by those functions, and we have a long disquisition on the geometrical representation of the elliptic integral of the first kind by an algebraic curve; while there is no mention of the late John Riddle's discovery, that the arcs of the curves by which circles on the sphere are represented in Mercator's projection are directly given by, and absolutely co-extensive with, the elliptic integrals of the first kind, the amplitude being simply the longitude on the sphere. We think this quite as simple and as important as the discussion of the lemniscata. If we are to go into geometry at all, it might be as well also to make some allusion to Dr. Booth's discussion of the spherical conics, and to Mr. Roberts's integration of the Cartesians.

Then, again, we have an account of Jacobi's geometrical theorem in its original form, depending upon a family of circles having the same radical axis, while the corresponding theorem, depending upon circles having two inverse points in common, given by Chasles (see his "Géométrie Supérieure," cap. xxxi., p. 533), which much more directly represents both amplitudes and moduli, is not mentioned explicitly, although it is involved in the geometrical exposition given of Landen's theorem.

We have also been unable to find any account of Jacobi's reduction of the integral of the third kind to the form—

$$\int \alpha \phi. E \phi : \Delta \phi$$

The transformations of the functions are worked out with great completeness, the results being tabulated in some rather formidable-looking, but really very convenient, schedules. This part of the work is carried almost to an extreme.

On the whole the book is one of the most important contributions to mathematical literature which has appeared for a long time. It is well done, and covers ground that was previously but ill occupied. It is clearly printed, and the fact that the proof-sheets have been revised by Mr. J. W. L. Glaisher is a guarantee for the correctness of detail. C. W. MERRIFIELD

OUR BOOK SHELF

Instruction in Photography. By Capt. Abney, R.E., F.R.S., &c., Instructor in Chemistry and Photography at the School of Military Engineering, Chatham. Third Edition. (London: Piper and Carter, Gough Square, Fleet Street, E.C., 1876.)

WE are very glad to find that Capt. Abney did not carry out the intention which he mentions in the preface of not producing another edition of his well-known "Instruction in Photography." That the little volume is widely known and appreciated is shown by the fact of its having reached a third edition, and we can only say that it well deserves its success. A photographer of the author's well-known skill and repute could not fail to be able to instruct others in his art, but when in addition he has gained large experience by continued practical teaching in such a school as that at Chatham his lessons become additionally valuable.

Capt. Abney does not enter much into theory, though he gives very good and simple accounts, illustrated by chemical equations, of the principal changes occurring during the processes described. We observe that he announces the forthcoming publication of a "Photography" among Messrs. Longmans' Text-books of Science, in which he proposes to deal more fully with the theoretical part of the subject. We shall look forward to this

with considerable interest; meanwhile, for practical instruction in the art this little book distances all competitors. R. J. F.

LETTERS TO THE EDITOR

[The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, or to correspond with the writers of, rejected manuscripts. No notice is taken of anonymous communications.]

Just Intonation

THE errors and oversights—in my paper in NATURE, (vol. xv. p. 159)—with which Mr. Chappell charges me, are imaginary. To make the matter clearer, the vibration numbers of a diatonic scale started from $\frac{3}{2}$ as a tonic are—

$$\frac{3}{2}, \frac{27}{16}, \frac{15}{8}, 2, \frac{9}{4}, \frac{5}{2}, \frac{45}{16}, 3.$$

In order to keep to the same part of the keyboard, let the last five notes be depressed one octave, and we get this series:—

$$1, \frac{9}{8}, \frac{5}{4}, \frac{45}{32}, \frac{3}{2}, \frac{27}{16}, \frac{15}{8}, 2,$$

where $\frac{3}{2}$ is the tonic, and 1 or 2 the subdominant. A similar

explanation applies to the scale starting from $\frac{4}{3}$ as tonic. With respect to the "comma of Pythagoras," I am not aware of any "generally adopted miscalculation." Who was the real discoverer of that interval is a matter of no consequence; but in a system such as the Pythagorean, which was tuned by true fifths, it would have been not only a very natural but an essential inquiry, "What definite number of fifths corresponds with another number of octaves?" This, without at all necessitating the supposition of the existence among the Greeks of instruments of an immense range in octaves, would be but an easy arithmetical calculation resulting necessarily in the conclusion that there was no exact coincidence between fifths and octaves, but that twelve fifths differed by a very small quantity from seven octaves. This small difference is therefore very aptly termed the Pythagorean comma. Now in the equal temperament system twelve fifths just coincide with seven octaves, so that the despised comma of Pythagoras is really a measure of the error of the equal temperament fifth. In fact, putting P for the comma in question—

$$P = \left(\frac{3}{2}\right)^{12} 2^{-7} \therefore 2^{\frac{7}{12}} = \frac{3}{2} P^{-\frac{1}{12}}$$

so that if 2N be the vibration number of the lower tone of an E T fifth, that of the upper tone will be $3NP^{-\frac{1}{12}}$, which is in error by $3N(1 - P^{-\frac{1}{12}})$ vibrations, giving rise to a number of

$$\text{Beats} = 6N(1 - P^{-\frac{1}{12}})$$

per second in the E T fifth. Not, however, that it is necessary to allude to Pythagoras or seven octaves to get those beats.

The grounds upon which Mr. Chappell declines to accept 24, 27, 30, 32, 36, 40, 45, 48, as representing the vibration numbers of the diatonic scale are not very clear, certainly; and repudiating these, he can of course have no sympathy with Colin Brown's keyboard.

To revert shortly to this, the subject of my previous communication,—in his new and very interesting work on "Temperament," Mr. Bosanquet has given a description of Colin Brown's keyboard, but in so peculiar a manner that it is really difficult to recognise the instrument at all, and neither its elegance nor simplicity are brought out as I think they should be.

A. R. CLARKE

Ordnance Survey, Southampton, January 9

South Polar Depression of the Barometer

I THINK it probable that on this subject Mr. Murphy's views and my own might have appeared more in harmony if we had neither of us expressed them with so much brevity. In my letter on Ocean Currents in NATURE, vol. xv. p. 157, I was incidentally led to speak of the barometric depression round the South as greater than that round the North Pole. In speaking