Manufactures, are well worthy of attention, but as they deal with the artistic rather than with the scientific aspect of those industries, we cannot dwell upon them.

Amongst the instructions handed to each artisan reporter at the outset, were suggestions to ascertain the prices and cost of production, the relative amount of machinery employed in production, the hours of labour and the manner of living of the French artisans. Much useful information has been collected on most of these heads. Almost all the reports agree that while cost of living is perhaps a little cheaper in Paris than in London, wages are on the whole much lower; so that it is only by working longer hours and by thrift and steadiness that the French workman can live. The remark is almost universally made that drunkenness is extremely rare; while the absence of almost everything that constitutes home life is equally conspicuous in the habits of the Parisian workman.

In one or two points the volume before us is, from the nature of things, strangely defective. Almost all the reporters who mention the subject at all, appear to have misapprehended the nature and status of the *Cercles ouvrières* or *Corporations ouvrières*, which are the nearest approach in France to the Trades Unions of this country, and the comparisons drawn between the two are in consequence often irrelevant or incorrect. These bodies in France cannot legally extend beyond the limits of the "commune" or parish; they are usually semi-political or socialistic in character, and while they concern themselves with the conditions of labour, are not exclusively occupied in matters of wages and hours of work, and do not, from the local restriction on their operations, exercise an influence in any measure comparable to that exercised by the English Unions over the price or conditions of labour. Again it is impossible to derive from the reports any ideas upon the relation between skilled labour and the educational systems in operation in Paris or in the provinces of France, for the simple reason that not one of the reporters appears to have been made acquainted with those educational systems as a whole. A few of the more prominent technical schools, the *École d'Apprentis*, the *Horological Schools*, and the *Typographic School* of MM. Chaix and Co., are indeed mentioned; but beyond these exceptional institutions and a chance reference to the free evening schools of drawing and modelling which are to be found in every quarter of Paris, there is no reference to the educational systems of the country or to their influence on the artisan, the foreman, and the employer. Any account of the conditions of the skilled industries in France which leaves these out of consideration must be regarded as imperfect in the extreme.

One result is however unmistakable. The artisans who drew up these reports were fully alive to all the advantages of which accrue to an industry from the extension of labour-saving appliances, and from the dissemination of higher technical knowledge. They have faithfully pointed out those departments of industry in which we excel, and those in which we are excelled. They have in most cases stated their opinions as to the causes which have brought about these results. It will be our own fault if we do not strengthen the weak points and fill the gaps now revealed to us. The strides made by some of our foreign competitors are so great as to leave us no margin for indolence or wastefulness on our part. The less favoured nation may more than make up for the material disadvantage of having to import raw products and fuel by the superior thrift and the better training of its workmen. All these things point to the need at home to lose no opportunity of pushing forward the scientific and artistic culture of the workers and of their employers, so that their training may at least be not inferior to that of their Continental rivals.

SILVANUS P. THOMPSON

HOW TO COLOUR A MAP WITH FOUR COLOURS

SINCE the publication in the *American Journal of Pure and Applied Mathematics*, vol. ii. part 3, of the solution of this problem obtained by me, and referred to in NATURE, vol. xx. p. 275, I have succeeded in obtaining the following simple solution in which mathematical formulæ are conspicuous by their absence. It may be premised that the problem is to show how the districts of a map may be coloured with four colours, so that no two districts which have a common boundary or boundaries shall be of the same colour. The object of this colouring being to make the division of the map into districts clear without reference to boundary lines, which may be confused with rivers, &c., it is obvious that nothing will be lost if districts which are remote from each other, or touch only at detached points, are coloured the same colour.

The only parts of the map that it is necessary to consider are the districts, boundaries, and points of concourse, *i.e.*, points at which boundaries terminate. Two districts may have a single common boundary, or they may have two or more such boundaries. Any two districts which have more than one common boundary, inclose one or more groups of districts; in any one of these groups two districts which have more than one common boundary inclose one or more groups of districts, and so on. Proceeding in this way, we limit the area under consideration more and more at each step, and must finally come either to a group which has no pair of districts which have more than one common boundary, or to a single district having only two boundaries, one in common with each of its two surrounding neighbours. *Thus every map must have at least one pair of adjacent districts which have only one common boundary* (β).

