suddenly receded, leaving the rocks in the neighbourhood dry. The level of the water, it is said, fell to the extent of four or five feet. About a quarter of an hour after, the sea regained its former level with extreme violence, causing the *Touareg* and other vessels to roll frightfully. An alternate lowering and rising of the sea-level continued till 6 p.m. By 7 p.m. all had disappeared. On the morning of the 28th, however, there were still strong currents. In its issue of August 31, the same paper reports that, while traversing the pass between Round Island and the Coin de Mire, on the 27th, a Government boat, though running before a strong breeze, was stopped by a current from the opposite direction, and that the *patron* observed the sea receding precipitately from the vicinity of Gabriel Island, leaving the reefs dry, but that in a few minutes they were, by a sudden reflux, covered with water to the depth of six feet. The *Mercantile Record* of August 30 reported that on the 27th the sea in the Trou Fanfaron went down every twelve minutes, leaving all the boats moored in front of the harbour workshed dry, and then immediately rose again. The *Touareg* and *Stella* seemed to be in a boiling sea. On the 27th the sea in Tombeau Bay suddenly fell five feet below its usual level, and fish were caught on the dry beach. A quarter of an hour later the sea returned and rose nearly five feet above its ordinary level. Similar phenomena were observed at the Morne Brabant. On the same day, remarkable atmospheric and magnetic disturbances were recorded at the Royal Alfred Observatory, Pamplemousses. It would thus appear that from at least Flat Island, about eight miles north of the mainland of Mauritius, to Port Louis on the west coast, and thence round by the Morne to Souillac on the south coast, a distance in all of about forty-six miles, an unusual perturbation occurred with regard to the level and motion of the sea-water, and that on the same day meteorological and magnetic perturbations were recorded at the Observatory. The interest created by these occurrences was heightened by the report that vessels which arrived from the eastward on August 28 had passed through fields of pumice-stone.

Mr. Meldrum then gives a short account of what happened at the Observatory, and relates what has been told him by eye-witnesses of what occurred in the harbour and elsewhere.

1. *Barograms.*—The barogram sheet for the forty-eight hours ending at 8 a.m. on the 28th shows a remarkable disturbance in the atmospheric pressure between 11 a.m. and 5 p.m. on the 27th. Under ordinary conditions the barometer invariably falls from a maximum at about 9.30 a.m. to a minimum at about 3.30 p.m. But on August 27 last this was not the case. Soon after 11 a.m. a slight disturbance began, as indicated by small successive indentations in the barogram. At 11.55 a.m. the mercury stood at 29·996 inches, and at 0.06 p.m. at 29·918; it then rose to 29·961 at 0.20 p.m., after which it fell to 29·916 at 1.10 p.m. From 1.10 to 3.00 p.m. it rose, and at the latter hour stood at 29·968. In the interval from 2.50 to 3.55 p.m. there were five wavelets, and the mean interval between their lowest points was 16 minutes. Upon the whole, however, the mercury continued to rise after 1.10 p.m. The sudden fall from 11.55 a.m. to 0.06 p.m., and the rise from 0.06 to 0.20 p.m., are shown by a projecting peak. This peak, the undulations from 2.50 to 3.55 p.m., and the fact that the minimum occurred fully two hours earlier than usual, are the principal characteristics of the disturbance. After 5 p.m. there was no trace of disturbance.

A smaller disturbance occurred between 9 p.m. and midnight on the 28th.

2. *Magnetograms.*—Towards 9 a.m. on the 27th the north end of the declination magnet began to move towards the west, at first slowly and then more rapidly, and at 11 a.m. it attained its westerly maximum position, the movement since 9 a.m. amounting to 7′ 37″ of arc. An easterly movement then set in, and continued till 0h. 15m. p.m., the north end being then 13′ 18″ east of its position at 11 a.m. A slight westerly movement of 3′ 18″ then occurred up to 1h. 20m., after which there was a rapid movement towards the east till 2 p.m., the decrease in declination since 1h. 20m. being 10′ 37″. The magnet then moved towards the west, recovering its normal position about 5 p.m., and all traces of disturbance ceased. From 10 to 11 a.m., and especially from 11 a.m. to 1 p.m., there were several minor oscillations. The extreme range from the maximum westerly position at 11 a.m. to the maximum easterly position at 2 p.m. was 20′ 43″.

The dip, or vertical force magnetometer, as indicated by the curve, shows traces of disturbance between 8h. 15m. and 11h. a.m. on the 27th. At the latter hour a rapid decrease of force began and continued till 0h. 15m. p.m., the decrease amounting in parts of force to ·00086, and the south end of the magnet moving upward through an angle of 16′ 07″. From 0h. 15m. to 1h. 20m. a slight increase of force took place, amounting to ·00027, the south end of the magnet dipping to the extent of 5′ 06″. After 1h. 20m. a very rapid decrease set in, which continued till 1.50 p.m., the decrease amounting to ·00083, and the south end of the magnet moving upward through an angle of 15′ 38″. The force then gradually increased and recovered its normal value at 6 p.m., by which time it had increased to the extent of ·00104 parts of force since 1h. 50m., the south end of the magnet moving downward through an angle of 19′ 36″. The total decrease from 11 a.m. to 1h. 50m. p.m. was ·00142, during which interval the range of angular movement was 26′ 40″. There were several minor oscillations, particularly between 11 a.m. and 0h. 40m. p.m.

