They are, moreover, being issued in such numbers, under the present demand for popular education, that their very likeness to one another is fatiguing. They require also in their construction the rare faculty, whether intuitive or gained by long experience, of insight into a student's probable difficulties; for it seems desirable that they should rather aim at being employed as condensors and systematisers of knowledge already acquired generally from the study of larger and more diffuse treatises, than as independent works. It is in this respect that useful practical knowledge differs from "cram"; a distinction very real, though more difficult to define than to understand. The concentrated food offered by such compilations is less easy of digestion, and more readily expelled from the mental economy, than that which is more gradually administered and more completely assimilated.

The writer of the present manual has, for instance, only seventy pages to devote to Sound, one hundred and eighteen to Light, and ninety-one to Heat, exclusive of the Appendix. But it is remarkable how much he succeeds in compressing within these very restricted limits. The illustrative experiments are, as a rule, simple and well chosen, though occasionally trite, and even of doubtful accuracy; as is seen in the drawing of the periodic curve of a musical sound at p. 40, and that of dispersion of light on p. 135. On the other hand, the use of a long spiral steel spring to illustrate waves of compression and rarefaction, the description of the effects of Temperature on Sound-waves, and the chapters on Interference, Diffraction, and Polarisation of Light, especially in its Circular and Rotatory forms, are ingenious and easy to comprehend.

A few simple numerical examples are given of each important law, with their solutions, and the mode of working out; a method which probably tends more than any other to fix essential points on the memory of the student. W. H. STONE

LETTERS TO THE EDITOR

[*The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, or to correspond with the writers of, rejected manuscripts. No notice is taken of anonymous communications.*]

Postulates and Axioms

A STRONG committee, appointed, or rather re-appointed, for the purpose, reported last year to the British Association upon the Syllabus drawn up by the Association for the Improvement of Geometrical Teaching. I have only just seen a copy of the report, and I wish to point out that it incidentally touches in a misleading fashion upon a matter which, though primarily of only historical interest, is really of theoretical importance too, if not (in the strictest sense) for the special purpose of the committee; I mean upon the different ways of distributing the fundamental assumptions under the two heads of postulate and axiom.

Let us stop for a moment at the historical point of view. It is well known that the received text of Euclid, which we may consider represented by David Gregory's edition (Oxford, 1703), misplaces the assumption about right angles, the assumption at the base of the theory of parallels, and the assumption that two straight lines do not inclose a space. That is to say, whereas in the correct text these are the 4th, 5th, and 6th postulates, the received text makes them the 10th, 11th, and 12th common notions, or, as we usually say, axioms.

Now, when the report speaks of Euclid in this connection, it means something nearly identical with the received text. Not quite, however; for, though the language is not clear in all respects, it clearly says thus much, that Euclid divided the axioms into general and specially geometrical. But this is not the case in either text; for in both texts the first seven common notions are general, the 8th geometrical, and the 9th general again, nor is the 8th distinguished from the rest by its grammatical form. But whether you follow the received text or depart from both, it is unhistorical to affirm of Euclid what is not true of the correct text.

Let us now consider the theoretical significance of the two distributions. The case is thus stated by De Morgan, under *Eucleides*, in Smith's "Dictionary of Greek and Roman Biography," p. 66b:—"The intention of Euclid seems to have been to distinguish between that which his reader must grant, or seek another system, whatever may be his opinion as to the propriety of the assumption, and that which there is no question everyone will grant. The modern editor merely distinguishes the assumed *problem* (or construction) from the assumed *theorem*." This latter distinction is at least as old as Proculus; but to De Morgan it is Euclid's, at least as concerns right angles and parallels, that "seems most reasonable; for it is certain," he continues, "that the first two assumptions can have no claim to rank among common notions or to be placed in the same list with 'the whole is greater than its part.'" We need not pursue the modern editor's distinction further; but Euclid's acquires a more definite significance in relation to those generalised conceptions of space which, since De Morgan wrote these words, have almost passed into popular science. This in its generality is a difficult subject, but for the present purpose it is enough to regard plane geometry as a particular case of the geometry of points and lines on a given surface.

In this view the postulates specify the attributes of the plane which make plane geometry what it is. Thus the first three, whatever else they do, provide that the power of drawing diagrams shall not be restricted by boundaries, and the fourth, "all right angles are equal," affirms that a complete rotation is the same in quantity at all points; thereby the first three exclude surfaces having such a singular locus as a cuspidal line, and the fourth excludes surfaces having such a point as the vertex of a cone. Again the fifth excludes anticlastic surfaces, and the sixth synclastic ones and any which, like the common cylinder, returns into itself. Nothing remains but the plane and such developable surfaces as the parabolic cylinder to which *mutatis mutandis* everything in plane geometry will equally apply.

The axioms, on the contrary, specify no property of any class of surfaces. This is crucially instanced in the one axiom (the 8th, that things congruent are equal) which does concern figures traced on surfaces of only a limited class. For this axiom merely says that if things coincide they are equal, not that figures in different places may be brought to coincide.

The question may be asked whether this last assumption ought not to be premised somewhere; that is, whether the method of superposition ought not to have been vindicated by expressly assuming that any plane figure may be laid down on any plane so as to coincide with a portion of it. The omission is an extremely curious fact—in Euclid, I mean, for it is not at all remarkable in his successors. On the one hand, express statement is superfluous in the sense that the assumption is implied in the last two postulates; for the fifth affirms that the "measure of curvature" of the plane is not negative, and the sixth that it is not positive; between them it is naught, and therefore constant; but this is the condition of superposableness. On the other hand, express statement is indispensable in the sense that the student cannot do without it, because the theory of measure of curvature does not belong to elementary geometry.

The fact is that Euclid has drawn the line with what is really remarkable accuracy, but is only seen to be so in virtue of principles not discerned, I believe, by any one before Gauss. Whatever may be the explanation of this phenomenon, to ignore it in speaking of Euclid's postulates and Euclid's axioms is to depart from history where adherence to history would be instructive in theory too.

It is of course another question whether this distinction of Euclid's ought to be preserved in books intended to supersede Euclid. C. J. MONRO

Hadley, Barnet

Just Intonation

THAT Mr. Chappell misunderstands me is due partly to his confounding vibration numbers with their ratios. Thus $\frac{27}{16}$ is the vibration number of the supertonic, where $\frac{3}{2}$ is that of the tonic; while 524288 is not the vibration number of any musical sound, though the ratio $524288 : 531441 = 2^{19} : 3^{12}$ expresses an interval that may be picked out fourteen times in each octave of Mr. Colin Brown's keyboard. A still more complex interval $2^{22} : 3^{14}$ is found seven times in each octave.

I have followed Mr. Chappell's advice and purchased his sixpenny pamphlet, and having read it with the care it deserves, I can only say I dissent from a great part of it, especially where