experiments before the year 1851 with different coals suitable for the Navy. These trials were conducted near London, under a small marine boiler at atmospheric pressure.

(3) At the English Government dockyards, various interesting experiments have been made under small marine boilers, and the results published in Blue-books.

(4) Messrs. Armstrong, Longridge, and Richardson published in 1858 an account of some valuable experiments they had made with the steam-coals of the Hartley district of Northumberland, under a small marine boiler, for the Local Steam Colliery Association.

(5) At Wigan many excellent experiments were made by Messrs. Richardson and Fletcher about 1867, to test the value of Lancashire and Cheshire steam-coals for use in marine boilers. The water was evaporated under atmospheric pressure from a small marine boiler. This station was afterwards abolished.

In none of the above do the gases of combustion appear to have been analyzed.

(6) A fuel-testing station was worked at Dantzig in 1863.

(7) An important station was opened at Brieg, on the Oder, by the colliery-owners of Lower Silesia in April 1878, with the primary object of testing the value as fuel of the important coal-seams of that province. After working with the most satisfactory results for two years, and establishing the superiority of the Lower Silesian coal, the experiments terminated in 1880. The testing boilers had each 40 square metres of heating surface. Gases and coals were analyzed.

Existing Continental Stations.—(8) The Imperial Naval Administration Coal-testing Station at Wilhelmshaven, Germany, was established in 1877.

(9) Dr. Bunte's coal-testing station, erected at Munich about 1878, particulars of which have been published in the Proceedings of the Institution of Civil Engineers, vol. lxxiii. Here some hundreds of trials have been reported on and published; much valuable work has been done, and many fuels tested, including coals of the Ruhr valley, Saar basin, Saxon and Bohemian coal-fields, and those of Silesia and Upper Bavaria. The boiler of the station has about 450 square feet of heating surface. The gases and coals are analyzed, and all particulars carefully noted. It is one of the most complete stations I have seen.

(10) In Belgium, near Brussels, there is a Government station for testing fuels, under the administration of the Belgium State railways; locomotive boilers are used. The establishment has been at work for the last two years, but no results are published, as they are considered the property of the Government. Private firms can, however, have their coals tested and reported upon.

(11) The Imperial Marine Station, Dantzig.

(12) Boiler Insurance Company at Magdeburg.

The above is a slight outline of the work already done in this direction.

With the view of obtaining the opinions of those interested in starting a fuel-testing station, I ask you kindly to give this letter publicity. If the necessary sum can be raised, we may hope to have before long a practical and useful establishment in London, and to gain from it many interesting practical results respecting the combustion of fuels. BRYAN DONKIN, JUN.

Bermondsey, S.E., June 11.

The Geometric Interpretation of Monge's Differential Equation to all Conics—the Sought Found.

THE question of the true geometric interpretation of the Mongian equation has been often considered by mathematicians. In the first place, we have the late Dr Boole's statement that "here our powers of geometrical interpretation fail, and results such as this can scarcely be otherwise useful than as a registry of integrable forms" ("Diff. Equ.," pp. 19–20). We have next two attempts to interpret the equation geometrically. The first of these propositions, by Lieut-Colonel Cunningham, is that "the eccentricity of the osculating conic of a given conic is constant all round the latter" (*Quarterly Journal*, vol. xiv. 229); the second, by Prof. Sylvester, is that "the differential equation of a conic is satisfied at the sextactic points of any curve" (*Amer. Journ. Math.*, vol. ix. p. 19). I have elsewhere considered both these interpretations in detail, and I have pointed out that both of them are irrelevant; the first of them is, in fact, the geometric interpretation, not of the Mongian equation, but of one of its five first integrals which I have actually calculated (Proc. Asiatic Soc. Bengal, 1888, pp. 74–86); the second is out of mark as failing to furnish such a property of the conic as would lead to a geometrical quantity which vanishes at every point of every conic (Journal Asiatic Soc. Bengal, 1887, Part 2, p. 143). In this note I will briefly mention the true geometric interpretation which I have recently discovered.

Consider the osculating conic at any point, P, of a given curve; the centre, O, of the conic is the centre of aberrancy at P, and as P travels along the given curve, the locus of O will be another curve, which we may conveniently call the *aberrancy curve*. Take rectangular axes through any origin; let (x, y) be the given point P, and α, β the co-ordinates of the centre of aberrancy. Then it can be shown without much difficulty that

$$\alpha = x - \frac{3qr}{3qs - 5r^2},$$

$$\beta = y - \frac{3q(pr - 3q^2)}{3qs - 5r^2},$$

whence

$$\frac{d\alpha}{dx} = \lambda T, \quad \frac{d\beta}{dx} = \mu T,$$

where

$$\lambda = \frac{r}{(3qs - 5r^2)^2}, \quad \mu = \frac{pr - 3q^2}{(3qs - 5r^2)^2},$$

$$T \equiv 9q^2t - 45qrs + 40r^3,$$

p, q, r, s, t being, as usual, the successive differential coefficients of y with respect to x.

If $d\psi$ be the angle between two consecutive axes of aberrancy, ds the element of arc, and ρ the radius of curvature of the aberrancy curve, we have

$$\rho = \frac{ds}{d\psi}, \quad ds^2 = d\alpha^2 + d\beta^2,$$

whence

$$\rho = (\lambda^2 + \mu^2)^{\frac{1}{2}} \cdot T \cdot \frac{dx}{d\psi}.$$

But it is easy to show that

$$\frac{d\psi}{dx} = \frac{q(3qs - 5r^2)}{r^2 + (rp - 3q^2)^2},$$

so that

$$\rho = T \cdot \frac{\{r^2 + (rp - 3q^2)^2\}^{\frac{3}{2}}}{q(3qs - 5r^2)^3}.$$

Now, $T = 0$ is Monge's differential equation to all conics, and when $T = 0$ we have $\rho = 0$. Hence, clearly, the true geometric interpretation of the Mongian equation is:

The radius of curvature of the aberrancy curve vanishes at every point of every conic.[1]

This geometrical interpretation will be found to satisfy all the tests which every true geometrical interpretation ought to satisfy, and I believe that this is the interpretation which, during the last thirty years, has been sought for by mathematicians, ever since Dr. Boole wrote his now famous lines. I will not take up the valuable space of these columns with the details of calculation: they will be found fully set forth in two of my papers which will be read next month before the Asiatic Society of Bengal, and will in due course be published in the Journal.

Calcutta, May 18. ASUTOSH MUKHOPADHYAY.

PERSONAL IDENTIFICATION AND DESCRIPTION.[2]

I.

IT is strange that we should not have acquired more power of describing form and personal features than we actually possess. For my own part I have

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[1] The differential equation of all parabolas,

$$3qs - 5r^2 = 0,$$

is also easily interpreted. viz. calling the distance OP between the given point and the centre of aberrancy the *radius of aberrancy*, and the reciprocal of this (= I) the *index of aberrancy*, we have, easily,

$$I = \frac{3qs - 5r^2}{3q\{r^2 + (rp - 3q^2)^2\}^{\frac{1}{2}}}$$

so that the interpretation is that *the index of aberrancy vanishes at every point of every parabola.*

[2] The substance of a Lecture given by Francis Galton, F R.S., at the Royal Institution on Friday evening, May 25, 1888.

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