The horizontal force curve also shows a well-marked disturbance, but it was less than in the case of the other curves.

The principal features of these disturbances were the unusually large ranges of the movements of the magnets, and the differences between the epochs of maximum and minimum and the average epochs.

Magnetic disturbances occurred also on August 28 and 29.

Disturbances in the Trou Fanfaron.—Capt. Ferrat states that at some time between 1.30 and 2 p.m. on the 27th the water rushed inwards from the harbour with great violence, and rose above its former level to the extent of fully three feet. An alternate ebb and flow then continued till nearly 7 p.m., the intervals in time between low and high water being about 15 minutes. There was no wave or billow, but a strong current, the estimated velocity of which was about three knots in ten minutes, or eighteen knots an hour. The current appeared to be strongest towards evening. Similar disturbances, but less marked, were observed on the morning of the 28th.

On the opposite side of the Trou Fanfaron another observer noticed, about 2 p.m. on the 27th, that around the *Stella*, which was moored within about 25 yards of him, the water had a "boiling appearance." The water then receded about 20 feet from the shore, leaving some boats near him partly on dry land. About a quarter of an hour after the water rushed back and advanced about six feet farther inland than where it was at 2 p.m. The water then receded, and a series of oscillations took place till about 6 p.m., the intervals between high and low water being from 15 to 20 minutes, and the extent of rise and fall which was at first about three feet, becoming less and less after 4 p.m.

These statements accord with others previously made to the Hon. Mr. Connal.

Similar phenomena, but of a less violent character, occurred between 2.30 and 6 p.m. on the 28th.

The Trou Fanfaron, it may be remarked, is a narrow inlet on the north-east side of the harbour. Near its mouth, or entrance, its direction is south-west and north-east; it then turns towards the east, and throughout the greater part of its length (about 1600 feet) it runs nearly east and west. Its breadth is generally from 200 to 300 feet.

Similar disturbances were observed at Arsenal and Tombeau Bays.

On August 11 and 12 the *Idomene*, in 6° to 8° S., and in 88° E., passed through fields of pumice-stone, which may have been ejected from a volcano near the Straits of Sunda. At all events, that pumice-stone had no immediate connection with what took place in Mauritius.

"I will at present," Mr. Meldrum concludes, "make no attempt to explain the phenomena in question. Magnetic disturbances, properly so-called, are not produced by earthquakes, but are generally ascribed, in a measure at least, to cosmical causes. The disturbances of the magnets on this occasion, however, may have been mechanical effects of earth-waves, although no permanent change of level took place. The difficulty is to refer all the phenomena to the same cause."

THE MOTION OF WATER[1]

1. *Objects and Results of the Investigation.*

THE results of this investigation have both a practical and a philosophical aspect.

[1] "An Experimental Investigation of the Circumstances which Determine whether the Motion of Water shall be Direct or Sinuous, and of the Law of Resistance in Parallel Channels." Abstract of Paper read at the Royal Society by Prof. Osborne Reynolds, F.R.S.

In their practical aspect they relate to the *law of resistance to the motion of water in pipes*, which appears in a new form, the law for all velocities and all diameters being represented by an equation of two terms.

In their philosophical aspect these results relate to the fundamental principles of fluid motion; inasmuch as they afford for the case of pipes a definite verification of two principles, which are *that the general character of the motion of fluids in contact with solid surfaces depends on the relation* (1) *between the dimensions of the space occupied by the fluid and a linear physical constant of the fluid;* (2) *between the velocity and a physical velocity constant of the fluid.*

The results as viewed in their philosophical aspect were the primary object of the investigation.

As regards the practical aspects of the results it is not necessary to say anything by way of introduction; but in order to render the philosophical scope and purpose of the investigation intelligible it is necessary to describe shortly the line of reasoning which determined the order of investigation.

2. *The Leading Features of the Motion of Actual Fluids.*—Although in most ways the exact manner in which water moves is difficult to perceive, and still more difficult to define, as are also the forces attending such motion, certain general features both of the forces and motions stand prominently forth as if to invite or defy theoretical treatment.

The relations between the resistance encountered by, and the velocity of a solid body moving steadily through, a fluid in which it is completely immersed, or of water moving through a tube, present themselves mostly in one or other of two simple forms. The resistance is generally proportional to the square of the velocity, and when this is not the case it takes a simpler form, and is proportional to the velocity.

Again, the internal motion of water assumes one or other of two broadly distinguishable forms—either the elements of the fluid follow one another along lines of motion which lead in the most direct manner to their destination, or they eddy about in sinuous paths, the most indirect possible.

