of the primary oscillator, and found that with certain arrangements the result corresponds with theory. I did not attain an exact result, but found that at great distances, and in regions of small intensity of force, disturbances due to the boundary of the space available were already too considerable to permit a safe verdict.

While the oscillator is at work, the energy vibrates in and out of the spherical surfaces surrounding the origin. More energy goes out, however, through any spherical surface during an oscillation than comes back ; and indeed the same excess quantity goes through all spherical surfaces. This extra quantity represents the loss of energy during the period of swing due to radiation. We can easily calculate its value for a spherical surface whose radius, ρ, is so great that it is permissible to employ a simplified formula. Thus the energy going out of the spherical zone between θ and $\theta + d\theta$ in the time dt will be—

$$2\pi\rho \sin\theta \, \rho d\theta \, dt . \frac{P}{4\pi A} (Z \sin\theta - R \cos\theta).$$

Putting into this the values of Z, P, and R, which are proper for great distances, and integrating from $\theta = 0$ to π, and from $t = 0$ to T, we get, as the energy going out through the whole sphere during every half complete swing,—

$$\tfrac{1}{3}E^2 l^2 m^3 n T = \frac{\pi^4 E^2 l^2}{3\lambda^3}.$$

Let us try to obtain an approximate estimate of the amount of this corresponding to our actual experiments. In those we charged two spheres of 15 centimetres radius in opposite senses up to a spark length of 1 centimetre about. We may estimate the difference of potential between these spheres as 120 C.G.S. electrostatic units, so each sphere was charged to half this potential, and its charge was therefore E = 900 C.G.S. units.

The total store of energy which the oscillator originally possessed amounted to 60 × 900 = 54,000 ergs, or 55 centimetre-grammes. The length of the oscillators, moreover, was 1 metre approximately, and the wave-length was about 480 centimetres.

So the loss of energy in half a swing comes out about 2400 ergs. It seems, therefore, that after eleven half-swings one-half of the energy must have gone in radiation. The quick damping which the experiments made manifest was therefore necessitated by radiation, and could not be prevented even if the resistance of conductor and spark were negligible.

A loss of energy of 2400 ergs in 1·5/100,000,000 of a second means a performance of work equal to 22 horse-power. The primary oscillator must be supplied with energy at at least this rate if the oscillation is to be permanently maintained at constant intensity in spite of the radiation. During the first few oscillations the intensity of the radiation at about 12 metres distant from the vibrator corresponds with the intensity of solar radiation at the surface of the earth.

(*To be continued.*)

GENERAL EQUATIONS OF FLUID MOTION.

THE general equations of the motion of a fluid can all be comprehended in a single form, which seems to be deserving of special notice.

Taking the ordinary notation, u, v, w, for the velocity-components at any point, P, of the fluid at any instant, and denoting the components of vortical spin at the point by ω_1, ω_2, ω_3, the usual Cartesian equations can be at once put into the form—

$$\frac{du}{dt} + \frac{d}{dx}\left(\tfrac{1}{2}q^2 + \int\frac{dp}{\rho}\right) + 2(w\omega_2 - v\omega_3) = X,$$

and two analogues, q being the resultant velocity. If through the point P we draw any curve whatever, the direction-cosines of whose tangent are l, m, n, and multiply the above and its two analogues, respectively, by l, m, n, we obtain by addition the equation—

$$\frac{d\dot{s}}{dt} + \frac{dU}{ds} + 12\Delta = S \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\alpha)$$

in which $\dot{s}$ stands for the component of velocity along the tangent to the curve, $U = \frac{1}{2}q^2 + \int\frac{dp}{\rho}$, S = component of external force-intensity along the tangent, and Δ is the volume of the tetrahedron formed by the vector drawn at P to represent q, the resultant velocity, the vector drawn to represent Ω, the resultant vortical spin, and the vector representing a unit length along the tangent to the curve at P. (Strictly speaking, the notation $\dot{s}$ is not a good one, but it is the best that presents itself.)

