The Geometric Interpretation of Monge's Differential Equation to all Conics.

NEITHER the note of Prof. Asutosh Mukhopadhyay in NATURE of the 11th inst. (p. 564), nor that of Lieut.-Colonel Allan Cunningham in the number of August 2 (p. 318), has satisfied me that the criticism implied in my short note (June 28, p. 197) on the Professor's first note (June 21, p. 173) is unfounded. Permit me, therefore, to develop that criticism a little more at large.

I have not yet had an opportunity of referring to the papers of the Professor in the Proceedings of the Asiatic Society, but from what I can gather as to their contents from his notes in NATURE, I am in no way disposed to underestimate the accuracy or the value of his results. It is only to his claim to find in them "*the* true interpretation of Monge's differential equation to any conic" that I demur.

To my apprehension the interpretation in question is a *truism*, not a *truth*. What has been put into the question as a *definition* emerges afterwards, as might have been anticipated, as an *interpretation*. If the Professor has given a definition of *aberrancy*, independent of a conic and its known properties, of course I am wrong; but I gather from his note that by *aberrancy* he merely means (if I may thus express it) *deviation from conicity*. Whatever measure of *aberrancy*, then, he adopts for curves generally, must necessarily become zero for a conic, which has, from the very meaning of the words, no "deviation from conicity."

The difference, as I conceive it, between an interpretation properly so called and an interpretation that is a mere *truism*, may be clearly illustrated by the case of the circle. The Professor tells us that "the differential equation of all circles $(1 + p^2)r - 3pq^2 = 0$, means that the angle of aberrancy vanishes at every point of every circle." If thus read, what I have said above applies, and the interpretation is but a *truism*. It admits, however, of a different reading. For it is easy to show that $(1 + p^2)r - 3pq^2 = (1 + p^2)^3 \frac{d^2\phi}{ds^2}$, where s, ϕ are the usual intrinsic co-ordinates of the curve, so that the differential equation is equivalent to $d^2\phi/ds^2 = 0$. Now $d\phi/ds$ is the measure of the curvature of a curve, defined as the rate of change, per unit of arc, of the inclination of the tangent to a fixed direction, a *definition which is quite independent of the circle*; and $d^2\phi/ds^2$ is the rate of change, per unit of arc, of the curvature. Hence the equation $d^2\phi/ds^2 = 0$, being true at every point of every circle, expresses the *truth* that in a circle there is no change of curvature from point to point—or, in other words, the property that the curvature of a circle is the same at every point. I submit that this, rather than the Professor's, involving the notion of *aberrancy*, has a right to be regarded as the true interpretation of the equation.

In like manner, the true interpretation of the differential equation to a conic, if it ever is discovered, will express that some magnitude or concept connected with a curve, and defined independently of the particular curves, the conic sections, vanishes at every point of every conic.

Even admitting the Professor's interpretation, I agree with Colonel Allan Cunningham that it has no prerogative right over others of the same character to be called *the* interpretation of the equation. To go no farther, any number of "aberrancy curves" may be imagined; as, for instance, the locus of the focus, instead of the centre, of the osculating conic, for which it will be true that "the radius of curvature of the aberrancy curve vanishes at every point of every conic"; for in fact, in this case the aberrancy curve degenerates into a single point, and to say that the radius of curvature vanishes, or that the curvature is infinite, at every point of a curve, is, to my apprehension, only a roundabout, and not very instructive, way of saying that the curve becomes reduced to a single point.

Harrow, October 13. R. B. H.

A Shadow and a Halo.

THE following notices of anthelia may be interesting to the readers of NATURE. Frances Ridley Havergal thus described a sunset on the Faulhorn: "At one juncture a cloud stood still, apparently about two hundred yards off, and we each saw our own shadow gigantically reflected on it, surrounded by a complete rainbow arch, a full circle of bright prismatic colours, a transfiguration of our own shadows almost startling; each, moreover, seeing only their own glorification" ("Swiss Letters and Alpine Poems").

Tennant, in his book on Ceylon, states that this curious phenomenon, which may probably have suggested to the early painters the idea of the glory surrounding the heads of beatified saints, is to be seen in singular beauty at early morning in Ceylon. When the light is intense, and the shadows proportionally dark, when the sun is near the horizon, and the shadow of a person is thrown on the dewy grass, each drop of dew furnishes a double reflection from its convex and concave surfaces; and to the spectator the shadow of his own figure, but more particularly the head, appears surrounded by a halo as vivid as if radiated from diamonds.

S. T. Coleridge described the phenomenon thus:—

"Such thou art, as when
The woodman winding westward up the glen
At wintry dawn, where o'er the sheep-track's maze
The viewless snow-mist weaves a glist'ning haze,
Sees full before him, gliding without tread,
An image with a glory round its head:
The enamoured rustic worships its fair hues,
Nor knows he makes the shadow he pursues."

