varieties following the indication of *Coccinella 10-punctata*, L., and 6 or 8 analogous varieties are appended to many other species of Ladybirds. Taking it as a whole, this excellent catalogue may serve as a model for compilers of lists of the Beetle (or other entomological) fauna of other districts.

LETTERS TO THE EDITOR

[*The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, or to correspond with the writers of, rejected manuscripts. No notice is taken of anonymous communications.*

[*The Editor urgently requests correspondents to keep their letters as short as possible. The pressure on his space is so great that it is impossible otherwise to ensure the appearance even of communications containing interesting and novel facts.*]

Equal Temperament of the Scale

In your number of November 8, 1877, p. 34, Mr. Chappell, F.S.A., has intimated that mathematicians who propose to divide the octave into twelve equal semitones instead of "equally tempered semitones," are deficient in musical ear. I have not noticed that any mathematician has replied to him.

Representing (with Mr. Chappell) the number of vibrations in the C of my piano by 1, and the octave *c* therefore by 2, and dividing the octave into 12 equal intervals, I obtain for the vibration-numbers—

C $= 1\cdot$	G $= 1\cdot4983 = 2^{\frac{7}{12}}$
C♯ $= 1\cdot0594 = 2^{\frac{1}{12}}$	G♯ $= 1\cdot5874 = 2^{\frac{8}{12}}$
D $= 1\cdot1224 = 2^{\frac{2}{12}}$	A $= 1\cdot6818 = 2^{\frac{9}{12}}$
D♯ $= 1\cdot1892 = 2^{\frac{3}{12}}$	B♭ $= 1\cdot7818 = 2^{\frac{10}{12}}$
E $= 1\cdot2599 = 2^{\frac{4}{12}}$	B $= 1\cdot8877 = 2^{\frac{11}{12}}$
F $= 1\cdot3348 = 2^{\frac{5}{12}}$	*c* $= 2\cdot$
F♯ $= 1\cdot4142 = 2^{\frac{6}{12}}$	

In these equal semitones each is equidistant from the preceding and following: as F is to F♯, so is F♯ to G, &c. Hence in whatever key I play a passage on my piano, the divergence from harmonic intervals will be alike at every point; the keys on my piano will have no distinctive character, the key of 3 sharps will not be more "brilliant" or less "plaintive" than that of 4 flats.

In the key of C, the harmonic third, fifth, and seventh will be, according to the above notation, 1·25, 1·5, and 1·75 respectively. As regards the fifth G it is a remarkable numerical coincidence that $2^{\frac{7}{12}}$ only differs from 1·5 by $\frac{1}{600}$, *i.e.* the equal temperament G only differs from the harmonic by its $\frac{1}{400}$ part, a difference so slight that it may be neglected. We tune fiddles by fifths therefore. This coincidence is the fundamental fact which enables us to modulate into various keys on a piano, and it is the reason why the scale must be divided into 12 (and not any other number of) semitones; for it will be found that, until you get to the unmanageably high number of 53, no other equal division of the scale has any note so near the harmonic G.

The crucial point of tempering arises on the third. The E of my piano is $2^{\frac{1}{3}} = \frac{126}{100}$, whereas the harmonic E is $= \frac{125}{100}$; my E is therefore by its $\frac{1}{80}$ part too sharp, *in the key of C*, a perceptible degree of error, unpleasant to many musicians. In ordinary pianoforte tuning, the E (by the plan in Hamilton's pianoforte tuner or some similar compromise) is tuned somewhere between $\frac{125}{100}$ and $\frac{126}{100}$, say $\frac{125\frac{1}{2}}{100}$, and the wolf between this E and the upper *c* is distributed.

This is all very simple so long as we remain in the key of C; indeed if we remain there, we want no tempering. But G♯ is the third to E, and *c* is the third to A♭; on the piano G♯ and A♭ are one. On my equal-semitone piano I have

$$c = 1 ;\ \mathrm{E} = 2^{\frac{1}{3}}\ (= \tfrac{126}{100} \text{ nearly} ;$$

$$\mathrm{G}\sharp = \mathrm{A}\flat = 2^{\frac{2}{3}}\ (= \tfrac{157}{100} \text{ nearly}) ;\ c = 2.$$