3. *Connection between the Leading Features of Fluid Motion.*—These leading features of fluid motion are well known, and are supposed to be more or less connected, but it does not appear that hitherto any very determined efforts have been made to trace a definite connection between them, or to trace the characteristics of the circumstances under which they are usually presented.

Certain circumstances have been definitely associated with the particular laws of force. Resistance as the square of the velocity is associated with motion in tubes of more than capillary dimensions, and with the motion of the bodies through the water at more than insensibly small velocities, while resistance as the velocity is associated with capillary tubes and small velocities.

The equations of hydrodynamics, although they are applicable to *direct motion*, *i.e.* without eddies, and show that then the resistance is as the velocity, have hitherto thrown no light on the circumstances on which such motion depends. And although of late years these equations have been applied to the theory of the eddy, they have not been in the least applied to the motion of water, which is a mass of eddies, *i.e.* in *sinuous motion*, nor have they yielded a clue to the cause of resistance varying as the square of the velocity. Thus, while as applied to waves and the motion of water in capillary tubes the theoretical results agree with the experimental, the theory of hydrodynamics has so far failed to afford the slightest hint why it should explain these phenomena, and signally failed to explain the law of resistance encountered by large bodies moving at sensibly high velocities through water, or that of water in sensibly large pipes.

This accidental fitness of the theory to explain certain of the phenomena, while entirely failing to explain others, affords strong presumption that there are some fundamental principles of fluid motion of which due account has not been taken in the theory; and several years ago it seemed to me that a careful examination as to the connection between these four leading features, together with the circumstances on which they severally depend, was the most likely means of finding the clue to the principles overlooked.

4. *Space and Velocity.*—The definite association of resistance as the square of the velocity with sensibly large tubes and high velocities, and of resistance as the velocity with capillary tubes and slow velocities, seemed to be evidence of the very general and important influence of some properties of fluids not recognised in the theory of hydrodynamics.

As there is no such thing as absolute space or absolute time recognised in mechanical philosophy, to suppose that the character of motion of fluids in any way depended on absolute size or absolute velocity would be to suppose such motion outside the pale of the laws of motion. If, then, fluids, in their motions, are subject to these laws, what appears to be the dependence of the character of the motion on the absolute size of the tube and on the absolute velocity of the immersed body must in reality be a dependence on the size of the tube as compared with the size of some other object, and on the velocity of the body as compared with some other velocity. What is the standard object and what the standard velocity which come into comparison with the size of the tube and the velocity of an immersed body are questions to which the answers were not obvious. Answers, however, were found in the discovery of a circumstance on which sinuous motion depends.

5. *The Effect of Viscosity on the Character of Fluid Motion.*—The small evidence which clear water shows as to the existence of internal eddies, not less than the difficulty of estimating the viscous nature of the fluid, appears to have hitherto obscured the very important circumstance that *the more viscous a fluid is the less prone is it to eddying or sinuous motion.* To express this definitely, if μ is the viscosity and ρ the density of the fluid, for water $\frac{\mu}{\rho}$ diminishes rapidly as the temperature rises; thus at 5° C. $\frac{\mu}{\rho}$ is double what it is at 45° C. What I observed was that the tendency of water to eddy becomes much greater as the temperature rises.

Hence, connecting the change in the law of resistance with the birth and development of eddies, this discovery limited further search for the standard distance and standard velocity to the physical properties of the fluid.

To follow the line of this search would be to enter upon a molecular theory of liquids, and this is beyond my present purpose. It is sufficient here to notice the well-known fact that $\frac{\mu}{\rho}$ is a quantity of the nature of a product of a distance and a velocity.

6. *Evidence from the Equations of Motion.*—In this article it is pointed out that the equations of motion afford definite evidence of a dependence of the dynamical equilibrium of a fluid on the value of $\frac{c\rho U}{\mu}$, c being the diameter of the pipe and U the mean velocity of the fluid.

7. *The Cause of Eddies.*—There appeared to be two possible causes for the change of direct motion into sinuous. These are best discussed in the language of hydrodynamics; but as the results of this investigation relate to both these causes, which, although the distinction is subtile, are fundamentally distinct and lead to distinct results, it is necessary that they should be indicated.

The general cause of the change from steady to eddying motion was, in 1843, pointed out by Prof. Stokes as being that, under certain circumstances, the steady motion becomes unstable, so that an indefinitely small disturbance may lead to a change to sinuous motion. Both the causes above referred to are of this kind, and yet they are distinct; the distinction lying in the part taken in the instability by viscosity. If we imagine a fluid free from viscosity and absolutely free to glide over solid surfaces, then comparing such a fluid with a viscous fluid in exactly the same motion—

(1.) The frictionless fluid might be unstable and the viscous stable. Under these circumstances the cause of eddies is the instability as a perfect fluid, the effect of viscosity being in the direction of stability.

