bitten by rabid animals, who, if they had not been subjected to this treatment would have died of that disease. The value of his discovery is, however, greater than can be estimated by its present utility, for it shows that it may be possible to avert other diseases besides hydrophobia by the adoption of a somewhat similar method of investigation and of treatment. This, though the last, is certainly not the least of the debts which humanity owes to the great French experimentalist. Here it might seem as if we had outstepped the boundaries of chemistry, and have to do with phenomena purely vital. But recent research indicates that this is not the case, and points to the conclusion that the microscopist must again give way to the chemist, and that it is by chemical rather than by biological investigation that the causes of diseases will be discovered, and the power of removing them obtained. For we learn that the symptoms of infective diseases are no more due to the microbes which constitute the infection than alcoholic intoxication is produced by the yeast-cell, but that these symptoms are due to the presence of definite chemical compounds, the result of the life of these microscopic organisms. So it is to the action of these poisonous substances formed during the life of the organism, rather than to that of the organism itself, that the special characteristics of the disease are to be traced; for it has been shown that the disease can be communicated by such poisons in entire absence of living organisms.

If I have thus far dwelt on the progress made in certain branches of pure science it is not because I undervalue the other methods by which the advancement of science is accomplished, viz. that of the application and of the diffusion of a knowledge of Nature, but rather because the British Association has always held, and wisely held, that original investigation lies at the root of all application, so that to foster its growth and encourage its development has for more than fifty years been our chief aim and wish.

Had time permitted I should have wished to have illustrated this dependence of industrial success upon original investigation, and to have pointed out the prodigious strides which chemical industry in this country has made during the fifty years of Her Majesty's reign. As it is I must be content to remind you how much our modern life, both in its artistic and useful aspects, owes to chemistry, and, therefore, how essential a knowledge of the principles of the science is to all who have the industrial progress of the country at heart.

This leads me to refer to what has been accomplished in this country of ours towards the diffusion of scientific knowledge amongst the people during the Victorian era. It is true that the English people do not possess, as yet, that appreciation of the value of science so characteristic of some other nations. Up to very recent years our educational system, handed down to us from the Middle Ages, has systematically ignored science, and we are only just beginning, thanks in a great degree to the prevision of the late Prince Consort, to give it a place, and that but an unimportant one, in our primary and secondary schools or in our Universities. The country is, however, now awakening to the necessity of placing its house in order in this respect, and is beginning to see that if she is to maintain her commercial and industrial supremacy the education of her people from top to bottom must be carried out on new lines. The question as to how this can be most safely and surely accomplished is one of transcendent national importance, and the statesman who solves this educational problem will earn the gratitude of generations yet to come.

In conclusion, may I be allowed to welcome the unprecedentedly large number of foreign men of science who have on this occasion honoured the British Association by their presence, and to express the hope that this meeting may be the commencement of an international scientific organization, the only means nowadays existing, to use the words of one of the most distinguished of our guests, of establishing that fraternity among nations from which politics appear to remove us further and further by absorbing human powers and human work, and directing them to purposes of destruction. It would indeed be well if Great Britain, which has hitherto taken the lead in so many things that are great and good, should now direct her attention to the furthering of international organizations of a scientific nature. A more appropriate occasion than the present meeting could perhaps hardly be found for the inauguration of such a movement.

But whether this hope be realized or not, we all unite in that one great object, the search after truth for its own sake, and we all, therefore, may join in re-echoing the words of Lessing:—"The worth of man lies not in the truth which he possesses, or believes that he possesses, but in the honest endeavour which he puts forth to secure that truth; for not by the possession of truth, but by the search after it are the faculties of man enlarged, and in this alone consists his ever-growing perfection. Possession fosters content, indolence, and pride. If God should hold in His right hand all truth, and in His left hand the ever-active desire to seek truth, though with the condition of perpetual error, I would humbly ask for the contents of the left hand, saying, 'Father, give me this; pure truth is only for Thee.'"

SECTION A.

MATHEMATICAL AND PHYSICAL SCIENCE.

OPENING ADDRESS BY SIR ROBERT S. BALL, LL.D., F.R.S., PRESIDENT OF THE SECTION.

A Dynamical Parable.

THE subject I have chosen for my address to you to-day has been to me a favourite topic of meditation for many years. It is that part of the science of theoretical mechanics which is usually known as the "Theory of Screws."

A good deal has been already written on this theory, but I may say with some confidence that the aspect in which I shall invite you now to look at it is a novel one. I propose to give an account of the proceedings of a committee appointed to investigate and experiment upon certain dynamical phenomena. It may appear to you that the experiments I shall describe have not as yet been made, that even the committee itself has not as yet been called together. I have accordingly ventured to call this address "A Dynamical Parable."

There was once a rigid body which lay peacefully at rest. A committee of natural philosophers was appointed to make an experimental and rational inquiry into the dynamics of that body. The committee received special instructions. They were to find out why the body remained at rest, notwithstanding that certain forces were in action. They were to apply impulsive forces and observe how the body would begin to move. They were also to investigate the small oscillations. These being settled, they were then to—— But here the chairman interposed; he considered that for the present, at least, there was sufficient work in prospect. He pointed out how the questions already proposed just completed a natural group. "Let it suffice for us," he said, "to experiment upon the dynamics of this body so long as it remains in or near to the position it now occupies. We may leave to some more ambitious committee the task of following the body in all conceivable gyrations through the universe."