This equation (α) is that which I propose, as typical of all fluid motion, and as including all the special Cartesian equations in current use.

Some simple results follow at once for the case of steady motion. Thus, if we integrate (α) between any two points, A, B, of the curve,

$$U_B - U_A + 12\int\Delta ds = \int S ds \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where U_B and U_A are the values of U at B and A.

Now, in particular, if the curve drawn at P is a stream-line, $\Delta = 0$ at every point of it ; also, if the curve is a vortex-line, $\Delta = 0$ at every point, and we have the simple result,

$$U_B - U_A = \int S ds \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

a result which has long been known for a stream-line, but, apparently, not so long known for a vortex-line. It holds also for an infinite number of curves that can be drawn through P, all lying on a certain surface, as is pointed out by Lamb ("Motion of Fluids," p. 173), the surface in question being formed of a network of stream- and vortex-lines. That such surfaces exist in the fluid when the external forces have a potential, is proved most satisfactorily by taking the integral of (α) along a circuit through P, of which a part consists of stream-line and a part of vortex-line; but into the details of this we need not enter.

I observe, also, that this equation (2) holds for the portion of any curve whatever connecting any two points, A, B, on a network surface, although this curve does not lie on the surface.

Another point to which I would call attention is an analytical expression of the state of non-vortical motion. The physical expression has, of course, reference to the non-rotation of the three principal axes of the little ellipsoid into which, at each instant, a small sphere is deforming. The analytical expression of the fact takes usually the form that there is a velocity potential, *i.e.* $\frac{du}{dy} = \frac{dv}{dx}$, with two Cartesian analogues. Here, again, I would suggest a single equation, having no reference to special axes. This equation is simply

$$\frac{d\dot{s}}{d\sigma} = \frac{d\dot{\sigma}}{ds} \quad \ldots\ldots\ldots\ldots (\beta)$$

where s and σ denote arcs of *any two curves whatever* drawn at the point P, and $\dot{s}$ and $\dot{\sigma}$ the component velocities of the fluid along them.

It is obvious that these contain the whole three of the usual Cartesian expressions. The proof is very easy.

Cooper's Hill. GEORGE M. MINCHIN.

UNIVERSITY AND EDUCATIONAL INTELLIGENCE.

OXFORD.—The following Examiners in Natural Science have been appointed for the Honour Examinations:—Mr. J. V. Jones and Mr. A. L. Selby (Physics); Prof. McLeod and Mr. V. H. Veley (Chemistry); Prof. Milnes Marshall and Mr. W. Hatchett Jackson (Morphology); Prof. Sanderson and Prof. Schäfer (Physiology); Prof. Boyd Dawkins and Prof. Green (Geology).

The conditions of tenure of the Burdett-Coutts Geological Scholarship are to be altered, so as to make it necessary for the holders to devote themselves to Geology, and to work with the Professor.

Scholarships in Natural Science are announced for competition at Merton and at New College. The examination begins on July 2.

SOCIETIES AND ACADEMIES.

LONDON.

Royal Society, February 21.—"The Influence of Bile on the Digestion of Starch. (1) Its Influence on Pancreatic Digestion in the Pig." By Sidney Martin, M.D., B.Sc., British Medical Association Scholar, and Dawson Williams, M.D. (From the Physiological Laboratory, University College, London.)

The experiments of the authors have shown that if pig's bile be added to a solution of starch with pancreatic extract the digestion goes on with greater rapidity than without the bile. The rapidity of digestion is increased with the addition of quantities up to 4 per cent. of dried bile (equivalent to at least 30 per cent. of fresh bile). The rapidity was tested by noticing when the iodine reaction of starch had disappeared. On further research, it was found that this property of the bile depended on the bile salts (hyoglycocholate of sodium). The increased rapidity of digestion was well seen if 0·6 to 2 per cent. of bile salts were added to the digestive mixtures.