(2.) The frictionless fluid might be stable, and the viscous fluid unstable; under which circumstances the cause of instability would be the viscosity.

It was clear to me that the conclusion I had drawn from the equations of motion immediately related only to the first cause. Nor could I then perceive any possible way in which instability could result from viscosity. All the same I felt a certain amount of uncertainty in assuming the first cause of instability to be general. This uncertainty was the result of various considerations, but particularly from my having observed that eddies apparently come on in very different ways, according to a very definite circumstance of motion, which may be illustrated.

When in a channel the water is all moving in the same direction, the velocity being greatest in the middle, and diminishing to zero at the sides, as indicated by the curve in Fig. 1, eddies showed themselves reluctantly and irregularly; whereas when the water on one side of the channel was moving in the opposite direction to that on the other, as shown by the curve in Fig. 2, eddies appeared in the middle regularly and readily.

8. *Methods of Investigation.*—There appeared to be two ways of proceeding, the one theoretical, the other practical.

The theoretical method involved the integration of equations for unsteady motion in a way that had not then been accomplished, and which, considering the general intractability of the equations, was not promising.

The practical method was to test the relation between U, $\frac{\mu}{\rho}$, and c; this, owing to the simple and definite form of the law, seemed to offer, at all events in the first place, a far more promising field of research.

The law of motion in a straight, smooth tube offered the simplest possible circumstances and the most crucial test.

The existing experimental knowledge of the resistance of

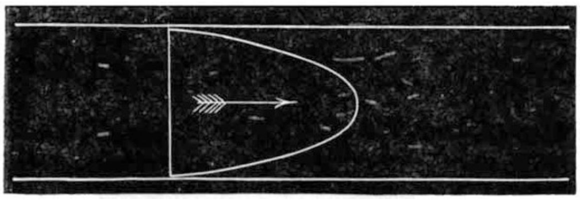


FIG. 1.

water in tubes, although very extensive, was in one important respect incomplete. The previous experiments might be divided into two classes—(1) those made under circumstances in which the law of resistance was as the square of the velocity, and (2) those made under circumstances in which the resistance varied as the velocity. There had not apparently been any attempt made to determine the exact circumstances under which the change of law took place.

Again, although it had been definitely pointed out that eddies would explain the resistance as the square of the velocity, it did not appear that any definite experimental evidence of the existence of eddies in parallel tubes had been obtained, and much less was there any evidence as to whether the birth of eddies was simultaneous with the change in the law of resistance.

These open points may be best expressed in the form of queries to which the answers anticipated were in the affirmative.

(1.) What was the exact relation between the diameters of the pipes and the velocities of the water at which the law of resistance changed; was it at a certain value of c U?

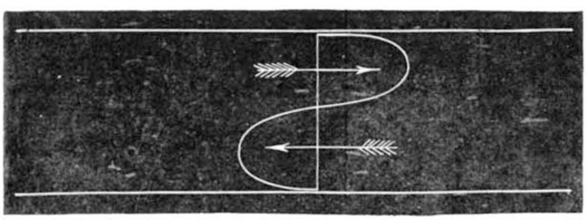


FIG. 2.

(2.) Did this change depend on the temperature, *i.e.* the viscosity of water; was it at a certain value of $\frac{U}{\mu}$?

(3.) Were there eddies in parallel tubes?

(4.) Did steady motion hold up to a critical value and then eddies come in?

(5.) Did the eddies come in at a certain value of $\frac{\rho c U}{\mu}$?

(6.) Did the eddies first make their appearance as small, and then increase gradually with the velocity, or did they come in suddenly?

The bearing of the last query may not be obvious; but, as will appear in the sequel, its importance was such that, in spite of satisfactory answers to all the other queries, a negative answer to this in respect of one particular class of motions led to the reconsideration of the supposed cause of instability, and eventually to the discovery of instability caused by fluid friction.

The queries as they are put suggest two methods of experimenting:—

(1.) Measuring the resistances and velocities for different diameters, and with different temperatures of water.

(2.) Visual observation as to the appearance of eddies during the flow of water along tubes or open channels.

Both these methods have been adopted, but as the question relating to eddies had been the least studied, the second method was the first adopted.

9. *Experiments by Visual Observations.*—The most important of these experiments related to water moving in one direction along glass tubes. Besides these, however, experiments on fluids flowing in opposite directions in the same tube were made; also a third class of experiments which related to motion in a flat channel of indefinite breadth.

These last-mentioned experiments resulted from an incidental observation during some experiments made in 1876 as to the effect of oil to prevent wind waves. As the result of this observation had no small influence in directing the course of this investigation, it may be well to describe it first.

10. *Eddies caused by the Wind beneath the Oiled Surface of Water.*—A few drops of oil on the windward side of a pond during a stiff breeze having spread over the pond and completely calmed the surface as regards waves, the sheet of oil, if it may be so called, was observed to drift before the wind, and it was then particularly noticed that close to, and at a considerable distance from, the windward edge, the surface presented the appearance of *plate glass*; further from the edge the surface presented that wavering appearance which has already been likened to that of sheet glass, which appearance was at the time noted as showing the existence of eddies beneath the surface.

