A rarer form is as follows :—

I have noticed that this latter form seems more difficult for the little musicians, one of whom in particular used to provoke me by singing the B most outrageously flat. I have been accustomed to imitate these birds by whistling, and they very readily answer my whistle. In this way the different forms of their theme have become fixed in my memory. W. L. GOODWIN.

Queen's University, Kingston, Canada, November 11.

Who was Mr. Charles King?

AMONG the ingenious in many considerable parts of the world, of whose undertakings, studies, and labours the Philosophical Transactions of the years 1700 *sqq.* gave some account, an able microscopist suddenly appears, of whose life and work one would like to have more accurate information than seems to be current. Perhaps a member of the Royal, or the Royal Microscopical, Society may be able to supply some particulars about this "Anglois anonyme," as Trembley calls him, and willing to assist in rescuing his name from an undeserved oblivion. His first contribution to the Philosophical Transactions—of very little importance indeed—is to be found in No. 266, for September and October 1700, pp. 672–673, under the title, "A Letter from Mr. Charles King to Mr. Sam. Doudy, F.R.S., concerning Crabs Eyes;" it is dated, "Little Wirley, Decemb. 14," and subscribed, "Ch. King." In the copy of the Transactions I have before me, a contemporary, who seems to have been tolerably well informed, has inserted divers MS. notes, remarks, and corrections; he added here the words, "Staffordsh^e." to the locality, and "Student of Ch. Ch. Oxon." to the subscription, which, as far as I know, does not recur in any of the subsequent Transactions. But under the title, "Two Letters from a Gentleman in the Country, relating to Mr. Leuwenhoeck's Letter in Transaction, No. 283, Communicated by Mr. C." (in No. 288, for November and December 1703, pp. 1494–1501, with eight figures, text and illustrations being both equally remarkable for the period), the same hand has again inscribed the name of "Mr. Charles King," and filled up the blanks left on pages 1494 and 1495 by the initials "W." and "W. Ch. Esq." with the additions of "irley par. Com̄. Stafford." and "Walter Chetw... of Ingestry Staffords." (the rest has been cut off by the binder of the volume), so that there remains no reasonable doubt as to the truth of the identification. Now we read in the second of these letters from the country, dated "July 5, 1703," p. 1501, "But of those" (viz. animalcula) "(among other things) I last year gave an account to Sir Ch. Holt, which I hear will shortly be publish'd in the Transactions." I don't think it is bold to conjecture that the account here alluded to had already been published, and is, in fact, the article printed in No. 284, for March and April 1703, pp. 1357(*bis*)–1372 (with excellent figures on the plate accompanying that number), under the title of "An Extract of some Letters sent to Sir C. H. relating to some Microspocal" (*sic*) "Observations. Communicated by Sir C. H. to the Publisher" (H. Sloane); and no doubt these epistles may also be ascribed to the same anonymous gentleman.

In all the above-mentioned letters we have some early and first-rate contributions to microscopical science, the importance of which had been shortly before so evidently demonstrated by the wonderful discoveries made by the improved magnifying-glasses.

Quæritur: Who was Mr. Charles King? S.

The Hague, November 27.

NOTE ON A PROPOSED ADDITION TO THE VOCABULARY OF ORDINARY ARITHMETIC.[1]

THE total number of distinct primes which divide a given number I call its Manifoldness or Multiplicity.

[1] Perhaps I may without immodesty lay claim to the appellation of the Mathematical Adam, as I believe that I have given more names (passed into general circulation) to the creatures of the mathematical reason than all the other mathematicians of the age combined.

A number whose Manifoldness is n I call an n-fold number. It may also be called an n-ary number, and for $n = 1, 2, 3, 4, \ldots$ a unitary (or primary), a binary, a ternary, a quaternary, ... number. Its prime divisors I call the *elements* of a number; the highest powers of these elements which divide a number its *components;* the degrees of these powers its *indices;* so that the indices of a number are the totality of the indices of its several components. Thus, we may say, a prime is a one-fold number whose index is unity.

