Class III. The development might just as well be in inverse order, though we have important reasons for believing it is not so.

The astronomy of the future must decide between these two alternatives. My object in undertaking this work was to facilitate this decision by giving as exact descriptions as possible of the spectra presented by the different stars of Class III. in the year 1880.

THE ART OF COMPUTATION FOR THE PURPOSES OF SCIENCE.[1]

II.

SOME few problems in astronomy and certain theories in pure mathematics require more than seven figures to be calculated. In these cases a large arithmometer is generally the most convenient. Ten-figure tables of logarithms may be obtained second-hand; or the required logarithms must be calculated.

The tables of Vlacq, re-edited by Vega in 1749, 1794, and 1797 are somewhat difficult to obtain and cumbrous to use. The logarithms of numbers up to 101,000 are given to ten figures with first and second differences. Thus to find log 10 542 482 375, from the table directly

$$
\begin{array}{lllr}
\log 10\,542 = & \cdot 0229\,230\,119 & \Delta_1 = & 411\,946 \\
 & \quad\;\; 198\,712{\cdot}3 & & 482\,375 \\
 & \qquad\qquad\;\; 5 & & \overline{1\,647\,784} \\
\text{log required} & \overline{\cdot 0229\,428\,836\,3} & & 329\,557\;+ \\
 & & & 8\,239\;+ \\
\multicolumn{2}{l}{\text{The true log of } 10\,542\,482\,375 \text{ is}} & & 1\,236\;+ \\
\multicolumn{2}{l}{\quad \cdot 022\,942\,883\,626\,562.} & & 288\;+ \\
 & & & 20\;- \\
 & & & \overline{198\,712\,3\,1} \text{ subtracted.}
\end{array}
$$

$$\Delta_2 = 40$$

$$\frac{\cdot 48(\cdot 48 - 1)(-40)}{2} = 4{\cdot}992.$$

In default of Vega, or if more places are required, the logarithm must be calculated, and this is by no means such a serious affair as one is led to think by the ordinary books on algebra. I am much indebted in what follows to the article by Mr. J. W. L. Glaisher on logarithms in the new edition of the "Encyclopædia Britannica," to which I refer my readers for further particulars in theory, restricting myself to practical details.

The easiest way to calculate a table of logarithms absolutely *de novo* would be by the method of differences, with some mechanical assistance, such as the difference-engine of Babbage or of Scheutz. It seems unlikely that larger tables will be calculated than those already in existence, since the cost increases with great rapidity. Mr. Sang has, however, recently calculated independently the logarithms of numbers from 100000 to 200000, where the ordinary tables are weakest.

Briggs used at least two methods for the calculation of logarithms which depended upon the extraction of a succession of roots. For instance, by taking the square root of 10 fifty-four times he found log $1{\cdot}(0)^{15}1\ 278\ 191\ 493$ to be $\cdot(0)^{15}0\ 555\ 111\ 512$. Whence assuming that very small numbers vary as their logarithms, log $1{\cdot}(0)^{15}1 = 555\ 111\ 512/1\ 278\ 191\ 493$, or log $1{\cdot}(0)^{15}1 = 0{\cdot}43\ 429\ 448 = \text{M}$, the modulus. And if x be small, log $1{\cdot}(0)^{15}x = x \times 0{\cdot}43\ 429\ 448$. To find log 2 he extracted the square root of the tenth power, 1024/1000 forty-seven times, and found $1{\cdot}(0)^{15}1\ 685\ 160\ 570$, which multiplied by M gave $(0)^{15}0\ 731\ 855\ 936$. This multiplied by 2^{47} gave log $1{\cdot}024$; adding 3 and dividing by 10 gives log 2. Another more simple method was to find a series of geometrical means between two numbers, such as 10 and 1, the logarithms of which are known. After taking 22 of these roots, log 5 is found to be 0·69897.