Subsequent observation confirmed this first view. At a sufficient distance from the windward edge of an oil-calmed surface there are always eddies beneath the surface even when the wind is light. But the distance from the edge increases rapidly as the force of the wind diminishes, so that at a limited distance (10 or 20 feet) the eddies will come and go with the wind.

Without oil I was unable to perceive any indication of eddies. At first I thought that the waves might prevent their appearance even if they were there, but by careful observation I convinced myself that they were not there. It is not necessary to discuss these results here, although, as will appear, they have a very important bearing on the cause of instability.

11. *Experiments by Means of Colour Bands in Glass Tubes.*—These were undertaken early in 1880; the final experiments were made on three tubes, Nos. 1, 2, and 3.

The diameters of these were nearly 1 inch, ½ inch, and ¼ inch. They were all about 4 feet 6 inches long, and fitted with trumpet mouthpieces, so that water might enter without disturbance.

The water was drawn through the tubes out of a large glass tank in which the tubes were immersed, arrangements being made so that a streak or streaks of highly-coloured water entered the tubes with the clear water.

The general results were as follows:—

(1.) When the velocities were sufficiently low, the streak of colour extended in a beautiful straight line through the tube (Fig. 3).

(2.) If the water in the tank had not quite settled to rest, at sufficiently low velocities the streak would shift about the tube, but there was no appearance of sinuosity.

(3.) As the velocity was increased by small stages at some point in the tube always at a considerable distance from the trumpet or intake, the colour band would all at once mix up with the surrounding water, and fill the rest of the tube with a mass of coloured water, as in Fig. 4.

Any increase in the velocity caused the point of breakdown to approach the trumpet, but with no velocities that were tried did it reach this.

On viewing the tube by the light of an electric spark, the mass of colours resolved itself into a mass of more or less distinct curls showing eddies, as in Fig. 5.

The experiments thus seemed to settle questions 3 and 4 in the affirmative—the existence of eddies and a critical velocity.

They also settled in the negative question 6 as to the eddies coming in gradually after the critical velocity was reached.

In order to obtain an answer to question 5 as to the law of the critical velocity, the diameters of the tubes were carefully measured, also the temperature of the water and the rate of discharge.

(4.) It was then found that with water at a constant temperature and the tank as still as could by any means be brought about, the critical velocities at which the eddies showed themselves were exactly in the inverse ratios of the diameters of the tubes.

(5.) That in all the tubes the critical velocity diminished as the temperature increased, the range being from 5° C. to 22° C., and the law of this diminution, so far as could be determined, was in accordance with Poiseuille's experiments.

Taking T to express degrees Centigrade, then by Poiseuille's experiments—

$$\frac{\mu}{\rho} \propto P = 1 + 0{\cdot}0336\,T + 0{\cdot}00221\,T^2,$$

Taking a metre as the unit, U_s the critical velocity, and D the diameter of the tube, the law of the critical point is completely expressed by the formula $U_s = \frac{1}{B_s}\frac{P}{D}$, where $B_s = 43{\cdot}7$. This is a complete answer to question 5.

During the experiments many things were noticed which cannot be mentioned here, but two circumstances should be mentioned as emphasising the negative answer to question 6. In the first place, the critical velocity was much higher than had been expected in pipes of such magnitude, resistance varying as the square of the velocity had been found at very much smaller velocities than those at which the eddies appeared when the water in the tank was steady. And in the second place it was observed that the critical velocity was very sensitive to disturbance in the water before entering the tubes, and it was only by the greatest care as to the uniformity of the temperature of the tank and the stillness of the water that consistent results were obtained. This showed that the steady motion was unstable for large disturbances long before the critical velocity was reached, a fact which agreed with the full blown manner in which the eddies appeared.

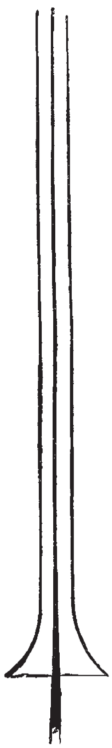


FIG. 3.

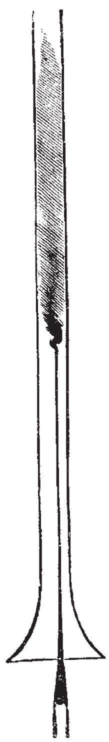


FIG. 4.

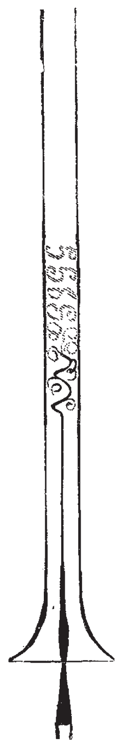


FIG. 5.