The committee was judiciously chosen. Mr. Anharmonic undertook the geometry. He was found to be of the utmost value in the more delicate parts of the work, though his colleagues thought him rather prosy at times. He was much aided by his two friends, Mr. One-to-One, who had charge of the homographic department, and Mr. Helix, whose labours will be seen to be of much importance. As a most respectable if rather old-fashioned member, Mr. Cartesian was added to the committee, but his antiquated tactics were quite out-manœuvred by those of Helix and One-to-One. I need only mention two more names. Mr. Commonsense was, of course, present as an *ex-officio* member, and valuable service was even rendered by Mr. Querulous, who objected at first to serve on the committee at all. He said that the inquiry was all nonsense, because everybody knew as much as they wished to know about the dynamics of a rigid body. The subject was as old as the hills, and had all been settled long ago. He was persuaded, however, to look in occasionally. It will appear that a remarkable result of the labours of the committee was the conversion of Mr. Querulous himself.

The committee assembled in the presence of the rigid body to commence their memorable labours. There was the body at rest, a huge amorphous mass, with no regularity in its shape—no uniformity in its texture. But what chiefly alarmed the committee was the bewildering nature of the constraints by which the movements of the body were hampered. They had been accustomed to nice mechanical problems, in which a smooth body lay on a smooth table, or a wheel rotated on an axle, or a body rotated around a point. In all these cases the constraints were of a simple character, and the possible movements of the body were obvious. But the constraints in the present case were of puzzling complexity. There were cords and links, moving axes, surfaces with which the body lay in contact, and many other geometrical constraints. Experience of ordinary problems in mechanics would be of little avail. In fact, the chairman truly appreciated the situation when he said that the *constraints were of a perfectly general type*.

In the dismay with which this announcement was received Mr. Commonsense advanced to the body and tried whether it could move at all. Yes, it was obvious that in some ways the body could be moved. Then said Commonsense, "Ought we not first to study carefully the nature of the freedom which the body possesses? Ought we not to make an inventory of every distinct movement of which the body is capable? Until this has been obtained I do not see how we can make any progress in the dynamical part of our business."

Mr. Querulous ridiculed this proposal. "How could you," he said, "make any geometrical theory of the mobility of a body without knowing all about the constraints? And yet you are attempting to do so with perfectly general constraints of which you know nothing. It must all be waste of time, for though I have read many books on mechanics, I never saw anything like it."

Here the gentle voice of Mr. Anharmonic was heard. "Let us try, let us simply experiment on the mobility of the body, and let us faithfully record what we find." In justification of this advice Mr. Anharmonic made a remark which was new to most members of the committee; he asserted that *though the constraints may be of endless variety and complexity there can be only a very limited variety in the types of possible mobility.*

It was therefore resolved to make a series of experiments with the simple object of seeing how the body could be moved. Mr. Cartesian, having a reputation for such work, was requested to undertake the inquiry and to report to the committee. Cartesian commenced operations in accordance with the well-known traditions of his craft. He erected a cumbrous apparatus which he called his three rectangular axes. He then attempted to push the body parallel to one of these axes, but it would not stir. He tried to move the body parallel to each of the other axes, but was again unsuccessful. He then attached the body to one of the axes and tried to effect a rotation around that axis. Again he failed, for the constraints were of too elaborate a type to accommodate themselves to Mr. Cartesian's crude notions.

We shall subsequently find that the movements of the body are necessarily of an exquisitely simple type, yet such was the clumsiness and the artificial character of Mr. Cartesian's machinery that he failed to perceive the simplicity. To him it appeared that the body could only move in a highly complex manner; he saw that it could accept a composite movement consisting of rotations about two or three of his axes and simultaneous translations also parallel to two or three axes. Cartesian was a very skilful calculator, and by a series of experiments even with his unsympathetic apparatus he obtained some knowledge of the subject, sufficient for purposes in which a vivid comprehension of the whole was not required. The inadequacy of Cartesian's geometry was painfully evident when he reported to the committee on the mobility of the rigid body. "I find," he said, "that the body can neither move parallel to x, nor to y, nor to z; neither can I make it rotate around x, nor y, nor z; but I could push it an inch parallel to x, provided that at the same time I pushed it a foot parallel to y and a yard backwards parallel to z, and that it was also turned a degree around x, half a degree the other way around y, and twenty-three minutes and nineteen seconds around z."

"Is that all?" asks the chairman. "Oh no," replied Mr. Cartesian, "there are other proportions in which the ingredients may be combined so as to produce a possible movement," and he was proceeding to state them when Mr. Commonsense interposed. "Stop! stop!" said he, "I can make nothing of all these figures. This jargon about x, y, and z may suffice for your calculations, but it fails to convey to my mind any clear or concise notion of the movements which the body is free to make."

Many of the committee sympathized with this view of Commonsense, and they came to the conclusion that there was nothing to be extracted from poor old Cartesian and his axes. They felt that there must be some better method, and their hopes of discovering it were raised when they saw Mr. Helix volunteer his services and advance to the rigid body. Helix brought with him no cumbrous rectangular axes, but commenced to try the mobility of the body in the simplest manner. He found it lying at rest in a position we may call A. Perceiving that it was in some ways mobile, he gave it a slight displacement to a neighbouring position, B. Contrast the procedure of Cartesian with the procedure of Helix. Cartesian tried to force the body to move along certain routes which he had arbitrarily chosen, but which the body had not chosen; in fact the body would not take any one of his routes separately, though it would take all of them together in the most embarrassing manner. But Helix had no preconceived scheme as to the nature of the movements to be expected. He simply found the body in a certain position, A, and then he coaxed the body to move, not in this particular way or in that particular way, but any way the body liked to any new position, B.