It was soon found that logarithms could be more easily calculated by the summation of various series, and many great mathematicians, such as Newton, Gregory, Halley, Cotes, exercised their ingenuity in discovering those most suitable for the purpose.

Though for practical purposes the use of series has been almost superseded, three very simple ones are still occasionally useful:—

$$\log (1 \pm x) = \text{M}\left(\pm x - \frac{x^2}{2} \pm \frac{x^3}{3} - \frac{x^4}{4} \pm \frac{x^5}{5} - \right)$$

which converges rapidly if x be small. M is a number depending upon the system of logarithms adopted, and constant for each system. If M be 1, the system is called the Naperian, or natural one; and if M = 0·434 &c., the system is the common one. Unless otherwise stated M will be assumed to be 1, or the logarithms will be natural ones.

Thus to calculate log $1{\cdot}1 = 1 + \frac{1}{10}$, omitting M:—

$$\log 1{\cdot}1 = \frac{1}{10} - \frac{1}{200} + \frac{1}{3000} - \frac{1}{40000} + \frac{1}{500000} -, \text{\&c.},$$
$$= 0{\cdot}1003\ 3534 - 0{\cdot}0050\ 2517 = 0{\cdot}0953\ 1017.$$

Suppose x be small, $\log (1 \pm x) = \pm \text{M}x$ nearly. Thus if log $1{\cdot}(0)^{10}9$ be required to twenty decimals, it is

$$(0)^{10}9 - \frac{1}{2}\,(\cdot 9 \times 10^{-10})^2,$$

or the error caused by omitting this and all subsequent terms is only 4 in the twenty-first decimal place. Using common logarithms the multiplication by M reduces the error by one-half. This result is of great importance in calculating logarithms by Flower's method, since the factors which have to be dealt with are only half the number of decimal places in the required logarithm.

Writing $\frac{1}{x}$ for x in the above series, we obtain—

$$\log (1 + x) - \log x = \text{M}\left(\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} - \frac{1}{4x^4} + \frac{1}{5x^5} - \right)$$

which converges rapidly when x is large. Various artifices may be used to render x large, even when the number the logarithm of which is required is small. Thus, Prof. J. C. Adams has calculated (NATURE, vol. xxxv. p. 381) log 2, log 3, log 5, log 7, $\frac{1}{\text{M}}$ and M to 270 places of decimals.

Another very valuable series is—

$$\log(a \pm x) = \log a \pm 2\text{M}\left\{\frac{x}{2a+x} + \frac{1}{3}\left(\frac{x}{2a+x}\right)^3 + \frac{1}{5}\left(\frac{x}{2a+x}\right)^5 + \text{\&c.}\right\}$$

Thus, supposing log 219 known, to calculate log 2198:—

$$\log 2198 = 7{\cdot}6916\ 5682\ 2810 + 2\left\{\frac{4}{2194} + \frac{1}{3}\left(\frac{4}{2194}\right)^3 + \text{\&c.}\right\}$$

$$
\begin{array}{rl}
\cdot 0036\ 4630\ 8113 & \qquad \dfrac{4}{2194} = \cdot 0018\ 2315\ 40565 \\
4039 & \\
\hline
\log 2198 = 7{\cdot}6593\ 0313\ 4962 & \qquad \dfrac{2}{3}\left(\dfrac{1823}{10^6}\right)^3 = \cdot 0^8 4039
\end{array}
$$

Using common logarithms, the third term of the series is $< \frac{1}{27{\cdot}6}\left(\frac{x}{a}\right)^3$, that is less than 5 in the ninth place when $\frac{x}{a} < \frac{1}{200}$. Hence, with a table giving the logarithms of 100–1000 to eight figures the third term may be neglected, or the required difference is $\pm \frac{2\text{M}x}{2a + x}$, or, writing $\log (a + x) - \log a = y$,

$$x = \frac{2ay}{2\text{M} - y}.$$

The given numbers may also be broken up into factors by the aid of such a table as Burkhard's, which gives the factors of all numbers up to 3,036,000. The logarithms of the factors may then be found from tables and added together. Of all tables for this purpose, that of Wolfram is the most valuable; it gives the natural logarithms to forty-eight places of *all* numbers up to 2200, and of all which are not easily divisible up to 10,009.

