The Geometric Interpretation of Monge's Differential Equation to all Conics.

NEITHER the note of Prof. Asutosh Mukhopadhyay in NATURE of the 11th inst. (p. 564), nor that of Lieut.-Colonel Allan Cunningham in the number of August 2 (p. 318), has satisfied me that the criticism implied in my short note (June 28, p. 197) on the Professor's first note (June 21, p. 173) is unfounded. Permit me, therefore, to develop that criticism a little more at large.

I have not yet had an opportunity of referring to the papers of the Professor in the Proceedings of the Asiatic Society, but from what I can gather as to their contents from his notes in NATURE, I am in no way disposed to underestimate the accuracy or the value of his results. It is only to his claim to find in them "*the* true interpretation of Monge's differential equation to any conic" that I demur.

To my apprehension the interpretation in question is a *truism*, not a *truth*. What has been put into the question as a *definition* emerges afterwards, as might have been anticipated, as an *interpretation*. If the Professor has given a definition of *aberrancy*, independent of a conic and its known properties, of course I am wrong; but I gather from his note that by *aberrancy* he merely means (if I may thus express it) *deviation from conicity*. Whatever measure of *aberrancy*, then, he adopts for curves generally, must necessarily become zero for a conic, which has, from the very meaning of the words, no "deviation from conicity."

The difference, as I conceive it, between an interpretation properly so called and an interpretation that is a mere *truism*, may be clearly illustrated by the case of the circle. The Professor tells us that "the differential equation of all circles $(1 + p^2)r - 3pq^2 = 0$, means that the angle of aberrancy vanishes at every point of every circle." If thus read, what I have said above applies, and the interpretation is but a *truism*. It admits, however, of a different reading. For it is easy to show that $(1 + p^2)r - 3pq^2 = (1 + p^2)^3 \frac{d^2\phi}{ds^2}$, where s, ϕ are the usual intrinsic co-ordinates of the curve, so that the differential equation is equivalent to $d^2\phi/ds^2 = 0$. Now $d\phi/ds$ is the measure of the curvature of a curve, defined as the rate of change, per unit of arc, of the inclination of the tangent to a fixed direction, a *definition which is quite independent of the circle*; and $d^2\phi/ds^2$ is the rate of change, per unit of arc, of the curvature. Hence the equation $d^2\phi/ds^2 = 0$, being true at every point of every circle, expresses the *truth* that in a circle there is no change of curvature from point to point—or, in other words, the property that the curvature of a circle is the same at every point. I submit that this, rather than the Professor's, involving the notion of *aberrancy*, has a right to be regarded as the true interpretation of the equation.

In like manner, the true interpretation of the differential equation to a conic, if it ever is discovered, will express that some magnitude or concept connected with a curve, and defined independently of the particular curves, the conic sections, vanishes at every point of every conic.

Even admitting the Professor's interpretation, I agree with Colonel Allan Cunningham that it has no prerogative right over others of the same character to be called *the* interpretation of the equation. To go no farther, any number of "aberrancy curves" may be imagined; as, for instance, the locus of the focus, instead of the centre, of the osculating conic, for which it will be true that "the radius of curvature of the aberrancy curve vanishes at every point of every conic"; for in fact, in this case the aberrancy curve degenerates into a single point, and to say that the radius of curvature vanishes, or that the curvature is infinite, at every point of a curve, is, to my apprehension, only a roundabout, and not very instructive, way of saying that the curve becomes reduced to a single point.

Harrow, October 13. R. B. H.

A Shadow and a Halo.

THE following notices of anthelia may be interesting to the readers of NATURE. Frances Kidley Havergal thus described a sunset on the Faulhorn: "At one juncture a cloud stood still, apparently about two hundred yards off, and we each saw our own shadow gigantically reflected on it, surrounded by a complete rainbow arch, a full circle of bright prismatic colours, a transfiguration of our own shadows almost startling; each, moreover, seeing only their own glorification" ("Swiss Letters and Alpine Poems").

Tennant, in his book on Ceylon, states that this curious phenomenon, which may probably have suggested to the early painters the idea of the glory surrounding the heads of beatified saints, is to be seen in singular beauty at early morning in Ceylon. When the light is intense, and the shadows proportionally dark, when the sun is near the horizon, and the shadow of a person is thrown on the dewy grass, each drop of dew furnishes a double reflection from its convex and concave surfaces; and to the spectator the shadow of his own figure, but more particularly the head, appears surrounded by a halo as vivid as if radiated from diamonds.

S. T. Coleridge described the phenomenon thus:—

"Such thou art, as when
The woodman winding westward up the glen
At wintry dawn, where o'er the sheep-track's maze
The viewless snow-mist weaves a glist'ning haze,
Sees full before him, gliding without tread,
An image with a glory round its head:
The enamoured rustic worships its fair hues,
Nor knows he makes the shadow he pursues."

Benvenuto Cellini saw, probably, this phenomenon, and supposed it peculiar to himself. F. Robertson cites it as a proof of inordinate vanity. He says: "Conceive a man gravely telling you that a vision of glory encircled his head through life, visible on his shadow, especially on the dewy grass at morning, and which he possessed the power of showing to a chosen few" ("Life and Letters of F. Robertson," vol. ii. p. 192).

Bardsea, October 22. EDWARD GEOGHEGAN.

I HAVE frequently, on the South Downs, seen a halo round the shadow of my head, as described in your last number by Mr. A. S. Eve. I have noticed that the further off the shadow, the brighter is the halo. I have also observed, when looking at my shadow in the sea, that rays of light appear to surround the shadow of my head. CHARLES CAVE.

Ditcham Park, Petersfield, October 22.

On the Grass Minimum Thermometer.

THE average readings of the self-recording grass minimum thermometer for every month during the past three years have been compared with the average minimum *damp* bulb temperatures, obtained from the means of hourly readings, and the following figures show the corrections to be applied to the latter in order to obtain the former:—January −0°·3, February +0°·3, March −0°·3, April −0°·8, May −0°·2, June −1°·1, July −1°·1, August −0°·9, September +0°·2, October +1°·4, November +1°·9, December +0°·4.

The grass minimum is nearly a degree below the damp bulb minimum in the wet season, and nearly 2° above it in the driest month. The comparison between the minimum air temperature and the minimum on grass does not measure the terrestrial radiation, although the difference is to some extent influenced by radiation. Moreover, the epochs of the two minima need not coincide—*e.g.* in Hong Kong the early morning hours are more cloudy than the evening hours.

During the daytime in summer the thermometer, exposed an inch above the short grass, shows as a rule temperatures rising to 120° or 130°, especially in calm weather; but even when it is not perfectly calm, the force of the wind is not felt so near the ground, from which the air rises laden with minute particles of dust, which are observed adhering to the cloth of damp bulbs and other objects cooled by evaporation, and which may occasionally be smelt in the air. At night such minute particles would of course tend to return to the ground, and the unhealthy character of the ground-fog during early morning hours in tropical countries may be intensified by this circumstance.

Hong Kong Observatory, September 10. W. DOBERCK.

ON THE ELECTROMOTIVE VARIATIONS WHICH ACCOMPANY THE BEAT OF THE HUMAN HEART.

THE observation of these variations is extremely easy, the only requisite being a sufficiently sensitive capillary electrometer.[1]

[1] The electrometers I used were made by Mr. Dean, glass-blower, 8 Cross Street, Hatton Garden.