It was also found that not only was the change of starch into dextrine hastened, but also the change into sugar; and that the amount of dextrine and sugar formed when bile-salts wer present was one-fifth more than when they were absent. Fo the methods used in estimating the amount of dextrine and sugar, the original paper must be consulted.

"The Innervation of the Renal Blood-vessels." By J. Rose Bradford, M.B., D.Sc., George Henry Lewes Student. Communicated by E. A. Schäfer, F.R.S. (From the Physiological Laboratory of University College, London.)

The research was undertaken in order to map out the origin, cause, and nature of the renal nerves in the dog more accurately than had hitherto been attempted. The method employed consisted in exciting the roots of the spinal nerves, and observing simultaneously the effects produced on the general blood-pressure and on the volume of the kidney, the latter being investigated by means of Roy's oncometer. The anæsthetics used were chloroform and morphia. The general results were shortly as follows:—

No efferent vasomotor fibres were found in the posterior roots.

The efferent vasomotor fibres for the blood-vessels of the kidney leave the cord in the anterior roots of the nerves, extending from the second dorsal to the second lumbar. The renal nerves are, however, most abundant in the tenth, eleventh, twelfth, and thirteenth dorsal nerves.

In individual cases, however, there may be small variations in the number of fibres going on the one hand to the kidney, and on the other hand to the other abdominal viscera.

When quick rates of excitation are used, only contraction of the kidney and increase of general blood-pressure are observed, *i.e.* the vaso-constrictor fibres are excited.

With slow rates, however, expansion of the kidney with no increase of blood-pressure occurs, *i.e.* the vaso-dilator fibres are stimulated.

Hence the renal vessels not only receive constrictor fibres, but also dilator, and these are also most abundant in the eleventh, twelfth, and thirteen dorsal nerves.

Similarly when the peripheral end of the divided splanchnic nerve is excited with slow rates, a fall of blood-pressure is observed instead of the rise seen with quick rates.

Hence the splanchnic contains not only vaso-constrictor fibres for the abdominal vessels, but also vaso-dilators.

The results of reflex excitation can be summed up shortly by saying that the excitation of an afferent nerve causing a rise of blood-pressure is accompanied by a renal contraction, unless the nerve is one of what may be called the renal area. In this case the rise of blood-pressure is accompanied, as a rule, by either a renal expansion or else by a mixed kidney effect.

The main conclusion of this communication is the demonstration of dilator fibres in the splanchnic and in the renal nerves, and also the fact that these vaso-dilator fibres reach the kidney by the same paths as the constrictor fibres.

Chemical Society, February 7.—Mr. W. Crookes, F.R.S., in the chair.—The following papers were read:—Researches on the constitution of azo- and diazo-derivatives; compounds of the naphthalene-β-series (continued), by Prof. R. Meldola, F.R.S., and Mr. G. T. Morgan.—The action of nitric acid on anthracene, by Mr. A. G. Perkin. Hitherto, only anthraquinone and nitro-anthraquinones have been obtained by treating anthracene with nitric acid; the author, however, finds that nitro- and dinitro-anthracene can readily be prepared by the action of nitric acid upon anthracene if care is taken at once to decompose any nitrous acid which may be formed.—The preparation of glyceric acid, by Dr. Lewkowitsch.—The relation of cobalt to iron as indicated by absorption-spectra, by Dr. W. J. Russell, F.R.S., and Mr. W. J. Orsman, Junr. It is well known that when examined spectroscopically, some coloured metallic compounds are found only to produce a general absorption, but from previous observations it seemed possible to the authors that in some cases at least this might be resolved into bands by employing more powerful chemical agents than are generally used in such cases; experience had indicated that the chloride is usually the most suitable salt, and that it should be dissolved in chlorhydric acid and the liquid saturated with hydrogen chloride, also that, if possible, ether should be taken as solvent. Applying these views to iron, it was found that ferric chloride gave a banded spectrum strikingly similar to that of cobalt chloride. Irons of all kinds were examined: pig-iron, commercial cast-iron, and various manufactured articles; steel in the form of