Every boundary is either continuous like a circle, or has two ends which lie at the same or at different points of concourse. Every point of concourse may be called *triple*, *quadruple*, &c., according to the number of *lines* radiating from it. I expressly say *lines* and not *boundaries*, because if two ends of any boundary lie at the same point of concourse two of the lines radiating from the latter will belong to only one boundary. If a boundary whose ends lie at two different points of concourse be rubbed out, the number of lines radiating from each of those points of concourse will be reduced by one, thus if the two points were each triple points, they will become double points, *i.e.*, they will no longer be points of concourse, the two remaining lines which radiate from each becoming one boundary. The result is that rubbing out a single boundary *may* reduce the number (B) of boundaries in the map by three. It can, however, never cause a greater reduction, and may cause a smaller, *e.g.*, rubbing out a continuous boundary, or one which ends in two quadruple points reduces the number of boundaries by one only.

Now the obliteration of the boundary β causes the two districts it separates to become one, thus reducing the number of districts (D) in the map by one, and the map still remains a map, and has therefore a pair of districts having only one common boundary. Obliterate this common boundary, and so on. We finally get a blank sheet, *i.e.* a single district and no boundary, and each reduction of D by one cannot involve a reduction of B by more than three; thus 3D must be greater than B, consequently 6D must be greater than 2B; but 2B is the number which would be arrived at if we counted both sides of every boundary, *i.e.*, the number which would be arrived at if we counted the number of boundaries to each district and added them all together; thus the number arrived at by the latter computation must be less than 6D, *i.e.*, it is impossible that every district can have as many as six boundaries, *i.e.*, *every map contains at least one district with less than six boundaries.*

We can therefore reduce a map to a single district by successive operations of throwing two districts into one by rubbing out the boundary or boundaries between two districts of which one has less than six boundaries. Conversely we can develop a map, starting from a single district and adding boundaries, at each stage dividing a district into two, one of which has less than six boundaries. Suppose at any stage of its development by this process a map can be coloured with four colours [red, blue, green, and yellow]. Let these colours be indicated by coloured wafers placed on the districts. Proceed to the next operation, this divides a wafered district into two districts. Shift its wafer on to the district of these two which is not the one which has less than six boundaries: if both have less than six boundaries shift the wafer on to either. If the district (W) which is left without a wafer is only touched by three colours it can be coloured the fourth, and a wafer may be put on it representing that colour. But if it is touched by all four colours we must take another step. This can only be necessary if W has four or five adjacent districts. These may either all surround or all be surrounded by or some surround and some be surrounded by it. Take first the case in which *four* districts are adjacent, all surrounding or surrounded by W. Let *a b c d* be the districts, taking them in the order in which they stand. Let *a* be red, *b* blue, *c* green, and *d* yellow. If, starting from *a*, we can get to *c*, going only through red and green districts, and not passing through any points of concourse, we cannot, starting from *b*, get to *d*, going similarly only through blue and yellow districts, for otherwise two tracks which pass through different districts would cross. Thus *b* forms one of a group (G) of blue and yellow districts which are cut off from the rest of the map by encircling red and green ones. We can accordingly interchange the blue and yellow wafers in G without changing any others; this makes *b* yellow, and we can put a blue wafer on W. Similarly, if we cannot pass from *a* to *c*, *a* belongs to an isolated group of red and green districts. Interchanging the wafers in these, *a* becomes green, and a red wafer can be put on W. Precisely similar reasoning applies in the case of *five* surrounding or surrounded districts, viz., *e* red, *f* blue, *g* green, *h* blue (two must of course be the same colour), and *k* yellow. Here either *e* belongs to an isolated group of red and green districts, or *k* to one of yellow and green districts, or *f* to one of blue and yellow, and *h* to one of blue and red districts. In the first case, interchanging wafers as before, *e* becomes green, and a red wafer can be put on W; in the second *k* becomes green, and a yellow wafer can be put on W; in the third *f* becomes yellow, and *h* red, and a blue wafer can be put on W. In all cases before putting the wafer on W we can interchange the colours of districts, *e.g.*, we can put red wafers in the place of all the green ones and *vice versâ*. Thus we can make the three colours adjacent to W any three we please. If therefore the districts adjacent to W belong to different groups of districts surrounding and surrounded by W, and so detached from each other, we can rearrange the wafers in each group so that only three colours in all shall be adjacent to W, which can therefore have a wafer of the fourth colour placed on it. Thus in any case the district W can be wafered with a wafer of one of the four colours. Thus if the map can be coloured as developed at any stage it can be coloured at the next. Hence since it can obviously be coloured at the first stage when there is only one district, it can be coloured at the last.