Benvenuto Cellini saw, probably, this phenomenon, and supposed it peculiar to himself. F. Robertson cites it as a proof of inordinate vanity. He says: "Conceive a man gravely telling you that a vision of glory encircled his head through life, visible on his shadow, especially on the dewy grass at morning, and which he possessed the power of showing to a chosen few" ("Life and Letters of F. Robertson," vol. ii. p. 192).

Bardsea, October 22. EDWARD GEOGHEGAN.

I HAVE frequently, on the South Downs, seen a halo round the shadow of my head, as described in your last number by Mr. A. S. Eve. I have noticed that the further off the shadow, the brighter is the halo. I have also observed, when looking at my shadow in the sea, that rays of light appear to surround the shadow of my head. CHARLES CAVE.

Ditcham Park, Petersfield, October 22.

On the Grass Minimum Thermometer.

THE average readings of the self-recording grass minimum thermometer for every month during the past three years have been compared with the average minimum *damp* bulb temperatures, obtained from the means of hourly readings, and the following figures show the corrections to be applied to the latter in order to obtain the former:—January $-0^\circ.3$, February $+0^\circ.3$, March $-0^\circ.3$, April $-0^\circ.8$, May $-0^\circ.2$, June $-1^\circ.1$, July $-1^\circ.1$, August $-0^\circ.9$, September $+0^\circ.2$, October $+1^\circ.4$, November $+1^\circ.9$, December $+0^\circ.4$.

The grass minimum is nearly a degree below the damp bulb minimum in the wet season, and nearly 2° above it in the driest month. The comparison between the minimum air temperature and the minimum on grass does not measure the terrestrial radiation, although the difference is to some extent influenced by radiation. Moreover, the epochs of the two minima need not coincide—*e.g.* in Hong Kong the early morning hours are more cloudy than the evening hours.

During the daytime in summer the thermometer, exposed an inch above the short grass, shows as a rule temperatures rising to 120° or 130°, especially in calm weather; but even when it is not perfectly calm, the force of the wind is not felt so near the ground, from which the air rises laden with minute particles of dust, which are observed adhering to the cloth of damp bulbs and other objects cooled by evaporation, and which may occasionally be smelt in the air. At night such minute particles would of course tend to return to the ground, and the unhealthy character of the ground-fog during early morning hours in tropical countries may be intensified by this circumstance.

Hong Kong Observatory, W. DOBERCK.
September 10.

ON THE ELECTROMOTIVE VARIATIONS WHICH ACCOMPANY THE BEAT OF THE HUMAN HEART.

THE observation of these variations is extremely easy, the only requisite being a sufficiently sensitive capillary electrometer.[1]

[1] The electrometers I used were made by Mr. Dean, glass-blower, 8 Cross Street, Hatton Garden.

The successful issue of the observations is so certain that they can be best described in the form of directions to a person who should be desirous of seeing them for himself, followed by the prediction of what will be observed by him.

§ I. Two vessels of salt solution are to be prepared, and connected with the capillary electrometer by electrodes. The various extremities of the observer are to be dipped into the salt solution, while the capillary column is watched. Electrical variations, apparently synchronous with the heart's pulse, will be observed with certain combinations rather than with others, and the results (on a normal person with the heart pointing to the left) will be as follows :—

Connect with electrometer—

1. Left hand and right hand	Electrical variations
2. Left hand and left foot	Little or no variations
3. Left hand and right foot	Little or no variations
4. Right hand and left foot	Electrical variations
5. Right hand and right foot	Electrical variations
6. Right foot and left foot	No electrical variations

will be apparent.

Further observations may be made with the mouth used as a leading-off point in connection with each of the four extremities. To lead off from the mouth a silver electrode coated with silver chloride is kept under the tongue. The results will be as follows :—

Connect with electrometer—

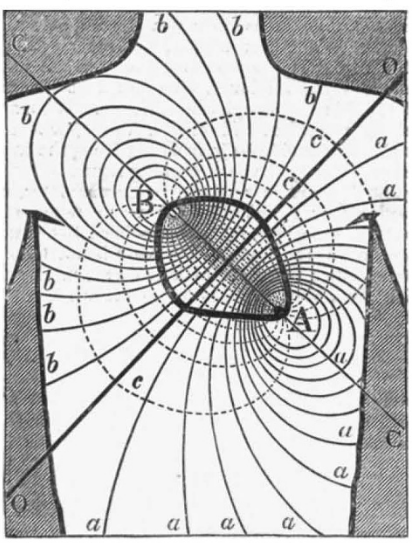
7. Mouth and left hand	Electrical variations
8. Mouth and right hand	Little or no variations
9. Mouth and left foot	Electrical variations
10. Mouth and right foot	Electrical variations

will be apparent.

