Dr. J. P. Gram (*Mémoires de l'Académie Royale de Copenhague*, 6me. série, vol. ii. p. 191) refers to a paper by Mertens ("Ein Beitrag zur analytischen Zahlentheorie," *Borchardt's Journal*, Bd. 78), as one in which the truth of the first of the two theorems is demonstrated—"fuldstoendigt Bevis af Mertens" are Gram's words.[1]

Assuming this to be the case, we shall easily find when N is indefinitely great, so that S_N becomes $\frac{\pi}{4}$,

$$Q_N S_N = \frac{1}{(1 - \frac{1}{2})(1 - \frac{1}{3}) \ldots \left(1 - \frac{1}{N}\right)},$$

which, according to Legendre's empirical law (Legendre, "Théorie des Nombres," 3rd edition, vol. ii. p. 67, art. 397), is equal to $\frac{2 \log N}{K}$, where $K = 1·104$; and as we have written $Q_N S_N = \log N + (V - R)$, we may deduce, upon the above assumptions,

$$V - R = \left(\frac{2}{K} - 1\right) \log N = 0·811 \ldots \log N.$$

R, we know, is demonstrably less than $\left(1 - \frac{\pi}{4}\right) \log N$, consequently V must be less than $(0·812 + 0·215) \log N$, *i.e.* less than $1·027 \log N$, and *a fortiori* the portion of the omnipositive aggregate Q_N, which consists of terms whose denominators exceed N, when N is indefinitely great, cannot be less than $\frac{4}{\pi}\left(1 - \frac{\pi}{4}\right) \log N$, *i.e.* $0·273 \log N$.

Before concluding, let me add a word on Legendre's empirical formula for the value of

$$(1 - \tfrac{1}{3})(1 - \tfrac{1}{5}) \ldots \left(1 - \frac{1}{p_{N.p}}\right),$$

referred to in the early part of this article.

If N is any odd number, the condition of its being a prime number is that when divided by any odd prime less than its own square root, it shall not leave a remainder zero. Now if N (an unknown odd number) is divided by p, its remainder is equally likely to be 0, 1, 2, 3, ... or $(p - 1)$. Hence the chance that it is not divisible by p is $\left(1 - \frac{1}{p}\right)$, and, if we were at liberty to regard the like thing happening or not for any two values of p within the stated limit as independent events, the expectation of N being a prime number would be represented by

$$(1 - \tfrac{1}{3})(1 - \tfrac{1}{5})(1 - \tfrac{1}{7})(1 - \tfrac{1}{11}) \ldots \left(1 - \frac{1}{p_{N^{\frac{1}{2}}.p}}\right),$$

which, according to the formula referred to, for infinitely large values of N is equal to $\frac{1·104}{\log N^{\frac{1}{2}}}$. It is rather more convenient to regard N as entirely unknown instead of being given as odd, on which supposition the chance of its being a prime would be $\frac{1·104}{2 \log N^{\frac{1}{2}}}$ or $\frac{1·104}{\log N}$.

Hence for very large values of N the sum of the logarithms of all the primes inferior to N might be expected to be something like (1·104)N. This does not contravene Tchebycheff's formula (Serret, "Cours d'Algèbre Supérieure," 4me ed., vol. ii. p. 233), which gives for the limits of this sum AN and BN, where $A = 0·921292$, and $B = \frac{6A}{5} = 1·10555$; but does contravene the narrower limits given by my advance upon Tchebycheff's method (see *Am. Math. Journal*, vol. iv. Part 3), according to which for A, B, we may write A_1, B_1, where

$$A_1 = 0·921423,\ B_1 = 1·076577.^1$$

That the method of probabilities *may* sometimes be successfully applied to questions concerning prime numbers I have shown reason for believing in the two tables published by me in the *Philosophical Magazine* for 1883.[2]

New College, June 10. J. J. SYLVESTER.

[1] It always seems to me absurd to speak of a complete proof, or of a theorem being rigorously demonstrated. An incomplete proof is no proof, and a mathematical truth not rigorously demonstrated is not demonstrated at all. I do not mean to deny that there are mathematical truths, morally certain, which defy and will probably to the end of time continue to defy proof, as, *e.g.*, that every indecomposable integer polynomial function must represent an infinitude of primes. I have sometimes thought that the profound mystery which envelops our conceptions relative to prime numbers depends upon the limitation of our faculties in regard to time, which like space may be in its essence poly-dimensional, and that this and such sort of truths would become self-evident to a being whose mode of perception is according to *superficially* as distinguished from our own limitation to *linearly* extended time.