Take then two copies, P and Q, of the map we wish to colour, one of which, Q, is on a slate or in pencil, so that the boundaries can easily be obliterated. Pick out a district with less than six boundaries. Rub out in Q the boundary or boundaries (if there be more than one) between this and any other district which is adjacent to it. Number with a (1) the corresponding boundary or boundaries in P. Repeat the operation, numbering the boundary or boundaries in P this time (2). Continue the process until a map is arrived at which can obviously be coloured with four colours. This will generally happen long before we reduce the number of districts to four. Put wafers on the middle of the districts of Q, representing the colours. Proceeding as before shown with the process of adding boundaries in the order indicated by the numbering of P taken backwards, and of shifting the wafers so as to be able to add a wafer to the W of each stage, we finally arrive at a stage when Q is in its original state. The map can then be coloured as indicated by the colours of the wafers.

This method applies equally to maps drawn on the surface of globes, but fails in the case of surfaces which are not necessarily divided into two parts by an endless line, these in general requiring more than four colours.

A. B. KEMPE

THE LIPARI ISLANDS

ON inquiring in Rome for the Stato Maggiore map of the Lipari Islands, I was told that it was out of print, and when afterwards I succeeded in getting one in Florence, I found that owing to the large scale, the islands from Vulcano to Stromboli, in a north-easterly direction, and from Vulcano to Alicudi, to the north-west, were given on three separate sheets, too unwieldy to use for practical purposes, except in their disconnected form. Our own Admiralty chart (scale $\frac{1}{100000}$) constructed from the maps of a French hydrographer, M. Darondeau, gives all the islands, save Ustica, at one view, accompanied by soundings, and a general diagrammatic view of the principal group. The Comitato Geologico of Rome has not yet published a geological map of the islands, and the only complete one that exists, as far as I know, is that to be found in the antiquated "Vulkanen-Atlas" of N. C. von Leonhard, which is taken from the survey of Fr. Hoffmann. In this map Alicudi, Felicudi, Salina, and the major part of Lipari are represented as composed of tuff with porphyritic lava. Panaria, with the surrounding islets Dattolo, Lisia Bianca, &c., is stated to consist entirely of trachyte. The greater part of Vulcano, and about half Stromboli are given as old felspathic lava, while the craters of Vulcano, Vulcanello, and Stromboli are described as *nochforldauernde vulk. Bildungen*. Pumice and obsidian are shown in various parts of Lipari.

Since Admiral Smyth visited the Æolian Island in 1815, numerous observers have followed in his footsteps. He has devoted thirty pages of a quarto volume on "Sicily and its Islands" to this subject, and two of the three Admiralty charts which relate to these islands, contain engravings executed from his drawings.

In 1874 Prof. J. W. Judd visited the islands, and he has embodied the results of his observations in some valuable memoirs contributed to the *Geological Magazine*, accompanied by reproductions of drawings made on the spot. To him we are indebted for the accompanying view of Vulcano and Vulcanello.

We were surprised at the complete ignorance manifested both by the Romans and the Neapolitans, in regard to everything connected with Lipari and the members of its group. Everybody said "You must tell us all about them when you come back." In fact the islands are very little visited; communication with the mainland is at the best only twice a week; the boats are small and inconvenient, and they start at midnight; and worse than all, the most important island of the group (Lipari) is cursed by the presence of some 400 convicts, who are sent to this penal settlement, much as we sent ill-disposed persons to Botany Bay forty years ago. We left the harbour of Messina at midnight, having on board ten of these manacled *manutengoli*, guarded by carabinieri. At six the next morning we were off Lipari, and soon afterwards