So, too, we may say that all the components but one of an odd perfect number must have even indices, and that the excepted one must have its base and index each of them congruous to 1 to modulus 4.

Again, a remarkable theorem of Euler, contained in a memoir relating to the Divisors of Numbers ("Opuscula Minora," vol. ii. p. 514), may be expressed by saying that *every even perfect number is a two-fold number, one of whose* components *is a prime, and such that when augmented by unity it becomes a power of* 2, *and double the other component.*[1]

Euler's function $\phi(n)$, which means the number of numbers not exceeding n and prime to it, I call the *totient* of n; and in the new nomenclature we may enunciate that *the totient of a number is equal to the product of that number multiplied by the several excesses of unity above the reciprocals of its elements.* The numbers prime to a number and less than it, I call its *totitives.*

Thus we may express Wilson's generalized theorem by saying that any number is contained as a factor in the product of its totitives increased by unity if it is the number 4, or a prime, or the double of a prime, and diminished by unity in every other case.

I am in the habit of representing the totient of n by the symbol $\tau n, \tau$ (taken from the initial of the word it denotes)

[1] It may be well to recall that a perfect number is one which is the half of the sum of its divisors. The converse of the theorem in the text, viz. that $2^n(2^{n+1} - 1)$, when $2^{n+1} - 1$ is a prime, is a perfect number, is enunciated and proved by Euclid in the 36th (the last proposition) of the 9th Book of the "Elements," the second factor being expressed by him in the sum of a geometric series whose first term is unity and the common ratio 2. In Isaac Barrow's English translation, published in 1660, the enunciation is as follows:—"If from a unite be taken how many numbers soever 1, A, B, C, D, in double proportion continually, untill the whole added together E be a prime number; and if this whole E multiplying the last produce a number F, that which is produced F shall be a perfect number."

The direct theorem that every even perfect number is of the above form could probably only have been proved with extreme difficulty, if at all, by the resources of Greek Arithmetic. Euler's proof is not very easy to follow in his own words, but is substantially as follows:

Suppose P (an even perfect number) $= 2^n$A. Then, using in general $\int$X to denote the sum of the divisors of X,

$$2 = \frac{\int P}{P} = \frac{\int 2^n . \int A}{2^n A} = \frac{2^{n+1} - 1}{2^n} . \frac{\int A}{A}.$$

Hence $$\frac{\int A}{A} = \frac{2^{n+1}}{2^{n+1} - 1}, \text{ say } = \frac{Q + 1}{Q}.$$

Hence $A = \mu Q$, and $\int A = 1 + \mu + Q + \mu Q + \ldots$ (if μ be supposed > 1).

Hence unless $\mu = 1$ and at the same time Q is a prime

$$\int A > \mu(Q + 1),$$

i.e. $\frac{\int A}{A}$ is greater than itself.

Hence an even number P cannot be a perfect number if it is not of the form $2^n(2^{n+1} - 1)$, where $2^{n+1} - 1$ is a prime, which of course implies that $n + 1$ must itself be a prime.

It is remarkable that Euler makes no reference to Euclid in proving his own theorem. It must always stand to the credit of the Greek geometers that they succeeded in discovering a class of perfect numbers which in all probability are the only numbers which are perfect. Reference is made to *so-called* perfect numbers in Plato's "Republic," H, 546 B, and also by Aristotle, Probl. I E 3 and "Metaph." A 5, which he attributes to Pythagoras, but which are purely fanciful and entitled to no more serious consideration than the late Dr. Cummings's ingenious speculations on the number of the Beast. Mr. Margoliouth has pointed out to me that Muhamad Al-Sharastani, in his "Book of Religious and Philosophical Sects," Careton, 1856, p. 267 of the Arabic text, assigns reasons for regarding all the numbers up to 10 inclusive as perfect numbers. My particular attention was called to perfect numbers by a letter from Mr. Christie, dated from "Carlton, Selby," containing some inquiries relative to the subject.

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