[1] Viz. $A_1 = \frac{51072}{50999}A$, and $B_1 = \frac{59595}{50999}A$, the values of which are incorrectly stated in the memoir. Strange to say, Dr. Gram, in his prize essay, previously quoted, on the number of prime numbers under a given limit, has omitted all reference to this paper in his bibliographical summary of the subject, which is only to be accounted for by its having escaped his notice; a narrowing of the asymptotic limits assigned to the sum of the logarithms of the prime numbers series being the most notable fact in the history of the subject since the publication of Tchebycheff's memoir. Subjectively, this paper has a peculiar claim upon the regard of its author, for it was his meditation upon the two simultaneous difference-equations which occur in it that formed the starting-point, or incunabulum, of that new and boundless world of thought to which he has given the name of Universal Algebra. But, apart from this, that the superior limit given by Tchebycheff as 1·1055 should be brought down by a more stringent solution of his own inequalities to only 1·076577—in other words, that the excess above the probable mean value (unity) should be reduced to little more than ⅔rds of its original amount—is in itself a surprising fact. Perhaps the numerous (or innumerable) misprints and arithmetical miscalculations which disfigure the paper may help to account for the singular neglect which it has experienced. It will be noticed that the mean of the limits of Tchebycheff is 1·01342, the mean of the new limits being 0·99900. The excess in the one case above and the defect in the other below the probable true mean are respectively 0·01342 and 0·00100.

[2] A principle precisely similar to that employed above if applied to determining the number of reduced proper fractions whose denominators do not exceed a given number n, leads to a correct result. The expectation of two numbers being prime to each other will be the product of the expectations of their not being each divisible by any the same prime number. But the probability of one of them being divisible by i is $\frac{1}{i}$, and therefore of two of them being not each divisible by i is $\frac{1}{i^2}$. Hence the probability of their having no common factor is

$$(1 - \tfrac{1}{4})(1 - \tfrac{1}{9})(1 - \tfrac{1}{25})(\tfrac{1}{121}) \ldots \textit{ad inf.}, \textit{ i.e.} \text{ is } \frac{6}{\pi^2}.$$

If, then, we take two sets of numbers, each limited to n, the probable number of relatively prime combinations of each of one set with each of the other should be $\frac{6n^2}{\pi^2}$, and the number of reduced proper fractions whose denominators do not exceed n should be the half of this or $\frac{3n^2}{\pi^2}$. I believe M. Césaro has claimed the prior publication of this mode of reasoning, to which he is heartily welcome. The number of these fractions is the same thing as the sum of the *totients* of all numbers not exceeding n. In the *Philosophical Magazine* for 1883 (vol. xv. p. 251), a table of these sums of totients has been published by me for all values of n not exceeding 500, and in the same year (vol. xvi. p. 231) the table was extended to values of n not exceeding 1000 In every case without any exception the estimated value of this totient sum is found to be intermediate between

$$\frac{3n^2}{\pi^2} \text{ and } \frac{3(n+1)^2}{\pi^2}.$$

Calling the totient sum to n, $T(n)$, I stated the exact equation

$$T(n) + T\left(\frac{n}{2}\right) + T\left(\frac{n}{3}\right) + T\left(\frac{n}{4}\right) + \ldots = \frac{n^2 + n}{2},$$

from which it is capable of proof, without making any assumption as to the form of Tn, that its asymptotic value is $\frac{3n^2}{\pi^2}$. The functional equation itself is merely an integration (so to say) of the well known theorem that any number is equal to the sum of the totients of its several divisors. The introduction to these tables will be found very suggestive, and besides contains an interesting bibliography of the subject of Farey series (*suites de Farey*), comprising, among other writers upon it, the names of Cauchy, Glaisher, and Sir G. Airy, the last-named as author of a paper on toothed wheels, published, I believe, in the "Selected Papers" of the Institute of Mechanical Engineers. The last word on the subject, as far as I am aware, forms one of the *interludes*, or rather the *postscript*, to my "Constructive Theory of Partitions," published in the *American Journal of Mathematics*.

SOCIETIES AND ACADEMIES.

LONDON.

Royal Society, May 3.—"Electro-Chemical Effects on Magnetizing Iron." Part II. By Thomas Andrews, F.R.S.E. Communicated by Prof. G. G. Stokes, P.R.S.