Finally, it is possible to add to the evidence obtained, by using the rectum as a lead off by means of a silver electrode. This, if tried, would give with

11. Rectum and mouth	Electrical variations
12. Rectum and left hand	Little or no variations
13. Rectum and right hand	Electrical variations
14. Rectum and left foot	Little or no variations
15. Rectum and right foot	Little or no variations.

These will have been the results ; the cases in which the mode of leading off has been favourable to the production



of electrical variations will be unmistakably distinguished from those in which the mode of leading off has been unfavourable.

The explanation of these facts is most shortly given in the diagram. C C is the axis of any current which must be produced if at any time the apex and base of the ventricles differ in potential. O O is the line of zero potential at right angles to C C.

a a a are equipotential lines round a supposed focus A. *b b b* are equipotential lines round a supposed focus B. Any lead off from two superficial points *a a* or *b b* is unfavourable. Any lead off from two points *a b* is favourable to the manifestation of electromotive differences originating at the heart. This will have been demonstrated by the experiments directed to be made.

§ II. On a quadruped (dog, cat, rabbit) the results will come out somewhat differently. The heart occupies an approximately median position, so that the asymmetry observed on man does not hold good with the above-named animals. In these the current axis will be along a median longitudinal line ; the line of zero potential will be at right angles to it, *i.e.* transverse.

This can be verified by trial with very little trouble. A quadruped is led off by the various extremities and orifices immediately after death before the heart has ceased to beat ; or a dog may be trained to stand quiet with his feet in dishes of salt solution (I have a large and well-disposed dog who will stand thus by the hour). However the test be made, the results will come out as follows :—

Connect with electrometer—

1. Left paw[1] and right paw	Little or no electrical variations
2. Left paw and left foot	Electrical variations
3. Left paw and right foot	Electrical variations
4. Right paw and left foot	Electrical variations
5. Right paw and right foot	Electrical variations
6. Right foot and left foot	Little or no electrical variations

will be apparent.

Extending the observations to mouth and rectum, the results will be thus :—

7. Mouth and left paw	Little or no electrical variations
8. Mouth and right paw	Little or no electrical variations
9. Mouth and left foot	Electrical variations
10. Mouth and right foot	Electrical variations
11. Mouth and rectum	Electrical variations
12. Rectum and left paw	Electrical variations
13. Rectum and right paw	Electrical variations
14. Rectum and left foot	Little or no electrical variations
15. Rectum and right foot	Little or no electrical variations.

§ III. Upon these two proofs may be piled a third proof of the correctness of the facts and of their explanation. Cases of *situs viscerum inversus* are to be found ; the viscera of such people are situated as those of a normal person seen in a mirror ; *i.e. inter alia*, the heart points to the right. I have examined two such cases, with results exactly as anticipated, viz. the favourable combinations, 4, 5, and 7, of a normal subject (§ I.) are unfavourable in the case of *situs inversus*, while the unfavourable combinations, 2, 3, and 8, are favourable. Combinations 1, 9, and 10 are favourable, and 6 is unfavourable in both cases, there being the notable peculiarity as regards 1 that the variations are reversed in direction in each of the two cases. The significance of this point will be obvious to the reader who has followed the facts up to this point : in both cases we have a favourable combination, but a reversal of points *a* and *b*.

§ IV. As regards the character and direction of each cardiac variation, it will be found to be composed of two phases, the first short, sharp, and difficult to read as regards direction, the second comparatively prolonged and easy to read. The second phase clearly indicates negativity of the heart's base, the first phase less clearly negativity of the heart's apex—facts which testify that the contraction begins at the apex and ends at the base of the ventricles. The auricular contraction does not affect any electrometer I have used.

[1] "Paw" is used as an abbreviation for anterior extremity ; "foot" for posterior extremity.

If I may venture to forecast the manner in which these statements may receive from independent sources that verification which any statement requires before it can be accepted as a correct representation of fact, I should say that as regards § I. no contradiction will arise unless the first case tested should happen to be that of a person with the heart occupying an unusually median position, when the favourable and unfavourable cases, though still distinguishable, may be less so than if the heart occupied its usual oblique position pointing to the left. In any case, however, the variation will be found more marked with a favourable than an unfavourable combination. As regards § II., the statements made can be verified as soon as tested upon a recently killed cat or upon a properly educated dog. The verification of § III. only requires that a suitable case should be discovered. As regards the character of the variation, it is probable that its diphasic character may be overlooked at the first glance, but (in a favourable case) this character will soon be apparent. As regards direction, that of the second phase will be determined without much difficulty, but that of the first will be found very difficult to seize. I was not able to make up my mind about it until I had obtained successful photographs of the movements on a quick-travelling sensitive plate.