12. *Experiments with Two Streams in Opposite Directions in the Same Tube.*—A glass tube 5 feet long and 1·2 inch in diameter, having its ends slightly bent up, as shown in Fig. 6, was half filled with bisulphide of carbon, and then filled up with water and both ends corked. The bisulphide was chosen as being a limpid liquid, but little heavier than water and completely insoluble, the surface between the two liquids being clearly distinguishable. When the tube was placed in a horizontal direction, the weight of the bisulphide caused it to spread along the lower half of the tube, and the surface of separation of the two liquids extended along the axis of the tube.

On one end of the tube being slightly raised, the water would flow to the upper end, and the bisulphide fall to the lower, causing opposite currents along the upper and lower halves of the tube, while in the middle of the tube the level of the surface of separation remained unaltered.

The particular purpose of this investigation was to ascertain whether there was a critical velocity at which waves or sinuosities would show themselves in the surface of separation. It proved a very pretty experiment and completely answered its purpose.

When one end was raised quickly by a definite amount, the opposite velocities of the two liquids, which were greatest in the middle of the tube, attained a certain maximum value depending on the inclination given to the tube. When this was small no signs of eddies or sinuosities showed themselves, but at a certain definite inclination waves (nearly stationary) showed themselves, presenting all the appearance of wind waves.

These waves first made their appearance as very small waves of equal lengths, the length being comparable to the diameter of the tube.

When by increasing the rise the velocities of flow were increased, the waves kept the same length but became higher, and when the rise was sufficient the waves would curl and break, the one fluid winding itself into the other in regular eddies.

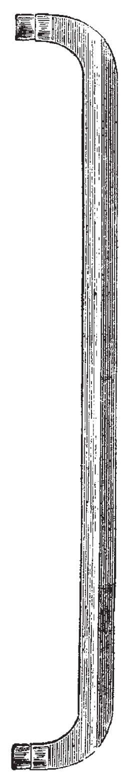


FIG. 6.

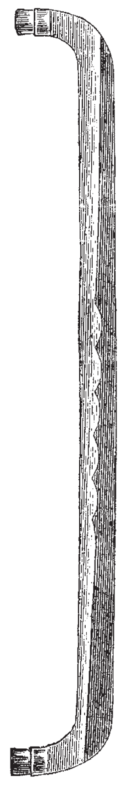


FIG. 7.

Whatever might be the cause, a skin formed slowly between the bisulphide and the water, and this skin produced similar effects to that of oil on water; the results mentioned are those which were obtained before the skin showed itself. When the skin first came on, regular waves ceased to form, and in their place the surface was disturbed as if by irregular eddies above and below, just as in the case of the oiled surface of water.

The experiment was not adapted to afford a definite measure of the velocities at which the various phenomena occurred, but it was obvious that the critical velocity at which the waves first appeared was many times smaller than the critical velocity in a tube of the same size when the motion was in one direction only. It was also clear that the critical velocity was nearly if not quite independent of any existing disturbance in the liquids. So that this experiment shows—

(1.) That there is a critical velocity, in the case of opposite flow, at which direct motion becomes unstable.

(2.) That the instability came on gradually and did not depend on the magnitude of the disturbances, or, in other words, that for this class of motion question 6 must be answered in the affirmative.

It thus appeared that there was some difference in the cause of instability in the two motions.

13. *Further Study of the Equations of Motion.*—Here the author explains that he had so far succeeded in integrating the equations of motion as to find that there must be two critical values of the velocity—the one that at which steady motion would break down into eddying motion, the other that at which, as the velocity diminished, previously existing eddies would die out; both these velocities depending on the relation $U \propto \frac{\mu}{\rho c}$.

14. *Results of Experiments on the Law of Resistance in Tubes.*—The existence of the critical velocity described in the previous article could only be tested by allowing water in a high state of disturbance to enter a tube, and after flowing a sufficient distance for the eddies to die out, if they were going to die out, to test the motion. As it seemed impossible to apply the method of colour bands, the test applied was that of the law of resistance as indicated in questions (1) and (2) in § 8. The result was very happy. Two straight lead pipes, No. 4 and No. 5, each 16 feet long, and having diameters of a quarter and half inch respectively, were used.

The water was allowed to flow through rather more than 10 feet before coming to the first gauge-hole, the second gauge-hole being 5 feet further along the pipe.

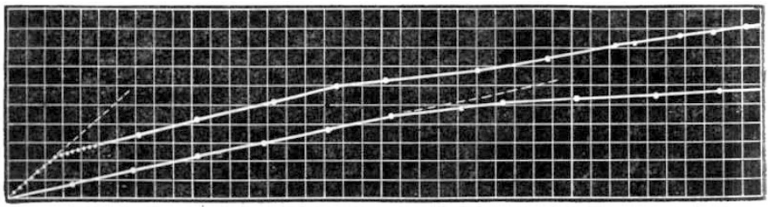


FIG. 8.

The results were very definite, and are partly shown in Fig. 8.