The present paper contains the results of a further study of the electro-chemical effects observed between a magnetized and an unmagnetized bar when in circuit in certain electrolytes, recorded in Part I. of this research. The method of experimentation was generally similar to that pursued and described in Part I., though it was necessary to introduce numerous modifications of detail and also new modes of experimentation. The bars experimented on were of specially prepared wrought-iron and cast-steel; all the rods were finely polished, and the general physical properties of the metals are given in Table B. Steel bars were employed in some of the experiments, because after magnetization by the coil their subsequent influence as permanent magnets could be observed. The reagents employed as electrolytes consisted of various solutions of bromine, ferric chloride, and chlorine water, ferrous sulphate, ferric chloride, cupric chloride, cupric sulphate, cupric nitrate, cupric acetate, cupric bromide, nickel chloride, hydrochloric, acid, nitric acid, and potassium chlorate. A pair of bars in each experiment were immersed as elements in the solution in the special apparatus employed, in circuit also with a delicate galvanometer, and after normal galvanic equilibrium had been obtained the bar within the coil was magnetized for various periods and the magneto-chemical effect observed. It was found to vary with the nature of the metal and solution employed, and also with the extent of the magnetization of the metals. The average results of many repeated experiments are given in numerous detailed tables, and it was generally found that a magnetized bar became electro-positive to an unmagnetized one. In Parts I. and II. a total of near 600 iron and steel bars have been experimented upon. Experiments were also made showing that local currents were developed in a magnetized bar between the more highly and less magnetized parts thereof, when the rod was immersed in suitable solutions acting chemically upon it.

Interesting experiments have also been made in connection with the influence of magnetization on the action of strong nitric acid on iron and steel. In course of the research the results of an extensive quantitative study of magneto-chemical phenomena have been recorded, the effect in connection with a considerable variety of typical reagents having been carefully observed; with some reagents the effect was found to be comparatively small, in other instances it was somewhat considerable. The general conclusion was that under the conditions recorded a magnetized bar was electro-positive to an unmagnetized one, when the two were immersed in suitable solutions, and that the extent of the result was in some degree dependent both on the nature and strength of the solution, and also on the extent of the magnetization of the metal.

June 7.—"Note on the Volumetric Determination of Uric Acid." By A. M. Gossage, B.A. Oxon.

It seemed improbable that the method recently proposed by Dr. Haycraft for the volumetric determination of uric acid in urine could be accurate, since both Salkowski and Maly had previously shown that the precipitate of silver urate obtained from urine contains variable quantities of other urates. To test the method, I examined samples of various urines both by his method and by that of Salkowski, which is universally acknowledged to be the most trustworthy. The mean percentages of uric acid found were as follow:—

Experiment	I.	II.	III.	IV.	V.
Haycraft's method	0·108	0·076	0·082	0·072	0·108
Salkowski's method	0·084	0·035	0·051	0·035	0·084

The results obtained by Haycraft's method were always considerably higher than those obtained by Salkowski's. The reason of this is that Dr. Haycraft has assumed that the silver precipitate from urine consists of a urate containing only 1 atom of silver in the molecule, whereas the proportion of silver in silver urate corresponds more nearly to 2 atoms in the molecule. Assuming, then, that there are 2 atoms of silver in all the molecules of the urate, and dividing the results obtained by Haycraft's method by two, we see that the results so obtained are usually lower than those obtained by Salkowski's method, and that the proportion between the results by the two methods varies, as would be expected from Salkowski's researches.

Edinburgh.

Royal Society, June 4.—Dr. John Murray, Vice-President, in the chair.—Dr. G. Sims Woodhead exhibited a series of photographs of large sections of the lung.—A paper by the Astronomer-Royal for Scotland on Scottish meteorology for the last thirty-two years was read.—Dr. E. Sang read a paper on John Leslie's computation of the ratio of the diameter to the circumference of a circle.—A paper by Lord Maclaren on the figure of aplanatic lenses was read.—Prof. Tait submitted some quaternion notes.

June 18.—The Hon. Lord Maclaren, Vice-President, in the chair.—The Secretary exhibited M. Amagat's photographs of the crystallization of chloride of carbon under pressure alone.—A paper by Prof. W. Carmichael McIntosh and Mr. E. E. Prince, St. Andrews' Marine Laboratory, was communicated.—A paper by Prof. Anglin on certain theorems mainly connected with alternants, was read.—Prof. Haycraft and Dr. R. T. Williamson gave a demonstration of a method, which can be used chemically, for estimating quantitatively the alkalinity of the blood.—A preliminary notice of a paper by Dr. G. N. Stewart on electrolytic decomposition of proteid substances was submitted.—Papers by Dr. A. B. Griffiths, on the Malpighian tubules of *Libellula depressa*, and on a fungoid disease in the roots of *Cucumis sativa*, were communicated.

Paris.