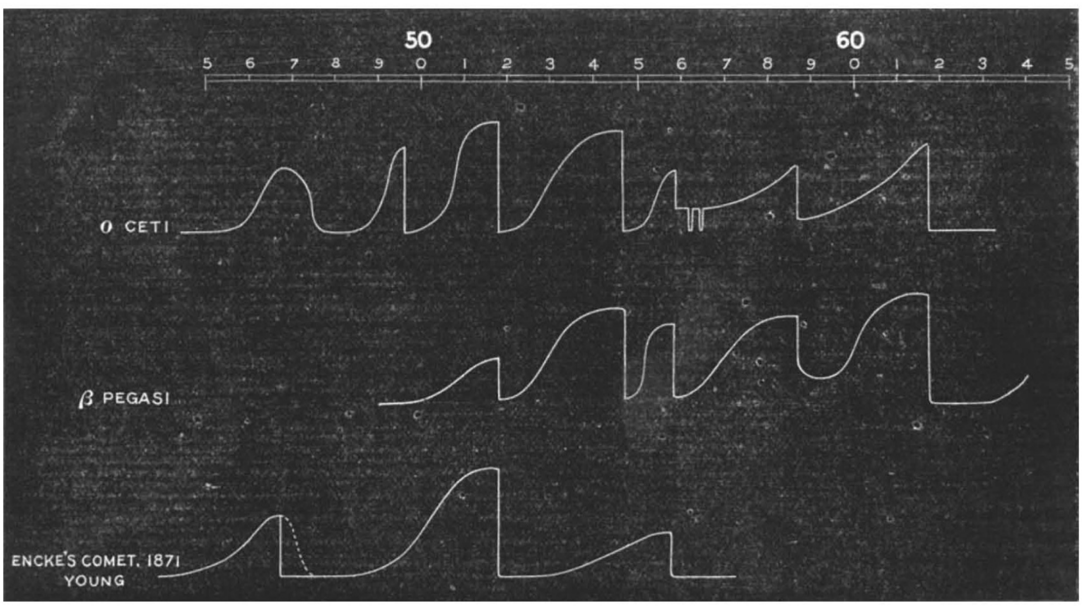
AUGUSTUS D. WALLER.

THE MAXIMUM OF MIRA CETI.

I AM anxious to call the attention of observers to the present spectrum of Mira, which arrived at its maximum brilliancy on the 15th inst. I pointed out recently (NATURE, May 24, p. 79) that stars of the group to which Mira belongs are sparse meteorite-swarms like comets, and that, when variable, the variability is produced by collisions between two swarms, the centres of which are nearest together (periastron passage) at maximum.

Broadly speaking, then, we may regard variables of this class as incipient double stars, or condensing swarms with double nuclei, the invisibility of the companion being due to its nearness to the primary, or to its faintness. It is obvious that variability will occur mostly in the swarms having a mean condensation, for the reason that at first the meteorites are too far apart for many collisions to occur, and that, finally, the outliers of the major swarm are drawn within the orbit of the smaller revolving one, so that it passes clear.

The present maximum of Mira tests my hypothesis, and its brightness is such that a small telescope and a Maclean's spectroscopic eye-piece are all that are necessary to see in how striking a manner the test is borne. The two brightest bands now visible are at λ 517 and λ 546, precisely where these are seen in the brightest comets. The former is the brightest carbon fluting seen in the spectrum of the Bunsen flame, or spirit-lamp,



and the other, at 546, is the citron carbon fluting beginning at 564, but modified by the masking effects of the manganese absorption fluting at 558, and also that of lead at 546.

The blackness of the spaces between the bright flutings shows that there can be very little continuous spectrum from the meteorites, and therefore that the absorption is that of the light of the carbon flutings.

The mean spectrum of Mira is that of a star like β Pegasi, which I have shown to consist of bright carbon flutings, and dark flutings of magnesium, manganese, iron, lead, and barium. In β Pegasi, as in Mira under mean conditions, the carbon is somewhat faint, but in α Herculis it is very bright. The general effect of the conditions of maximum of Mira therefore seems to be that of changing its spectrum from one like that of β Pegasi to one like that of α Herculis.

I observed that the principal carbon fluting at λ 517 was somewhat brighter on the 14th than on the 17th inst. In variable stars of this class the proof is now complete that the increase of luminosity is accompanied by cometary conditions, and that it is due to the increased radiation of carbon.

In the accompanying figure the spectrum of Mira is compared with that of β Pegasi and Encke's comet. In some comets the carbon fluting is cut off at 546, exactly as it is in Mira. The observations of Mira were made by myself at Westgate, those of β Pegasi by Mr. Fowler at the Astronomical Laboratory at South Kensington.

J. NORMAN LOCKYER.