(1.) At the lower velocities the pressure was proportional to the velocity, and the velocities at which a deviation from this law first occurred were in the exact inverse ratio of the diameters of the pipes.

(2.) Up to these critical velocities the discharges from the pipes agreed exactly with those given by Poiseuille's formula for capillary tubes.

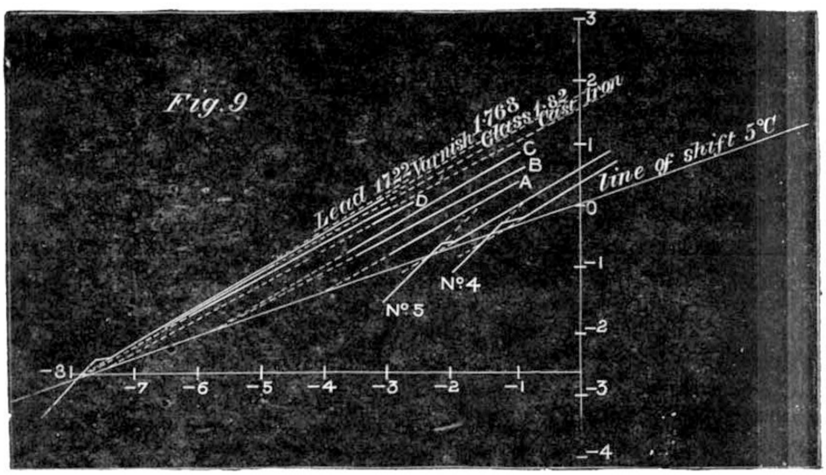
(3.) For some little distance after passing the critical velocity no very simple relations appeared to hold between the pressures and velocities; but by the time the velocity reached 1·3 (critical velocity) the relation became again simple. The pressure did not vary as the square of the velocity, but as 1·722 power of the velocity; this law held in both tubes, and through velocities ranging from 1 to 50, where it showed no signs of breaking down.

(4.) The most striking result was that not only at the critical velocity but throughout the entire motion the laws of resistance exactly corresponded for velocities in the ratio of $\frac{\mu}{\rho c}$. This last result was brought out in the most striking manner on reducing the results by the graphic method of logarithmic homologues as described in my paper on thermal transpiration.

Calling the resistance per unit of length as measured in the weight of cubic units of water i, and the velocity v, log i is taken for abscissa, and log v for ordinate, and the curve plotted.

In this way the experimental results for each tube are represented as a curve; these curves, which are shown as far as the small scale will admit in Fig. 9, present exactly the same shape, and only differ in position.

Either of the curves may be brought into exact coincidence with the other by a rectangular shift, and the horizontal shifts are



Pipe No. 4, Lead	0·00615 m. diameter	Pipe B, Cast Iron	0·188 m. diameter
,, ,, 5, ,, ...	0·0127 ,,	,, D, ,, ...	0·5 ,,
,, A, Glass	0·0496 ,,	,, C, Varnish	0·196 ,,

given by the difference of the logarithm of $\frac{D^3}{\mu^2}$ for the two tubes, the vertical shifts by the difference of the logarithm of $\frac{D}{\mu}$.

The temperatures at which the experiments had been made were nearly the same, but not quite, so that the effect of the variations of μ showed themselves.

15. *Comparison with Darcy's Experiments.*—The definiteness of these results, their agreement with Poiseuille's law, and the new form which they more than indicated for the law of resistance above the critical velocity, led me to compare them with the well-known experiments of Darcy on pipes ranging from 0·014 to 0·5 metre. Taking no notice of the empirical laws by which Darcy had endeavoured to represent his results, I had the logarithmic homologues plotted from his published experiments. If my law was general, then these log. curves, together with mine, should all shift into coincidence if each were shifted horizontally through $\frac{D^3}{P^2}$, and vertically through $\frac{D}{P}$.

In calculating these shifts there were some doubtful points.

Darcy's pipes were not uniform between the gauge points, the sections varying as much as 20 per cent., and the temperature was only casually given. These matters rendered a close agreement unlikely; it was rather a question of seeing if there was any systematic disagreement. When the curves came to be shifted, the agreement was remarkable; in only one respect was there any systematic disagreement, and this only raised another point; it was only in the slopes of the higher portions of the curves. In both my tubes the slopes were as 1·722 to 1; in Darcy's they varied according to the nature of the material, from the lead pipes, which were the same as mine, to 1·92 to 1 with the cast iron. This seems to show that the nature of the surface of the pipe has an effect on the law of resistance above the critical velocity.

16. *The Critical Velocities.*—All the experiments agreed in giving $v^c = \frac{1}{278} \frac{P}{D}$ as the critical velocity, to which correspond as the critical pressure $i_c = \frac{1}{47700000} \frac{P^2}{D^3}$, the units being metres and degrees Centigrade. It will be observed that this value is much less than the critical velocity at which steady motion broke down.