Let the constraints be what they may—let the position B lie anywhere in the close neighbourhood of A—Helix found that he could move the body from A to B by an extremely simple operation. With the aid of a skilful mechanic he prepared a screw with a suitable pitch, and adjusted this screw in a definite position. The rigid body was then attached by rigid bonds to a nut on this screw, and it was found that the movement of the body from A to B could be effected by simply turning the nut on the screw. A perfectly definite fact about the mobility of the body has thus been ascertained. It is able to twist to and fro on a certain screw.

Mr. Querulous could not see that there was any simplicity or geometrical clearness in the notion of a screwing movement; in fact he thought it was the reverse of simple. Did not the screwing movement mean a translation parallel to an axis *and* a rotation around that axis? Was it not better to think of the rotation and the translation separately than to jumble together two things so totally distinct into a composite notion?

But Querulous was instantly answered by One-to-One. "Lamentable, indeed," said he, "would be a divorce between the rotation and the translation. Together they form the unit of rigid movement. Nature herself has wedded them, and the fruits of their happy union are both abundant and beautiful."

The success of Helix encouraged him to proceed with the experiments, and speedily he found a second screw about which the body could also twist. He was about to continue when he was interrupted by Mr. Anharmonic, who said, "Tarry a moment, for geometry declares that a body free to twist about two screws is free to twist about a myriad of screws. These form the generators of a graceful ruled surface known as the cylindroid. There may be infinite variety in the conceivable constraints, but there can be no corresponding variety in the character of this surface. Cylindroids differ in size, they have no difference in shape. Let us then make a cylindroid of the right size, and so place it that two of its screws coincide with those you have discovered; then I promise you that the body can be twisted about every screw on the surface. In other words, if a body has two degrees of freedom the cylindroid is the natural and the perfect general method for giving an exact specification of its mobility."

A single step remained to complete the examination of the freedom of the body. Mr. Helix continued his experiments, and presently detected a third screw, about which the body can also twist in addition to those on the cylindroid. A flood of geometrical light then burst forth and illuminated the whole theory. It appeared that the body was free to twist about ranks upon ranks of screws all beautifully arranged by their pitches on a system of hyperboloids. After a brief conference with Anharmonic and One-to-One, Helix announced that sufficient experiments of this kind had now been made. By the single screw, the cylindroid, and the family of hyperboloids, every conceivable information about the mobility of the rigid body can be adequately conveyed. Let the body have any constraints, however elaborate, yet the definite geometrical conceptions just stated will be sufficient.

With perfect lucidity Mr. Helix expounded the matter to the committee. He exhibited to them an elegant fabric of screws, each with its appropriate pitch, and then he summarized his labours by saying, "About every one of these screws you can displace the body by twisting, and what is of no less importance it will not admit of any movement which is not such a twist." The committee expressed their satisfaction with this information. It was both clear and complete. Indeed, the chairman remarked with considerable force that *a more thorough method of specifying the freedom of the body was inconceivable.*

The discovery of the mobility of the body completed the first stage of the labours of the committee, and they were ready to commence the serious dynamical work. Force was now to be used, with the view of experimenting on the behaviour of the body under its influence. Elated by their previous success the committee declared that they would not rest satisfied until they had again obtained the most perfect solution of the most general problem.

"But what is force?" said one of the committee. "Send for Mr. Cartesian," said the chairman, "we will give him another trial." Mr. Cartesian was accordingly requested to devise an engine of the most ferocious description wherewith to attack the rigid body. He was promptly ready with a scheme, the weapons being drawn from his trusty but old-fashioned armoury. He would erect three rectangular axes, he would administer a tremendous blow parallel to each of these axes, and then he would simultaneously apply to the body a forcible couple around each of them; this was the utmost he could do.

"No doubt," said the chairman, "what you propose would be highly effective, but, Mr. Cartesian, do you not think that while you still retained the perfect generality of your attack, you might simplify your specification of it? I confess that these three blows all given at once at right angles to each other, and these three couples which you propose to impart at the same time, rather confuse me. There seems a want of unity somehow. In short, Mr. Cartesian, your scheme does not create a distinct geometrical image in my mind. We gladly acknowledge its suitability for numerical calculation, and we remember its famous achievements, but it is utterly inadequate to the aspirations of this committee. We must look elsewhere."

Again Mr. Helix stepped forward. He reminded the committee of the labours of Mathematician Poinsot, and then he approached the rigid body. Helix commenced by clearing away Cartesian's arbitrary scaffolding of rectangular axes. He showed how an attack of the most perfect generality could be delivered in a form that admitted of concise and elegant description. "I shall," he said, "administer a blow upon the rigid body from some unexpected direction, and at the same instant I shall apply a vigorous couple in a plane perpendicular to the line of the blow."

