surrounded by an atmosphere containing sufficient hydrogen sulphide, the opposite effect occurs. Now the needle, when vitreously electrified, turns from copper to iron; when resinously, from iron to copper. The conclusion to be drawn from these results seems to be that the electrical behaviour of metals in contact is almost, if not entirely, due to the difference of their affinities for one of the elements of such compound gases as may be in the atmosphere surrounding them. This would be entirely analogous to their behaviour in electrolytes containing these same elements, *e.g.* iron is positive to copper in an oxidising electrolyte such as water, because of the affinity of oxygen for iron being greater than for copper, while iron is negative to copper in potassium sulphide solution, because of the affinity of sulphur being greater for copper than for iron.

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Solar Halo

The following was noticed at Bordeaux on April 4, at 11 A.M.:—1. A well-defined and very complete circumzenithal circle (80° in diameter), of a brilliant white light, passing through the sun. 2. An iridescent circle, larger than the first, and cutting it at two points 60° distant from the sun. The second circle showed more especially the red rays on its concavity (*i.e.* towards the sun), except at the parhelia, where it was bright iridescent. Near the western parhelion the brilliancy of the mock sun was quite insufferable to naked eyes.

The morning was very warm, but the night had been very cold.

E. Rodier

29, Rue Saubat, Bordeaux, April 20

FLOATING MAGNETS

The extract from the *American Journal of Science* describing experiments with floating magnets by Mr. Alfred M. Mayer to illustrate the equilibrium of mutually-repellent molecules each independently attracted towards a fixed centre, which appeared in Nature, vol. xvii. p. 487, must have interested many readers.

It has interested me particularly because the mode of experimenting there described, with a slight modification, gives a perfect mechanical illustration (easily realised with satisfactory enough approximateness) of the kinetic equilibrium of groups of columnar vortices revolving in circles round their common centre of gravity, which formed the subject of a communication I had made to the Royal Society of Edinburgh on the previous Monday. In Mr. Mayer's problem the horizontal resultant repulsion between any two of the needles varies according to a complicated function of their mutual distance readily calculable if the distribution of magnetism in each needle were accurately known. Suppose the distributions to be precisely similar in all the bars and in each to be according to the following law:—Let the intensity of magnetisation be rigorously uniform throughout a very large portion, C D, of the whole length of the bar (Fig. 1), and let it vary uniformly from C and D to the two ends A and B. The bar will act as if for its magnetism were substituted ideal magnetic matter,[1] or polarity, as it may be called, uniformly distributed through the end portions C A and D B; the whole quantity in D B to be equal in amount and opposite in kind to that of C A. For example, suppose true northern polarity in A B and true southern in B D. The lengths of C A and D B need not be equal. Let now A′C′D′B′ be another bar with an exactly similar distribution of magnetism to that of A C D B, and let the two be held parallel to one another. The mutual repulsion will vary inversely as the distance, if the distance be infinitely small in comparison with D B or C A, and if each of these be infinitely small in comparison with C D. If the true south pole S of a powerful bar-magnet be held in a line midway between B A and B′A′, at a distance from the ends B and B′ infinitely great in comparison with B B′, and comparable with the length of each needle, the horizontal component of its effect on each magnet will be a force varying directly as its distance from the central axis. Under these conditions Mr. Mayer's experiments will show configurations of equilibrium of two, or three, or four, or any multitude of ideal points in a plane, repelling one another with forces inversely as the mutual distances, and each independently attracted towards a fixed centre with a force varying directly as the distance. This, as I showed in my communication to the Royal Society of Edinburgh, is the configuration of the group of points in which a multitude of straight columnar vortices with infinitely small cores is cut by a plane perpendicular to the columns; the centre of inertia of a group of ideal particles of equal mass placed at these points being the fixed centre in the static analogue.