17. *General Law of Resistance.*—The log. homologues all consist of two straight branches, the lower branch inclined at 45°, and the upper one at n horizontal to 1 vertical, except for the small distance beyond the critical velocity these branches constitute the curves. These two branches meet in a point, O, on the curve at a definite distance below the critical pressure, so that, ignoring the small portion of the curve above the point before it again coincides with the upper branch, the logarithmic homologues give for the law of resistance for all pipes and all velocities $A \frac{D^3}{P^2} i = \left(B \frac{D}{P} v\right)^n$, where n has the value unity as long as either member is below unity, and then takes the value of the slope n to 1 for the particular surface of the pipe.

If the units are metres and degrees Centigrade—

$$A = 67,700,000$$
$$B = 398$$
$$P = 1 + 0·0336\,T + 0·000221\,T^2.$$

This equation then, excluding the region immediately about the critical velocity, gives the law of resistance in Poiseuille's tubes, those of the present investigation, and Darcy's, the range of diameters being from 0·000013 metres (Poiseuille, 1843), to 0·5 metres (Darcy, 1857); and the range of velocities from 0·0026 to 7 metres per sec., 1883.

This algebraical formula shews that the experiments entirely accord with the theoretical conclusions. The empirical constants are A, B, P, and n; the first three relate solely to the dimensional properties of the fluid which enter into the viscosity, and it seems probable that the last relates to the properties of the surface of the pipe.

Much of the success of the experiments is due to the care and skill of Mr. Foster of Owens College, who has constructed the apparatus and assisted me in making the experiments.

SOCIETIES AND ACADEMIES

Paris

Academy of Sciences, October 15.—M. Blanchard, president, in the chair.—Note on a formula of Hansen in connection with the mechanism of the heavens, by M. F. Tisserand.—On the measurement of the forces brought into play in the various actions of human locomotion (continued), three illustrations, by M. Marey. By combining the indications obtained from the dynamometer with those yielded by instantaneous photography, a continuous comparison may be made of the forces brought into action with the movements resulting from them. The various applications of these two methods will form the subject of future experiments.—On a memoir by M. Raoult, entitled: "Loi générale de Congélation des Dissolvants,"—report by MM. Cahours, Berthelot, and Debray. Water holding saline bodies in solution freezes at a lower temperature than pure water, and the English physicist Blagden had shown in 1788 that the lowering of the freezing-point due to this cause is in many cases in proportion to the quantity of matter held in solution. This principle is now generalised by M. Raoult, who arrives at the conclusion that the freezing-point of any liquid compounds capable of solidification is lowered by all solid, fluid, or gaseous bodies dissolved in them. The reporters agree with the author that his methods will be found useful in supplying new means for ascertaining by a simple process the degree of purity of given substances.—Trial trip of an electric screw balloon made by MM. A. and G. Tissandier, note by M. G. Tissandier. This preliminary experiment took place at Auteuil on October 8, and was attended by a certain measure of success, although the apparatus proved powerless to prevent the spinning motion of the balloon when heading against aërial currents. The trip will be renewed as soon as certain improvements have been made in the electromotor suggested by this experiment.—Studies made on the summit of the Pic du Midi, with a view to the establishment of a permanent astronomic station, note by MM. Thollon and Trépied.—On the transformation of certain equations of the second degree to two independent variables, and on some integrations thence deducible, by M. R. Liouville.—On a method of isolating the calorific from the luminous and chemical rays, by M. F. van Assche.—On the form and characters of the reflex muscular contraction, by M. H. Beaunis.—On the resisting power of a ring whose outer surface supports a normal pressure constant as to unity of length of its mean axis, by M. J. Boussinesq.—On surfaces whose total curve is constant, by M. G. Darboux.—Indices of refraction of fluate of lime for the rays of different wave-lengths as far as the extreme ultra-violet, by M. Ed. Sarasin.—Note on a new method of insulating the metallic wires used in telegraphy and telephony, by M. C. Widemann.—Note on the determination of the equivalents of metals by means of their sulphates, by M. H. Baubigny.—On the process at present employed to determine the glucose in cane-sugar, by M. P. Lagrange. The object of this paper is to show that the quantitative analysis of glucose, made on a liquor whether treated or not with subacetate of lead, is liable to serious errors.—Analysis of a specimen of guano from the Cape Verde Islands, by M. A. Andouard.—Zoological dredgings and thermometric soundings in the lakes of Savoy, by M. F. A. Forel.—On the organisation of the *Spadella Marioni*, a new species from the Gulf of Marseilles, by M. P. Gourret.—On some peculiarities in the structure of Tunicata, by M. L. Roule.—Fresh studies on the fossil ruminants of Auvergne, by M. Depéret.—On the treatment of strabismus by means of the capsular "advancement," by M. L. de Wecker.—On the part played by the ligneous vessels in the upward movement of the sap, by M. J. Vesque.—Note on a lunar mirage observed on the night of October 11, by M. Virlet d'Aoust.

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