A happy inspiration here seized upon Mr. Anharmonic. He knew, of course, that the efficiency of a couple is measured by its moment—that is, by the product of a force and a linear magnitude. He proposed, therefore, to weld Poinsot's force and couple into the single conception of a *wrench* on a screw. The force would be directed along the screw while the moment of the couple would equal the product of the force and the pitch of the screw. "A screw," he said, "is to be regarded merely as a directed straight line with an associated linear magnitude called the pitch. The screw has for us a dual aspect of much significance. No small movement of the body is conceivable which does not consist of a twist about a screw. No set of forces could be applied to the body which were not equivalent to a wrench upon a screw. Every one remembers the two celebrated rules that forces are compounded like rotations and that couples are compounded like translations. These may now be replaced by the single but far more compendious rule which asserts that wrenches and twists are to be compounded by identical laws. Would you unite geometry with generality in your dynamics? It is by screws, and screws only, that you are enabled to do so."

These ideas were rather too abstract for Cartesian, who remarked that as D'Alembert's principle provided for everything in dynamics screws could not be needed. Mr. Querulous sought to confirm him by saying that he did not see how screws helped the study either of Foucault's Pendulum or of the Precession of the Equinoxes.

Such absurd observations kindled the intellectual wrath of One-to-One, who rose and said, "In the development of the natural philosopher two epochs may be noted. At the first he becomes aware that problems exist. At the second he discovers their solution. Querulous has not yet reached the first epoch, he cannot even conceive those problems which the 'Theory of Screws' proposes to solve. I may however inform him that the 'Theory of Screws' is not a general dynamical calculus. It is the discussion of a particular class of dynamical problems which do not admit of any other enunciation except that which the theory itself provides. Let us hope that ere our labours have ended Mr. Querulous may obtain some glimmering of the subject."

The chairman happily assuaged matters. "We must pardon," he said, "the vigorous language of our friend Mr. One-to-One. His faith in geometry is boundless. In fact he is said to believe that the only real existence in the universe is anharmonic ratio. It is also his opinion that if a man travelled sufficiently far along a straight line in one direction he will ultimately arrive at the point from which he started. The committee would be glad to see Mr. Querulous making the trial."

It was obvious that screws were indispensable alike for the application of the forces and for the observation of the movements. Special measuring instruments were devised by which the positions and pitches of the various screws could be carefully ascertained. All being ready the first experiment was commenced.

A screw was chosen quite at random, and a great impulsive wrench was administered thereon. In the infinite majority of cases this would start the body into activity, and it would commence to move in the only manner possible—*i.e.* it would begin to twist about some screw. It happened, however, that this first experiment was unsuccessful; the impulsive wrench failed to operate, or at all events the body did not stir. "I told you it would not do," shouted Querulous, though he instantly subsided when One-to-One glanced at him.

Much may often be learned from an experiment which fails, and the chairman sagaciously accounted for the failure, and in doing so directed the attention of the committee to an important branch of the subject. The mishap was due, he thought, to some reaction of the constraints which had neutralized the effect of the wrench. He believed it would save time in their future investigations if these reactions could be first studied and their number and position ascertained.

To this suggestion Mr. Cartesian demurred. He urged that it would involve an endless task. "Look," he said, "at the complexity of the constraints: how the body rests on these surfaces here; how it is fastened by links to those points there; how there are a thousand-and-one ways in which reactions might originate." Mr. Commonsense and other members of the committee were not so easily deterred, and they determined to work out the subject thoroughly. At first they did not see their way clearly, and much time was spent in misdirected attempts. At length they were rewarded by a curious and unexpected discovery, which suddenly rendered the obscure reactions perfectly transparent.

A trial was being made upon a body which had only one degree of freedom; was, in fact, only able to twist about a single screw, X. Another screw, Y, was speedily found, such that a wrench thereon failed to disturb the body. It now occurred to the committee to try the effect of interchanging the relation of these screws. They accordingly arranged that the body should be left only free to twist about Y, while a wrench was applied on X. Again the body did not stir. The importance of this fact immediately arrested the attention of the more intelligent observers, for it established the following general law: If a wrench on X fails to move a body only free to twist about Y, then a wrench on Y must be unable to move a body only free to twist about X. It was determined to speak of two screws when related in this manner as *reciprocal.*

Some members of the committee did not at first realize the significance of this discovery. Their difficulty arose from the restricted character of the experiments by which the law of reciprocal screws had been suggested. They said, "You have shown us that this law is observed in the case of a body only free to twist about one screw at a time; but how does this teach anything of the general case in which the body is free to twist about whole shoals of screws?" Mr. Commonsense immediately showed that the discovery could be enunciated in a quite unobjectionable form. "The law of reciprocal screws," he said, "does not depend upon the constraints or the limitations of the freedom. It may be expressed in this way: *Two screws are reciprocal when a small twist about either can do no work against a wrench on the other.*"

This important step at once brought into view the whole geometry of the reactions. Let us suppose that the freedom of the body was such that it could twist about all the screws of a system which we shall call U. Let all the possible reactions form wrenches on the screws of another system, V. It then appeared that every screw upon U is reciprocal to every screw upon V. A body might therefore be free to twist about every screw of V and still remain in equilibrium, notwithstanding the presence of a wrench on every screw of U. A body free to twist about all the screws of V can therefore be only partially free. Hence V must be one of those few types of screw system already discussed. It was, accordingly, found that the single screw, the cylindroid, and the set of hyperboloids completely described every conceivable reaction from the constraints just as they described every conceivable kind of freedom. The committee derived much encouragement from these discoveries; they felt that they must be following the right path, and that the bounty of Nature had already bestowed on them some earnest of the rewards they were ultimately to receive.