The consideration of stability referred to by Mr. Mayer has occupied me much in the numerical problem, and it is remarkable that the criterion of stability or instability is identical in the static and kinetic problems. In the static problem it is of course that the potential energy of the mutual forces between the particles, together with that of the attraction towards a fixed centre is less for the configuration of stable equilibrium than for any configuration differing infinitely little from it. The potential energy of the attractive force is a function of distance from the central axis, diminishing as the distance increases, and the statement of the criterion may be conveniently modified to the following:—

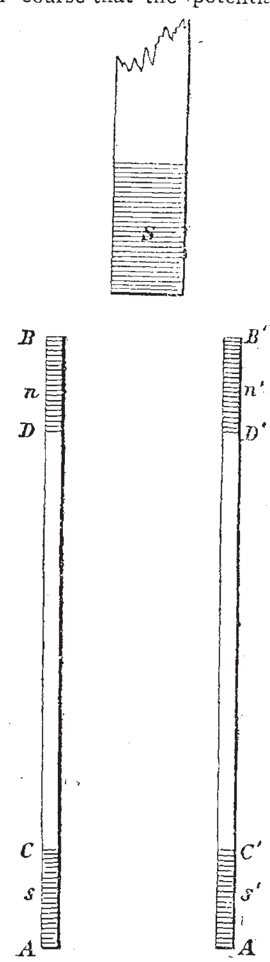

Fig. 1.

For a given value of this function the mutual potential energy of the atoms must be a minimum for stable equilibrium. When, as supposed above, the attractive force varies directly as the distance its potential energy is:—

$$C - \tfrac{1}{2} c \, \Sigma r^2$$

where C, c, denote constants, and Σr^2 the sum of the squares of the distances of all the particles from the attractive centre. And when the law of force between the particles is the inverse distance, their mutual potential energy is equal to—

$$K - k \log. (D D' D'' \ldots)$$

where K, k, denote constants, and D, D′, D″, &c., denote the mutual distances between the particles. Thus the condition of stable equilibrium becomes that the product of the mutual distances between the particles must be a true maximum for a given value of the sum of the squares of their distances from the attractive centre. A first conclusion from this condition must be that the centre of gravity of the particles must be the attractive centre. Now the condition of kinetic equilibrium of a group of vortex columns, that is to say the condition that they may revolve in circles round their common centre of inertia is, as proved in my communication to the Royal Society of Edin-

[1] Reprint of papers on Electrostatics and Magnetism, § 469 (W. Thomson).

burgh, that the product of their mutual distances must be a maximum or minimum or a maximum-minimum for a given value of the sum of the squares of their distances from the common centre of gravity;[1] and the condition

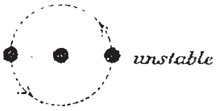

FIG. 2.

FIG. 3.

that this kinetic equilibrium may be stable is that the product be a true minimum for a given value of the sum of the squares of their distances from the centre of inertia. Taking for example a triad of vortices (or of the little magnetic needles of Mr. Mayer's problem), it is thus obvious the equilibrium is unstable in the case represented by Fig. 2, and stable in the case represented by Fig. 3. The arrow-heads in Figs. 2 and 3 represent the motions of the vortex columns round their centre of gravity. It must be understood that the core of each column revolves also round its centre of gravity in the same direction as the group round the common centre of gravity of all with enormously greater angular velocities.

I have farther considered the problem of oscillations in the neighbourhood of configuration of stable equilibrium. The general problem which it represents for mathematical analysis has a very easy and simple solution for the case of a triad of equal vortex columns in the neighbourhood of the angles of an equilateral triangle.