The multiplication by M to convert into common logarithms is tedious, and it is frequently better to dispense with it in heavy calculations. If necessary, a table of the first ninety-nine multiples of M should be prepared, and Oughtred's short method of multiplication used.

If any of my readers desire to test themselves and their tables

[1] Continued from p. 239.

by a long but easy calculation, the amount of £1 laid up at 5 per cent. compound interest for a thousand years will be found not to differ very much from £1,546,318,920,731,927,238,982. An answer of this sort is of course of no practical utility whatever, but it brings vividly before us an important point in political economy—the accretion of wealth in the hands of corporations. It was computed that just before the Revolution more than half the soil of France was owned by the Church. Looking at this array of figures, and remembering that since the Church could never alienate its property all surplus income must be regarded as at compound interest, we can only wonder that it was the half and not the whole.

The first table for facilitating the computation of logarithms was one given by Long (Phil. Trans., 1724) of the decimal powers of 10 to nine figures. Thus, to find the number the logarithm of which is

$$30103 = 10^{\cdot 3} \times 10^{\cdot 001} \times 10^{\cdot 00003} = 1{\cdot}99526231 \times 1{\cdot}00230523 \times 1{\cdot}00006908 = 1{\cdot}99999997, \text{ or } 2.$$

This method is cumbrous, but it is perhaps one of the most simple for explaining the calculation of logarithms to beginners.

A much more convenient method has been well worked out by M. Namur, but, unfortunately, only his twelve-figure table seems to be still in print. The table contains the logarithms of numbers from 433300 to 434300 to twelve figures, and the numbers corresponding to logarithms from 637780 to 638860. By the aid of certain factors which are tabulated with their complementary logarithms, any number or logarithm can be reduced between these limits.

Thus, to find log π—

	314	159	265	359		$\times$ 1·3
	94	247	779	607	7	
	408	407	044	956	7	$\times$ 1·063
	24	504	422	698	0	
	1	225	221	134	9	
	434	136	688	799	6	
log from table	637	625	800	474		Δ = 1·000364
				206	4	
				41	3	
				2	4	
	637	626	489	524		
	973	466	735	477		} complementary logs of
	886	056	647	693		} 1·3 and 1·063
	497	149	872	694		= log π.

The last method I shall mention is generally known by the name of Weddle; it was probably used by Briggs, and published by Flower in 1771. It consists in multiplying the given number by a series of factors of the form $1 \pm \frac{x}{10^n}$ until it is reduced to one. The complement of the sum of the logarithms of the factors is the required logarithm. The logarithms of the factors are easily calculated by the first series; they have been tabulated to about thirty places.

Thus to find log 3550·26:—

355026 $\times$ 2	30103	
	11394 3	
710052 $\times$ 1·3	3342 4	
2130156	130 1	
	3 9	
9230676 $\times$ 1·08		
738454	44973 7	
	55026	complement.
9969130 $\times$ 1·003		
29907		
9999037 $\times$ 1·00009		

Hence log 3550·26 = 3·55026, or we have a number which is expressed by the same figures as its logarithm.

It is the present fashion, while depreciating our own country men, to extol all Germans in matters connected with education, and especially to award them the palm for patient plodding. It will be some time before a German rivals Prof. Adams, and even then there is a height beyond. Of all monuments of calculation the value of π, or the number of times the circumference is longer than the diameter of a circle, is most astounding. Archimedes found it to be $\frac{22}{7}$, Wolf calculated it to 16 places, Van Ceulen to 35, Machin to 100, Beerens de Haan to 250, Richter to 500. But in 1853 Mr. Shanks threw all these results into the shade, and excited the admiration even of De Morgan by calculating π to 530 places, "throwing aside as an unnoticed chip the 219th power of 9"! Two printers' errors were pointed out by Mr. John Morgan, which Mr. Shanks corrected from his manuscript, and in 1873 gave a new result to 707 places. Hence the value of π is known to within $\frac{1}{3 \times 10^{707}}$, an exactness which is useless from the inability of the human mind to comprehend the figures which express it.