It was with eager anticipation that they now approached the great dynamical question. They were to see what would happen if the impulsive wrench were not neutralized by the reactions of the constraints. The body would then commence to move—that is, to twist about some screw which it would be natural to call the instantaneous screw. To trace the connexion between the impulsive screw and the corresponding instantaneous screw was the question of the hour. Before the experiments were commenced, some shrewd member remarked that the issue had not yet been presented with the necessary precision. "I understand," he said, "that when you apply a certain impulsive wrench, the body will receive a definite twist velocity about a definite screw; but the converse problem is ambiguous. Unless the body be quite free, there are myriads of impulsive screws corresponding to but one instantaneous screw." The chairman perceived the difficulty, and not in vain did he appeal to the geometrical instinct of Mr. One-to-One, who at once explained the philosophy of the matter, dissipated the fog, and disclosed a fresh beauty in the theory.

"It is quite true," said Mr. One-to-One, "that there are myriads of impulsive screws, any one of which may be regarded as the correspondent to a given instantaneous screw, but it fortunately happens that among these myriads there is always one screw so specially circumstanced that we may select it as *the* correspondent, and then the ambiguity will have vanished."

As several members were not endowed with the geometrical insight possessed by One-to-One, they called on him to explain how this special screw was to be identified; accordingly he proceeded:—"We have already ascertained that the constraints permit the body to be twisted about any screw of the system, U. Out of the myriads of impulsive screws corresponding to a single instantaneous screw it always happens that one, but never more than one, lies on U. This is the special screw. No matter where the impulsive wrench may lie throughout all the realms of space, it may always be exchanged for a precisely equivalent wrench lying on U. Without the sacrifice of a particle of generality, we have neatly circumscribed the problem. For one impulsive screw there is one instantaneous screw, and for one instantaneous screw there is one impulsive screw."

The experiments were accordingly resumed. An impulsive screw was chosen, and its position and its pitch were both noted. An impulsive wrench was administered, the body commenced to twist, and the instantaneous screw was ascertained by the motion of marked points. The body was brought to rest. A new impulsive screw was then taken. The experiment was again and again repeated. The results were tabulated, so that for each impulsive screw the corresponding instantaneous screw was shown.

Although these investigations were restricted to screws belonging to the system which expressed the freedom of the body, yet the committee became uneasy when they reflected that the screws of that system were still infinite in number, and that consequently they had undertaken a task of infinite extent. Unless some compendious law should be discovered, which connected the impulsive screw with the instantaneous screw, their experiments would indeed be endless. Was it likely that such a law could be found—was it even likely that such a law existed? Mr. Querulous decidedly thought not. He pointed out how the body was of the most hopelessly irregular shape and mass, and how the constraints were notoriously of the most embarrassing description. It was therefore, he thought, idle to search for any geometrical law connecting the impulsive screw and the instantaneous screw. He moved that the whole inquiry be abandoned. These sentiments seemed to be shared by other members of the committee. Even the resolution of the chairman began to quail before a task of infinite magnitude. A crisis was imminent—when Mr. Anharmonic rose.

"Mr. Chairman," he said, "Geometry is ever ready to help even the most humble inquirer into the laws of Nature, but Geometry reserves her most gracious gifts for those who interrogate Nature in the noblest and most comprehensive spirit. That spirit has been ours during this research, and accordingly Geometry in this our emergency places her choicest treasures at our disposal. Foremost among these is the powerful theory of homographic systems. By a few bold extensions we create a comprehensive theory of homographic screws. All the impulsive screws form one system, and all the instantaneous screws form another system, and these two systems are homographic. Once you have realized this, you will find your present difficulty cleared away. You will only have to determine a few pairs of impulsive and instantaneous screws by experiment. The number of such pairs need never be more than seven. When these have been found, the homography is completely known. The instantaneous screw corresponding to every impulsive screw will then be completely determined by geometry both pure and beautiful." To the delight and amazement of the committee, Mr. Anharmonic demonstrated the truth of his theory by the supreme test of fulfilled prediction. When the observations had provided him with a number of pairs of screws, one more than the number of degrees of freedom of the body, he was able to predict with infallible accuracy the instantaneous screw corresponding to any impulsive screw. Chaos had gone. Sweet order had come.

A few days later the chairman summoned a special meeting in order to hear from Mr. Anharmonic an account of a discovery he had just made, which he believed to be of signal importance, and which he was anxious to demonstrate by actual experiment. Accordingly the committee assembled, and the geometer proceeded as follows:—

"You are aware that two homographic ranges on the same ray possess two double points, whereof each coincides with its correspondent; more generally when each point in space, regarded as belonging to one homographic system, has its correspondent belonging to another system; then there are four cases in which a point coincides with its correspondent. These are known as the four double points, and they possess much geometrical interest. Let us now create conceptions of an analogous character suitably enlarged for our present purpose. We have discovered that the impulsive screws and the corresponding instantaneous screws form two homographic systems. There will be a certain limited number (never more than six) of double screws common to these two systems. As the double points in the homography of point systems are fruitful in geometry, so the double screws in the homography of screw systems are fruitful in dynamics."

