how it is related to the entity called matter, is no less mysterious than how subjectivity may be a property of matter. Energy moreover may be divided, Why may not subjectivity? Energy exists in isolated as well as in grouped and combined forms, Why may it not be so with subjectivity? Energy exists in a potential form when opposing forces neutralise each other—that condition of matter we call rest. May not subjectivity exist in a potential form when opposite kinds of subjective states tend to establish themselves in the same material mass—so constituting that condition of matter which we call unconsciousness? Energy is only kinetic or active, that is, only shows itself in the form best realised by us when one force has of all the others the ascendency, or is the expression of their united tendencies. May it not be so of subjectivity that it only develops into its active and best recognised form when one kind of subjective state has the ascendency, or is the expression of the united subjective states? Thus, as energy potential is rest, so subjectivity potential is unconsciousness. As kinetic energy is motion, so active subjectivity is consciousness.

In this way while all matter is subjective or susceptible of consciousness, this subjectivity seems to exist in the potential form only in all but organisms possessed of a nervous system. In the active nerve fibre the subjectivity of matter appears alone to be active or conscious, while the complex organisation of the nervous systems of the higher animals alone permits of matter rising to the powers of mind by harmoniously combining many subjective states so as to build them up into perception, understanding, memory, imagination, reason, invention, and judgment.

WM. S. DUNCAN

Stafford, November 9

Meteor

I OBSERVED from a high point overlooking the Weald, on the night of November 6, about the time Mr. Nostro mentions, a large meteor fall from a point a little below the zenith in the northern sky. It burst twice, emitting bluish sparks in doing so, once shortly before it disappeared, and the second time on its disappearing. Could it have been the same meteor seen from different positions?

I could not be positive to a point or two as to its exact magnetic bearing, but I do not think I am far wrong in saying it fell almost due north from where I observed it.

CECIL H. SP. PERCEVAL

Pulborough, November 18

THE PRESENT STATE OF MATHEMATICAL SCIENCE

AT the meeting of the Mathematical Society on November 9, Prof. H. J. S. Smith gave an address on this subject, in which he excluded all reference to applied mathematics. "I shall regard it," he said, "as a fortunate circumstance if my successor when he, in his turn, is looking round for a subject for his own presidential address, should be attracted by a domain on which I must myself decline to enter, but of which he, better perhaps than anyone among us, is fitted to give us a clear and comprehensive view." He professed to offer only fragmentary remarks, "hoping that even such fragmentary remarks may not be without their use if they serve to remind us of the vastness of our science, and yet of its unity; of its unceasing development, rapid at the present time, promising to be still more rapid in the immediate future, and yet deriving strength and vitality from roots which strike far back into the past, so that the organic continuity of its gigantic growth has been preserved throughout. In every science there is a time and place for general contemplations, as well as time for minute investigations. And it is a rule of sound philosophy that neither of these shall be neglected in its proper season ('itaque alternandæ sunt istæ contemplationes,' says Lord Bacon, 'et vicissim sumendæ ut intellectus reddatur simul penetrans et capax')."

Touching upon a charge brought against the *Proceedings* of the Society that its memoirs "have shown and still continue to show a certain partiality in favour of one or two great branches of mathematical science to the comparative neglect and possible disparagement of others," it might be rejoined "with great plausibility that ours is not a blamable partiality but a well-grounded preference. So great (we might contend) have been the triumphs achieved in recent times by that combination of the newer algebra with the direct contemplation of space which constitutes the modern geometry—so large has been the portion of these triumphs, which is due to the genius of a few great English mathematicians—so vast and so inviting has been the field thus thrown open to research, that we do well to press along towards a country which has, we might say, been 'prospected' for us, and in which we know beforehand we cannot fail to find something that will repay our trouble, rather than adventure ourselves into regions where, soon after the first step, we should have no beaten tracks to guide us to the lucky spots, and in which (at the best) the daily earnings of the treasure-seeker are but small, and do not always make a great show, even after long years of work. Such regions, however, there are in the realm of pure mathematics, and it cannot be for the interest of science that they should be altogether neglected by the rising generation of English mathematicians. I propose, therefore, in the first instance, to direct your attention to some few of these comparatively neglected spots."