Academy of Sciences, July 2.—M. Janssen, President, in the chair.—Reply to Mr. Douglas Archibald's strictures on the subject of storms, by M. H. Faye. The storm laws, as established by the observations of Capper, Piddington, Reid, and Redfield, are declared to be one of the greatest discoveries of the century, and their truth is here vindicated against the recent attacks of Prof. Loomis, Dr. Meldrum, and especially Mr. E. Douglas Archibald, in NATURE for June 14 (p. 149). Archibald's diagram of the Manilla cyclone of October 20, 1882, is here reproduced, and it is contended that these highly characteristic phenomena can be explained only by admitting a *descending* motion in the central part of the cyclone. But on the opposite supposition it is precisely here that the *ascending* current should be strongest, for this central region corresponds exactly to the minimum of barometric pressure. The error in this theory of his opponents is attributed to a confusion between two quite distinct kinds of depressions, a confusion which has for fifty years impeded the progress of meteorological science and increased the perils of navigation.—On the cultivation of Bœmaria in Provence, by M. Naudin. The author reports that the white species (*B. nivea*), lately introduced from China, thrives well in the Antibes district, where the green variety (*B. utilis*) has long been acclimatized. The foliage makes excellent fodder for cattle.—Automatic control of the velocity in machinery of variable action, by M. H. Léauté. An apparatus, the result of many years' study, is here described, by means of which the action of engines may easily be regulated, even when required to work at varying rates of speed.—On a compass enabling the observer to find the meridian on land or water despite the disturbing influence of iron, by M. Bisson. An ingenious apparatus is described by means of which the compass may be prevented from deviating more than one-tenth of a millimetre, even in the neighbourhood of iron. It has been tested with satisfactory results on board several French ironclads, and works equally well by land or sea.—On the snows, ice, and waters of Mars, by M. Flammarion. In reply to some recent remarks on the meteorological condition of this planet, it is pointed out that the varying state of the polar ice-caps has long been carefully observed by Maedler, Schiaparelli, and others, the inference being that Mars is not in a state of glaciation. On the contrary its temperature is equal to, if not higher, than that of the earth, and its polar snows melt periodically to a far greater extent than on our planet.—On the graphic representation of numerical divisors, by M. Saint-Loup. By adopting a rectangular distribution of the numerals, the author arrives at some practical results on the general grouping of the prime numbers.—On the determination of the constants and of the dynamic coefficient of elasticity for steel, by M. E. Mercadier. By the method already indicated (*Comptes rendus*, July and August, 1887), the author here determines the relation $\frac{\lambda}{\mu}$ of the constants for steel. In a future paper will be given the results of the experiments undertaken to determine the coefficients of electricity.—On the mechanism of electrolysis by the process of alternative currents, by MM. J. Chappuis and G. Maneuvrier. The recognized impossibility of electrolysing the sulphate of copper by alternative currents is explained by the theory that the copper deposited on each electrode by one of the currents is immediately dissipated by the inverse current. This explanation is here justified by the authors' experiments, which render *visible* the decomposition of the sulphate of copper, as they had previously done for acidulated water. From this experimental study they hope to deduce the general principles for the practical application of alternative currents in the process of electrolysis.—Application of Carnot's principle to endothermic reactions, by M. Pellat. By distinguishing between the temperature of the bodies giving rise to the endothermic reaction and that of the source supplying in the form of heat the energy needed for the reaction, the author is led by the application of Carnot's principle to a law analogous to that of Potier, but of a more general character.—On the hydrochlorate of cupric chloride, by M. Paul Sabatier. The author admits the priority of M. Engel's researches on the properties and preparation of this substance, but points out that this chemist gives it a very different composition from that which he has himself obtained, and which is represented by the formula CuCl, HCl, 5HO.—On the artificial reproduction of the micas and of scapolite, by M. Doelter. A process is described, by means of which the author has artificially reproduced the chief minerals of the mica group, as well as of natural scapolite. He has already effected the synthesis of biotite, phlogopite, muscovite, and lepidolite (zinnwaldite variety).—Fresh physiological researches on the organic substance which has the property of hydrogenating sulphur, by M. J. de Rey-Pailhade. During his further study of this substance, to which he has given the name of philothion, the author has determined several new facts, amongst others that when the yeast is treated by reagents, the death of the organism always precedes the destruction of this organic substance. Philothion is generated by the physiological development of the yeast, and combines with sulphur according to an equation of which sulphuretted hydrogen is a factor. Acting as a diastase, it adds a fresh proof to M. Berthelot's theory of fermentation. Lastly, it is the first known instance of a substance extracted from a living organism which has the property of hydrogenizing sulphur.—Prof. Langley has been elected by a large majority to succeed the late M. Roche as Corresponding Member of the Academy on the Section of Astronomy.