A question for experimental inquiry could now be distinctly stated. Does a double screw possess the property that an impulsive wrench delivered thereon will make the body commence to move by twisting about the same screw? This was immediately tested. Mr. Anharmonic, guided by the indications of homography, soon pointed out the few double screws. One of these was chosen; a vigorous impulsive wrench was imparted thereon. The observations were conducted as before: the anticipated result was triumphantly verified, for the body commenced to twist about the identical screw on which the wrench was imparted. The other double screws were similarly tried, and with a like result. In each case the instantaneous screw was identical both in pitch and in position with the impulsive screw.

"But surely," said Mr. Querulous, "there is nothing wonderful in this. Who is surprised to learn that the body twists about the same screw as that on which the wrench was administered? I am sure I could find many such screws. Indeed, the real wonder is not that the impulsive screw and the instantaneous screw are ever the same, but that they are ever different."

And Mr. Querulous proceeded to illustrate his views by experiments on the rigid body. He gave the body all sorts of impulses, but, in spite of all his endeavours, the body invariably commenced to twist about some screw which was *not* the impulsive screw. "You may try till Doomsday," said Mr. Anharmonic, "you will never find any besides the few I have indicated."

It was thought convenient to assign a name to these remarkable screws, and they were accordingly designated the *principal screws of inertia*. There are, for example, six principal screws of inertia when the body is perfectly free, and two when the body is free to twist about the screws of a cylindroid. The committee regarded the discovery of the principal screws of inertia as the most remarkable result they had yet obtained.

Mr. Cartesian was very unhappy. The generality of the subject was too great for his comprehension. He had an invincible attachment to the x, y, z, which he regarded as the *ne plus ultra* of dynamics. "Why will you burden the science," he sighs, "with all these additional names? Can you not express what you want without talking about cylindroids, and twists, and wrenches, and impulsive screws, and instantaneous screws, and all the rest of it?" "No," said Mr. One-to-One, "there can be no simpler way of stating the results than that natural method we have followed. You would not object to the language if your ideas of the natural phenomena had been sufficiently capacious. We are dealing with questions of perfect generality, and it would involve a sacrifice of generality were we to speak of the movement of a body except as a twist, or of a system of forces except as a wrench."

"But," said Mr. Commonsense, "can you not as a concession to our ignorance tell us something in ordinary language which will give an idea of what you mean when you talk of your 'principal screws of inertia?' Pray for once sacrifice this generality you prize so much and put the theory into some extreme shape that ordinary mortals can understand."

Mr. Anharmonic would not condescend to comply with this request, so the chairmrn called upon Mr. One-to-One, who somewhat ungraciously consented. "I feel," said he, "the request to be an irritating one. Extreme cases generally make bad illustrations of a general theory. That zero multiplied by infinity may be anything is surely not a felicitous exhibition of the perfections of the multiplication table. It is with reluctance that I divest the theory of its flowing geometrical habit, and present it only as a stiff conventional guy from which true grace has departed.

"Let us suppose that the rigid body, instead of being constrained as heretofore in a perfectly general manner, is subjected merely to a special type of constraint. Let it, in fact, be only free to rotate around a fixed point. The beautiful fabric of screws, which so elegantly expressed the latitude permitted to the body before, has now degenerated into a mere horde of lines all stuck through the point. Those varieties in the pitches of the screws which gave colour and richness to the fabric have also vanished, and the pencil of degenerate screws have a monotonous zero of pitch. Our general conceptions of mobility have thus been horribly mutilated and disfigured before they can be adapted to the old and respectable problem of the rotation of a rigid body about a fixed point. For the dynamics of this problem the wrenches assume an extreme and even monstrous type. Wrenches they still are, as wrenches they ever must be, but they are wrenches on screws of infinite pitch; they have ceased to possess definite screws as homes of their own. We often call them couples.

"Yet so comprehensive is the doctrine of the principal screws of inertia that even to this extreme problem the theory may be applied. The principal screws of inertia reduce in this special case to the three principal axes drawn through the point. In fact, we see that the famous property of the principal axes of a rigid body is merely a very special application of the general theory of the principal screws of inertia. Everyone who has a particle of mathematical taste lingers with fondness over the theory of the principal axes. Learn, therefore," says One-to-One in conclusion, "how great must be the beauty of a doctrine which comprehends the theory of principal axes as the merest outlying detail."

Another definite stage in the labours of the committee had now been reached, and accordingly the chairman summarized the results. He said that a geometrical solution had been obtained of every conceivable problem as to the effect of impulse on a rigid body. The impulsive screws and the corresponding instantaneous screws formed two homographic systems. Each screw in one system determined its corresponding screw in the other system, just as in two anharmonic ranges each point in one determines its correspondent in the other. The double screws of the two homographic systems are the principal screws of inertia. He remarked, in conclusion, that the geometrical theory of homography and the present dynamical theory mutually illustrated and interpreted each other.

There was still one more problem which had to be brought into shape by geometry, and submitted to the test of experiment.