Clerk Maxwell proposed, possibly in irony, to take the wave-length of a certain light as the universal unit of length. Choosing for this purpose about the middle of the violet, a mile would be expressed by $60000 \times 63360 = 3{\cdot}8 \times 10^9$ units nearly. Suppose that Sirius, the brightest star in our firmament, has an annual parallax of $\frac{1}{5}''$, a quantity perceptible, but barely measurable, by our best telescopes, the distance of the sun from Sirius is about $5 \times 206{,}265 \times 92{,}300{,}000$ miles, or $3{\cdot}5 \times 10^{23}$ units. Assume again that Kant's fanciful conjecture is correct, and that the sun revolves round Sirius in a circle the length of which is expressed by $7 \times 10^{23} \times \pi$ units. Make the still greater assumption that all our measures are correct, and our arithmetic as it ought to be, so that the only possible error would be in the evaluation of π. The greatest possible error according to Mr. Shanks's determination would be $\frac{7 \times 10^{23}}{3 \times 10^{707}}$ or $\frac{1}{4{\cdot}3 \times 10^{683}}$ of a wave-length of violet light. Whatever metaphysicians may say, I think we have here reached, if not surpassed, the limits of the human understanding.

SYDNEY LUPTON.

SOCIETIES AND ACADEMIES.

PARIS.

Academy of Sciences, January 2.—M. Janssen, President, in the chair.—On an objection made to the employment of electro-magnetic regulators in a system of synchronous time-pieces, by M. A. Cornu. This is a reply to M. Wolf's recent communication, in which several objections were urged against the apparatus in question. It is shown (1) that such a regulator does not necessarily tend to stop the system to which it is applied; (2) that in any case the stoppage may be prevented without complication or expense; and (3) that in a public time-distributing service the stoppage should not only not be prevented, but efforts should be made to bring it about whenever the synchronizing system gets out of order. The paper was followed by some further remarks on the part of M. Wolf, who reiterated his objections, and treated M. Cornu's third point as somewhat paradoxical.—Remarks on Père Dechrevens's letter regarding the artificial reproduction of whirlwinds, by M. H. Faye. The author complains that, like other partisans of the prevailing ideas on the subject of tornadoes, typhoons, and cyclones, M. Dechevrens endeavours to suit the facts to the exploded theory of an ascending motion in the artificial reproduction of these aërial phenomena.—On the meteorite which fell at Phû-Long, Cochin China, on September 22, 1887, by M. Daubrée. In supplement to M. Delauney's communication of December 19, the author adds that this meteorite was an oligosiderite of somewhat ordinary type, closely resembling those of Tabor (Bohemia), July 3, 1753; Weston (Connecticut), December 14, 1807; Limerick, September 10, 1813; and Ohaba (Transylvania), October 10, 1817.—Remarks in connection with the presentation of the "Annuaire du Bureau des Longitudes" for 1888, the "Connaissance des Temps" and the "Extrait de la Connaissance des Temps" for 1889, by M. Faye. Amongst the fresh matter added to the "Annuaire" this year are papers by M. Janssen on the age of the stars, by Admiral Mouchez on the progress of stellar photography, and by M. d'Abbadie on his recent expedition to the East in order to determine the elements of terrestrial magnetism in Egypt, Palestine, and Syria.—Observations of Olbers' comet made at the Observatory of Nice (Gautier's 0·38 m. equatorial), by M. Charlois. These observations are for December 25, 26, and 27, after the comet was discovered on December 23, when the nucleus was of the tenth magnitude, surrounded by a bright nebulosity, and with tail from 20′ to 25′ in length.—On the total eclipse of the sun