The foremost place is assigned, by Prof. Smith, to the Theory of Numbers. "Of all branches of mathematical knowledge this is the most remote from all practical application, and yet, perhaps more than any other, it has kindled an extraordinary enthusiasm in the minds of the greatest mathematicians. We have the examples of Fermat, of Euler, of Lagrange, Legendre, of Gauss, Cauchy, Jacobi, Lejeune Dirichlet, Eisenstein, without mentioning the names of others who have passed away, and of some who are still living. But, somehow, the practical genius of the English mathematician has in general given a different direction to his pursuits; and it would sometimes seem as if we measured the importance of the subject by what we find of it in our best treatises of algebra, or as if we accepted the denunciations of Auguste Comte, and regarded the votaries of the higher arithmetic as reprobate of positive science, as moving in a vicious circle of metaphysical ideas, and as guilty of a great crime against humanity in the pursuit of knowledge beyond the limits of the useful. . . . I would rather ask you to listen to what is recorded of the great master of this branch of science."

Gauss (we are told by his biographer) held mathematics to be the queen of the sciences, and arithmetic to be the queen of mathematics—"She sometimes condescends to render services to astronomy and other natural sciences (so spoke the great astronomer and physicist); but under all circumstances the first place is her due." A citation was also made from Jacobi's Life of Göpel: "Many of those who have natural vocation for pure mathematical contemplation find themselves in the first instance attracted by the higher parts of the theory of numbers."

Three great departments of arithmetic were instanced: The arithmetical theory of homogeneous forms (or quantics)—"It is a memorable fact that some of the greatest conceptions of modern algebra had their origin in connection with arithmetic, and not with geometry or even with the theory of equations." In the "Disquisitiones Arithmeticæ" are given for the first time the characteristic properties of an invariant and a contravariant (for ternary quadratic forms). "But the progress of modern algebra and of modern geometry has far outstripped the progress of arithmetic; and one great problem which arithmeticians have now before them is to endeavour to turn to account for their own science the great results which have been obtained in the sister sciences. How difficult this problem may prove is, perhaps, best attested by the little advance that has been made towards its solution." As an example, the researches of Cayley, Bachmann, and Hermite on the algebraical problem of the automorphies of a quadratic form, containing any number of indeterminates, were alluded to. Omitting many other points which were brought out, we can only mention the second department of arithmetic, the theory of congruences. In connection with this division, Prof. Smith also dwelt in detail upon the subject of complex numbers. "The last part of arithmetical theory to which I would wish to direct the attention of some of the younger mathematicians of this country is the determination of the mean values, or the asymptotic values of arithmetical functions. This is a field of inquiry which presents enormous difficulties of its own; it is certainly one in which the investigator will not find himself incommoded or crowded out by the number of his fellow-workers. 'Nemo est fere mathematicorum,' said Euler, in the last century; 'qui non magnam temporis partem inutiliter consumpserit in investigatione numerorum primorum;' but I do not think that (as a rule) the mathematicians of the present day have any reason to reproach themselves on this score." The speaker then pointed out what had been done in this direction since the days of Euler. "I do not know that the great achievements of such men as Tchebychef and Riemann can fairly be cited to encourage other and less highly gifted inquirers, but at least they may serve to show two things—first, that nature has fixed no impenetrable barrier to the further advancement of mathematical science in this direction; and secondly, that the boundary of our present knowledge lies so near us that at any rate the inquirer has no very long journey to take before he finds himself in the unknown land. It is this peculiarity, perhaps, which gives such perpetual freshness to the higher arithmetic. It is one of the oldest branches —perhaps the very oldest branch—of human knowledge, but yet its tritest truths lie close to some of its most abstruse secrets. I do not know that any more striking example of this could be furnished than by the theorem of M. Tchebychef. To understand his demonstration requires only such algebra and arithmetic as are at the command of many a schoolboy; and the method itself might have been invented by a schoolboy with the genius of Pascal or of M. Tchebychef."