The body is lying at rest though gravity and many other forces are acting upon it. These forces constitute a wrench which must lie upon a screw of the reciprocal system, inasmuch as it is neutralized by the reaction of the constraints. Let the body be displaced from its initial position by a small twist. The wrench will no longer be neutralized by the reaction of the constraints; accordingly when the body is released it will commence to move. So far as the present investigations are concerned these movements are small oscillations. Attention was therefore directed to these small oscillations. The usual observations were made, and Helix reported them to be of a very perplexing kind. "Surely," said the chairman, "you find the body twisting about some screw, do you not?" "Undoubtedly," said Helix; "the body can only move by twisting about some screw; *but*, unfortunately, this screw is not fixed, it is indeed moving about in such an embarrassing manner that I can give no intelligible account of the matter." The chairman appealed to the committee not to leave the interesting subject of small oscillations in such an unsatisfactory state. Success had hitherto guided their efforts. Let them not separate without throwing the light of geometry on this obscure subject.

Mr. Querulous here said he must be heard. He protested against further waste of time; there was nothing for them to do. Everybody knew how to investigate small oscillations; the equations were given in every book on mechanics. You had only to write down these equations, and scribble away till you got out something or other. But the more intelligent members of the committee took the same view as the chairman. They did not question the truth of the formulæ which to Querulous seemed all-sufficient, but they wished to see what geometry could do for the subject. Fortunately this view prevailed, and new experiments were commenced under the direction of Mr. Anharmonic. He first quelled the elaborate oscillations which had so puzzled the committee; he reduced the body to rest, and then introduced the subject as follows:—

"The body now lies at rest. I displace it a little, and I hold it in its new position. The wrench, which is the resultant of all the varied forces acting on the body, is no longer completely neutralized by the reactions of the constraints. Indeed, I can feel it in action. Our apparatus will enable us to measure the intensity of this wrench, and to determine the screw on which it acts."

A series of experiments was then made, in which the body was displaced by a twist about a screw, which was duly noted, while the corresponding evoked wrench was determined. The pairs of screws so related were carefully tabulated. When we remember the infinite complexity of the forces, of the constraints and of the constitution of the body, it might seem an endless task to determine the connexion between the two systems of screws. Here Mr. Anharmonic pointed out how exactly modern geometry was adapted to supply the wants of dynamics. The two screw systems were homographic, and when a number of pairs, one more than the degrees of freedom of the body, had been found, all was determined. This statement was put to the test. Again and again the body was displaced in some new fashion, but again and again did Mr. Anharmonic predict the precise wrench which would be required to maintain the body in its new position.

"But," said the chairman, "are not these purely statical results. How do they throw light on those elaborate oscillations which seem at present so inexplicable?"

"This I shall explain," said Anharmonic; "but I beg of you to give me your best attention, for I think the theory of small oscillations will be found worthy of it.

"Let us think of any screw, α, belonging to the system U, which expresses the freedom of the body. If α be an instantaneous screw, there will of course be a corresponding impulsive screw, θ, also on U. If the body be displaced from a position of equilibrium by a small twist about α, then the uncompensated forces produce a wrench, ϕ, which, without loss of generality, may also be supposed to lie on U. According as the screw α moves over U so will the two corresponding screws θ and ϕ also move over U. The system represented by α is homographic with both the systems of θ and of ϕ respectively. But two systems homographic with the same system are homographic with each other. Accordingly the θ system and the ϕ system are homographic. There will therefore be a certain number of double screws (not more than six) common to the systems θ and ϕ. Each of these double screws will of course have its correspondent in the α system, and we may call them α_1, α_2, &c., their number being equal to the degrees of freedom of the body. These screws are most curiously related to the small oscillations. We shall first demonstrate by experiment the remarkable property they possess."

The body was first brought to rest in its position of equilibrium. One of the special screws α having been carefully determined both in position and in pitch, the body was displaced by a twist about this screw and was then released. As the forces were uncompensated, the body of course commenced to move, but the oscillations were of unparalleled simplicity. With the regularity of a pendulum the body twisted to and fro on this screw, just as if it were actually constrained to this motion alone. The committee were delighted to witness a vibration so graceful, and, remembering the complex nature of the ordinary oscillations, they appealed to Mr. Anharmonic for an explanation. This he gladly gave, not by means of complex formulæ, but by a line of reasoning that was highly commended by Mr. Commonsense, and such that even Mr. Querulous could understand.

"This pretty movement," said Mr. Anharmonic, "is due to the nature of the screw α_1. Had I chosen any screw at random, the oscillations would, as we have seen, be of a very complex type; for the displacement will always evoke an uncompensated wrench, in consequence of which the body will commence to move by twisting about the instantaneous screw corresponding to that wrench; and of course this instantaneous screw will usually be quite different from the screw about which the displacement was made. But you will observe that α_1 has been chosen as a screw in the instantaneous system, corresponding to one of the double screws in the θ and ϕ systems. When the body is twisted about α_1 a wrench is evoked on the double screw, but as α_1 is itself the instantaneous screw, corresponding to the double screw, the only effect of the wrench will be to make the body twist about α_1. Thus we see that the body will twist to and fro on α_1 for ever. Finally, we can show that the most elaborate oscillations the body can possibly have may be produced by compounding the simple vibrations on these screws α_1, α_2, &c."

Great enlightenment was now diffused over the committee, and even Mr. Querulous began to think there must be something in it. Cordial unanimity prevailed among the members, and it was appropriately suggested that the screws of simple vibration should be called *harmonic screws*. This view was adopted by the chairman, who said he thought he had seen a similar expression in "Thomson and Tait."