A mechanism for producing it kinematically is represented in Fig. 4, showing three circular discs of cardboard pivotted on pins through their centres at the angles of an equilateral triangle rotating in a vertical plane. The plane carrying these three centres may be conveniently made of a circular disc of stiff cardboard, or of light wood pivotted on a fixed pin through its centre. Each of the small discs or epicycles is prevented from rotation

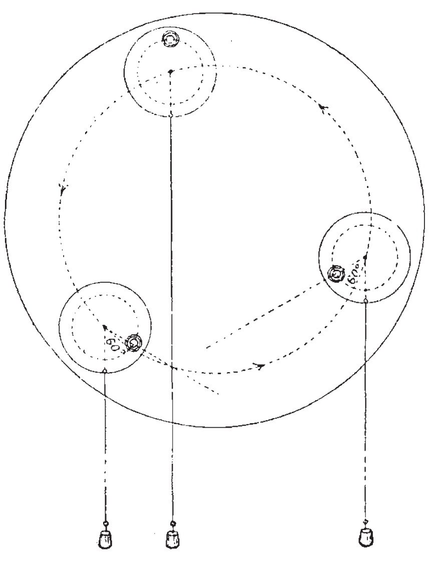

FIG. 4.

by a fine thread bearing a weight, and attached to a point of its circumference; and on each of them is marked, by a small dark shaded circle, the section of one of the vortex cores in proper position.

[1] Helmholtz proved that whatever be the complication of motions due to mutual influences among the vortices, their centre of gravity must remain at rest.

The rule for placing the vortices on their epicycles is as follows:—Each vortex keeps a constant distance from its mean position (this being the centre of the epicycle, carrying it in the mechanism); each of the radius vectors drawn from the centres of the epicycles to the centres of the vortices keeps an absolutely fixed direction, while the equilateral triangle of the centres of the epicycles rotates uniformly; and these three fixed directions are inclined to one another at equal angles of 120° measured backwards relatively to the order in which we take the three vortices. It is easily verified that when the distances of the vortices from their mean positions are infinitely small (that is to say, when the triangle of the triad is infinitely nearly equilateral), the product of its three sides remains constant in the movement actually given by the mechanism, and so does the sum of the squares of the distances of its three corners from its centre of gravity. From the stability of the equilateral triangle it follows[1] that there must be stability with three equal vortices at the corners of an equilateral triangle, and one (whether equal to them or not) at its centre,[2] . . For four equal vortices I have found that the square order, ..., also is stable. From the stability of the square follows (for vortices or for particles repelling according to inverse distance) the stability of four equals at the corners of the square, and one (whether equal to them or not) at its centre,[3] . . . I have not yet ascertained *mathematically* whether for a pentad of equal vortices there is stability also in the pentagonal arrangement, . . . But Mr. Mayer's experiment, showing it to be stable for the magnets, is an experimental proof that it must be stable for the vortices, for it is easily proved that if any of the figures is stable with mutual repulsion varying more rapidly (as is the case with the magnets in Mr. Mayer's experiment), than according to the inverse distance, *à fortiori*, it must be stable when the force varies inversely as the distance. From the stability of the pentagon I infer (for vortices and for particles repelling according to inverse distance) the stability of the configuration . . .

Mr. Mayer's figure [4] . . shows that the hexagonal order was unstable for his six magnets. I had almost convinced myself before seeing the account of his experiments in NATURE, that the hexagonal order is stable for six equal vortices; and Mr. Mayer's last figure shows that with his magnets the hexagonal order is rendered stable by the addition of one in the centre . . .

The instability of the hexagon of six magnets shows the simple polygon to be unstable for seven or any other number exceeding six. Thus Mr. Mayer's beautiful experiment brings us very near an experimental solution of a problem which has for years been before me unsolved—of vital importance in the theory of vortex atoms —to find the greatest number of bars which a vortex mouse-mill can have.

WILLIAM THOMSON

[1] In the case of vortices or of the static problem when the law of the mutual repulsions is the inverse distance, but not with the law of repulsion with ordinary proportions of linear dimensions and magnetic distributions, in Mr. Mayer's magnetic arrangement.

[2] In repetitions of Mr. Mayer's experiments, I have always found this configuration unstable, and for four only the square stable.

[3] This configuration of the floating magnets I have found stable, but with less wide limits of stability than the pentagon.

[4] I have not found this, nor any other configuration than the pentagon with centre, stable for six floating magnets.