BERLIN.

Physiological Society, June 22.—Prof. du Bois Reymond, President, in the chair.—Dr. H. Virchow spoke on the blood-vessels of the eye in Carnivora as worked at by Bellarminoff under his direction. The communication was illustrated by drawings and the exhibition of preparations. The points of most general interest which stand out from among the mass of details in this research are that the blood-vessels of the eye have a tendency to form rings from which a large number of fine branches pass posteriorly; further that the arrangement is often very different in different classes of animals, thus, for instance, the course of the arteries in the eye of a dog as compared with that of a rabbit is such that the dog's eye must be turned through an angle of 180° in order to make the course of its arteries correspond with that of the rabbit's eye.—Dr. Heymans communicated the results of his researches on the nerve-endings in the unstriated muscle-fibres of the medicinal leech. In the alimentary canal of the Hirudinea the muscle-fibres are placed both longitudinally and circularly; they consist of a contractile sheath and a protoplasmic axis containing the nucleus, and either have pointed ends or else divide into two or more branches, each of which then ends in a point. The muscle-fibres are separated from each other by large interstitial spaces filled with connective tissue, in which the nerve-plexus lies and sends fine nerve-branches into the muscle-fibres. The nerves end partly as extremely fine filaments and partly as round, flattened end-plates, and in no case does the nerve-ending penetrate the contractile sheath of the fibre so as to come into connection with the protoplasmic axis. In the vascular system of the leech the muscular layers are principally disposed in a circular fashion, but frequently the speaker noticed that at some point or another a circular fibre divided itself into two branches, and that the latter were then bent through a right angle so as now to pass in a longitudinal course in the wall of the blood-vessel. The nerve-endings in the fibres of the vascular system are the same as in those of the alimentary canal. Similarly, the muscle-fibres in the vascular system do not lie in close apposition to each other, but are separated by interstitial spaces; each fibre also contains only one nucleus.—Dr. van der Gehnchten, of Holland, gave a short abstract of his observations on the minute structure of striated muscles in Vertebrata and Arthropoda. He described the appearance of the muscles in the fresh conditions, after the coagulation of the myosin and after the solution of the amorphous proteid, and illustrated his statements by drawings. According to these researches the muscle-fibre of the Vertebrata consists of a network of doubly-refractive filaments, whose meshes are filled with the semi-fluid plasmatic substance. In Arthropoda the structure differs according as the muscle is taken from the wings or the legs; when taken from the latter the structure is extremely similar to that in the Vertebrata. In the discussion which followed, Dr. Benda pointed out that being engaged for years in studying the structure of striated muscle he had often obtained preparations similar in appearance to those of Dr. van der Gehnchten, but his interpretation of these appearances was very different. He pointed out, moreover, that he had often observed transitional forms between the muscles of the leg and wing in Arthropoda and those of Vertebrata. Without entering into any details, Dr. Benda gave it as his opinion that the network in a striated muscle-fibre must not be regarded as contractile, but as a connective-tissue interstitial substance, in whose interspaces the really contractile muscle fibrillæ lie.

IN the report of the meeting of the Physical Society in NATURE of June 21, p. 192, for "Dr. Lummer" (line 37 from the bottom) read "Prof. von Helmholtz."

BOOKS, PAMPHLETS, and SERIALS RECEIVED.

Geologische Studien ueber Niederlandische West Indien, 2te. Lief. Holländisch Guyana, K. Martin (Brill, Leyden).—Lectures on Geography: Lieut.-General R. Strachey (Macmillan).—British Dogs, No. 21: H. Dalziel (Gill).—Speaking Parrots, Part 3: Dr. K. Russ (Gill).—India in 1887: Prof. R. Wallace (Oliver and Boyd).—Annual Report of the Aëronautical Society of Great Britain for the years 1885–86 (Hamilton).—Beiblätter zu den Annalen der Physik und Chemie, 1886, No. 6 (Leipzig).—Geological Magazine, July (Trübner).—Journal of Anatomy and Physiology, July (Williams and Norgate).—Jahrbuch der Meteorologischen Beobachtungen der Wetterwarte der Magdeburgischen Zeitung, Jahrg. v., 1886 (Magdeburg).—Zeitschrift für Wissenschaftliche Zoologie, xlvi. Band, 4 Heft. (Leipzig).—Mind, July (Williams and Norgate).—Notes from the Leyden Museum, vol. x. No. 3 (Leyden).—Journal of the Chemical Society, July (Gurney and Jackson).

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