The final meeting showed that real dynamical enthusiasm had been kindled in the committee. Vistas of great mathematical theories were opened out in many directions. One member showed how the theory of screws could be applied not merely to a single rigid body but to any mechanical system whatever. He sketched a geometrical conception of what he was pleased to call a *screw-chain*, by which he said he could so bind even the most elaborate system of rigid bodies that they would be compelled to conform to the theory of screws. Nay, soaring still further into the empyrean, he showed that all the instantaneous motions of every molecule in the universe were only a twist about one screw-chain while all the forces of the universe were but a wrench upon another.

Mr. One-to-One expounded the "Ausdehnungslehre," and showed that the theory of screws was closely related to parts of Grassman's great work; while Mr. Anharmonic told how Plücker, in his celebrated "Neue Geometrie des Raumes," had advanced some distance towards the theory of screws, but still had never touched it.

The climax of mathematical eloquence was attained in the speech of Mr. Querulous, who, with new-born enthusiasm, launched into appalling speculations. He had evidently been reading his "Cayley," and had become conscious of the poverty of geometrical conception arising from our unfortunate residence in a space of an arbitrary and unsymmetrical description.

"Three dimensions," he said, "may perhaps be enough for an intelligent geometer. He may get on fairly well without a four-dimensioned space, but he does most heartily remonstrate against a flat infinity. Think of infinity," he cries, "as it should be, perhaps even as it is. Talk not of your scanty straight line at infinity and your miserable pair of circular points. Boldly assert that infinity is an ample quadric, and not the mere ghost of one; and then geometry will become what geometry ought to be. Then will every twist resolve itself into a right vector and a left vector, as the genius of Clifford proved. Then will the 'theory of screws' shed away some few adhering deformities, and fully develop its shapely proportions. Then will——" But here the chairman said he feared the discussion was beginning to enter rather wide ground. For his part he was content with the results of the experiments, even though they had been conducted in the vapid old space of Euclid. He reminded them that their labours were now completed, for they had ascertained everything relating to the rigid body which had been committed to them. He hoped they would agree with him that the inquiry had been an instructive one. They had been engaged in the study of Nature. They had approached the problems in the true philosophical spirit, and the rewards they had obtained proved that

> "Nature never did betray
> The heart that truly loved her."

NOTES.

At a public meeting held on Tuesday in Newcastle, under the presidency of the Mayor, Sir B. L. Brown, it was finally decided, on the motion of the Sheriff, Alderman W. H. Stephenson, seconded by Prof. Philipson, head of the medical staff at the Royal Infirmary, that a cordial invitation should be sent to the British Association to hold their annual meeting in Newcastle in 1889. It was stated that the necessary amount to cover expenses would be £4000, and of this £1700 had been already subscribed.

The New York meeting of the American Association for the Advancement of Science seems to have been very successful, although the attendance was not so large as had been expected. The next meeting will be held in Cleveland, O. An invitation from Toronto came just too late. The following are the officers for the next meeting:—President, J. W. Powell, of Washington; Vice-Presidents, Ormond Stone, of the University of Virginia, (Mathematics and Astronomy), A. A. Michelson, of Cleveland, (Physics), C. E. Munroe, of Newport, (Chemistry), Calvin M. Woodward, of St. Louis, (Mechanical Science), George H. Cook, of New Brunswick, (Geology and Geography), C. V. Riley, of Washington, (Biology), C. C. Abbot, of Trenton, (Anthropology), C. W. Smiley, of Washington, (Economic Science and Statistics); Permanent Secretary, F. W. Putnam, of Cambridge, (office, Salem, Mass.); General Secretary, J. C. Arthur, of La Fayette; Secretary of the Council, C. Leo Mees, of Athens; Secretaries of the Sections, C. L. Doolittle, of Bethlehem, (Mathematics and Astronomy), A. L. Kimball, of Baltimore, (Physics), William L. Dudley, of Nashville, (Chemistry), Arthur Beardsley, of Swarthmore, (Mechanical Science), George H. Williams, of Baltimore, (Geology and Geography), N. L. Britton, of New York, (Biology), Frank Baker, of Washington, (Anthropology), Charles S. Hill, of Washington, (Economic Science and Statistics).

The twenty-fourth annual meeting of the British Pharmaceutical Conference was opened on Tuesday in the Chemical Theatre of Owens College, Manchester. There was a large attendance of members of the Association. Mr. S. R. Atkins, of Salisbury, occupied the chair, and in his presidential address invited the attention of the Conference to "a brief review of the Victorian era as it more especially affected themselves as pharmacists."

The International Astronomical Congress met at Kiel on Monday, in the large hall of the University, under the presidency of Privy Councillor Dr. Auwers, of Berlin. There was a large assembly of astronomers, including delegates from Austria, France, Sweden and Norway, and America. The delegates were received on behalf of the Government by Herr Steinmann, Civil Governor of the province of Schleswig-Holstein, and on the part of the University by the Rector, Prof. Harsen. Dr. Auwers, in replying, thanked the Prussian Government for the interest which it had manifested in the Congress.

The Hygienic Congress, which will meet in Vienna next month, will be attended by over 1400 delegates from all countries. The programme includes excursions to the Kahlenberg, the Semmering, Buda-Pesth, and Abbazzia.

The Academy of Aërostation of France has presented a medal to M. Mendeleieff, in recognition of the pluck exhibited by him at Klin on August 19, when he went up alone