Passing on to other branches of analysis, Jacobi's method of approximation ("a natural extension of the theory of continued fractions"), Lejeune Dirichlet's researches on complex units and his great generalisation of the theory of the Pellian equation, Liouville's treatment of irrational quantities, Lambert's proofs that neither π nor π^2 nor ϵ are rational with M. Hermite's extensions, who, though he has proved that ϵ is a transcendental irrational, declines entering on a similar investigation for the number π, but leaves this to others, adding, "Nul ne sera plus heureux que moi de leur succès, mais, croyez m'en, il ne laissera pas que de leur en coûter quelques efforts"—all came in for a notice.

Another class of questions mentioned were those which relate to the transcendental or algebraic character of developments in the form of infinite series, products, or continued fractions. The theorem of Eisenstein and M. Hermite's recent investigation of it, lately communicated to the Society, "are amply sufficient to awaken the expectation of great future discoveries in this almost unexplored field of inquiry."

Amongst important objects for mathematicians to set before them were named the advancement of the integral calculus ("confessedly all important in the applications of mathematics to physics"). In this connection the theory of differential equations and of singular solutions came in for a detailed notice, as also did the subject of elliptic functions.

Towards the close of the address, Prof. Smith said: "I am convinced that nothing so hinders the progress of mathematical science in England as the want of advanced treatises on mathematical subjects. We yield the palm to no European nation for the number and excellence of our text-books of the second grade; I mean of such text-books as are intended to guide the student as far as the requirements of our University examinations in honours are concerned. But we want works suitable for the requirements of the student when his examinations are over—works which will carry him to the frontiers of knowledge in certain directions, which will direct him to the problems which he ought to select as the objects of his own researches, and which will free his mind from the narrow views which he is apt to contract while getting up work with a view to passing an examination, or, a little later in his life, in preparing others for examination. Can we doubt that much of the preference for geometrical and algebraical speculation which we notice among our younger mathematicians is due to the admirable works of Dr. Salmon; and can we also doubt that if other parts of mathematical science had been equally fortunate in finding an expositor, we should observe a wider interest in, and a juster appreciation of, the progress which has been achieved?

There are, of course, other works besides those of Prof. Cayley and Dr. Salmon to which I might refer; there is, for example, the work of Boole, on Differential Equations; and there are the great historical treatises of Mr. Todhunter so suggestive of research, and so full of its spirit; we have also a recent work by the same author on the functions of Laplace, Lamé, and Bessel. But the field is not nearly covered . . . There are at least three treatises which we sadly need, one on definite integrals, one on the theory of functions in the sense in which that phrase is understood by the school of Cauchy and of Riemann, and one (though he should be a bold man who would undertake the task) on the hyperelliptic and Abelian integrals.

Geometry, and some other subjects, were hardly more than mentioned.

> "Verum hæc ipse equidem spatiis exclusus iniquis
> Prætereo, atque aliis post memoranda relinquo."

"In these days, when so much is said of original research, and of the advancement of scientific knowledge, I feel that it is the business of our Society to see that, so far as our own country is concerned, mathematical science should still be in the vanguard of progress. I should not wish to use words which may seem to reach too far, but I often find the conviction forced upon me that the increase of mathematical knowledge is a necessary condition for the advancement of science, and, if so, a no less necessary condition for the improvement of mankind. I should tremble for the intellectual strength of any nation of men whose education was not based on a solid foundation of mathematical learning, and whose scientific conceptions, or in other words, whose notions of the world and of the things in it, were not braced and girt together with a strong framework of mathematical reasoning. It is something to know what proof is, and what it is not; and where can this be better learned than in a science which has never had to take one footstep backward, and which is the same at all times and in all places. . . . I shall be more than satisfied if anything that may have fallen from me may induce any one of us to think more highly than he has hitherto done of the first and greatest of the sciences, and more hopefully of the part which he himself may bear in its advancement."

The address, delivered in the author's effective style, was frequently applauded by an appreciative group of members. On the proposal of Prof. Cayley it was resolved (with the author's consent) that the address should be printed in the *Proceedings*.