I now ask the champion of "equally tempered semitones" what is the numerical value of his E and what of his G♯. If he gives them any other values than $2^{\frac{1}{3}}$ and $2^{\frac{2}{3}}$ respectively, it is clear that a *greater* error will be introduced in one part of the scale than is saved in another. Instead of algebraic proof I take an instance—suppose that Mr. Chappell tunes his E at $\frac{125\frac{1}{2}}{100}$; if he equally tempers his G♯ in the scale of E, it will be $\left(\frac{125\frac{1}{2}}{100}\right)^2 = \frac{31}{20}$ very nearly. Then when he puts down the common chord in the key of A♭, his third the *c* will be by its $\frac{1}{32}$ part too sharp, whereas on my equal temperament piano it would only be by its $\frac{1}{80}$ part too sharp. In other words, though the keys of C and E may be somewhat better on Mr. Chappell's piano than on mine, the key of A♭ will be *very much* worse. This is pretty nearly what occurs in practice. The point of my argument is that Mr. Chappell cannot move his E ever so little from the value $2^{\frac{1}{3}}$ without introducing a greater error somewhere else. The term "equally tempered semitone" is inaccurate; the semitones on my piano are all equal; and no one of them can be altered by a disciple of the "equally-tempered semitone" without making them unequal. The "equally-tempered semitones" are *not* equally tempered. Moreover if you "temper" *at all* you lose the effect of the harmonics; by moving E from $\frac{125}{100}$ to $\frac{125\frac{1}{2}}{100}$ you sacrifice harmonic coincidence.

The simple reason that unequal tempering is practised is because all keys are not used equally often. A piano is unequally tempered so that the keys C, G, A, F are fair, E, B♭, E♭ tolerable, the other keys being very much worse than on my equal-semitone piano. On most church organs, being unequally tempered, if you modulate even transiently into 4 or 5 flats, the effect is unendurable.

The crucial question in tuning is the question, if your E is not $2^{\frac{1}{3}}$ and your G♯ $2^{\frac{2}{3}}$, what values do you put them at? The question of the seventh is more complex; I may observe that though my equal-semitone seventh (1·7818) appears far away from the harmonic seventh (1·75), yet that the B♭ of tuners on the "equally-tempered semitone" system is not much nearer it. Their B♭ is $\frac{16}{9}$ or thereabout, or in other words, the sub-subdominant of C. Therefore, on the piano, you have not got the "harmonic-seventh" at all; the note which replaces it is one that suggests overpoweringly the key of F. This is the secret which underlies several of our rules in harmony. It is also the reason why valve-horn players play B♭ (though an open note) with valve *n.*2, or if they play without a key "lip it up" very carefully.

It is often supposed that the "wolf" has been introduced into music by that most useful though imperfect instrument the piano, and that the noble violin or human voice knows it not, except in so far as our natural good ear for harmonic intervals has been debauched by continually hearing tempered intervals. This is not so; the "wolf" is not only in the piano but in the scale. It is true that a violin can play in harmonic tune so long as the melody runs in one key, or if it modulates into a closely allied key, and *back again the same way*. But suppose my violin begins by rising from C to E harmonically, *i.e.* to $\frac{125}{100}$; then after playing awhile there proceeds to G♯ $(\frac{125}{100})^2$ harmonically, being then in 8 sharps; and then, after playing awhile in 8 sharps, proceeds to *c*; the *c* of the fiddle will then be $(\frac{125}{100})^3$ instead of 2, *i.e.* it will be $\frac{3}{128}$ out of tune. In this simple case the fiddle is supposed to play alone, unfettered by any harmonics but its own; in the case of a string-band, the agreeableness of many modulations actually depends upon some chords being harmonically out of tune, the note in the chord which performs the duty of G♯ to its preceding chord, performing the duty of A♭ to its succeeding chord.

The practical conclusion is that the best plan of tuning a piano for vulgar music and vulgar players is that now ordinarily practised by the tuners, and recommended by Mr. Chappell; but if the piano is to be used equally in all keys (or even frequently in 4 or 5 flats, 5 or 6 sharps) the best plan is to tune it in 12 mathematically equal semitones. C. B. Clarke

Animal Intelligence

In an excellent paper on "Animal Intelligence" (Nature, vol. xxvi. p. 523), Mr. C. Lloyd Morgan says that "The brute has to be contented with the experience he inherits or individually acquires. Man, through language spoken or written, profits by the experience of his fellows. Even the most savage tribe has traditions extending back to the father's father. May there not be, in social animals also, traditions from generation