volumes, the first of which is to appear in the present month, the second in the spring, and the third during the summer of 1876, and will be arranged under the following divisions:—(1) Motions of the Bodies of the Solar System; (2) Spherical Astronomy; (3) Theory of Instruments; (4) Stellar Astronomy; (5) Mathematics; (6) Geodesy; (7) Physics, and (8) Various. A portrait and short life of Bessel is to be attached to the first volume. Dr. Busch's complete list of Bessel's works, inclusive of astronomical notes in various scientific periodicals, as the *Monatliche Correspondenz* of Zach, the *Berliner Jahrbuch*, &c., which is appended to vol. xxiv. of the Königsberg Observations, contains 385 titles; but many of the shorter contributions being of minor or ephemeral interest, it is probable that the selection proposed will include all the writings of the illustrious astronomer which can now possess value.

AMONG THE CYCLOMETERS AND SOME OTHER PARADOXERS*

II.

MR. H. HARBORD, who hails from Hull, has put forth three letters, with which we have been favoured. "The Circle Squared" (in November 1867) has, we guess, been noticed by Prof. De Morgan. There is a nicely drawn diagram, two concentric circles, two squares, said to be their respective equivalents, all in black; an equilateral triangle and its circumscribing circle in red ink; the former is described on a side of the smaller square, and the red circle passes through the extremities of the same side. A statement is made, which appears to be a statement and nothing more, for it proves nothing. From "Squaring the Circle" (April 15, 1874) we learn that the writer has leisure (*fons et origo mali!*), and so has ventured to amuse himself by considering the relation of the equilateral triangle, the square, and the circle. He obtains the positive altitude of an equilateral triangle on a side of the square to be 7·75448559771125, and requires the exact side of the square and the proportion of the triangle to the square and the equivalent circle. He winds up, like many of his race, with the following reflections:—"I think if the learned in geometry, mathematics, and trigonometry, abandoned approximating theories, and would take the trouble to elucidate the above-stated propositions, they would undoubtedly be able to subvert all anomalous and vague theorems, free the study of geometry, &c., from ambiguity, enable tutors to explain correctly, remove burthens imposed on the mind of the pupil, and establish a system of teaching which shall be correct and intelligible, for it is evident the result of minute calculations proves there is no mystery in geometry, mathematics, or trigonometry; they are uniform, and may be more easily taught and comprehended with perfect truthfulness without approximation." To prevent trouble, this man of leisure appends the rule; it is: Add one-seventh to the altitude, and we get the base; and so on. Not satisfied with the above remarks, we have a note to the "learned" (see above); and it is the following curious sentence:—"It is worthy of remark, and more especially to those who are interested in the forthcoming 'Transit of Venus,' when the true distance of the earth from the sun is to be determined, and a difference of about three millions of miles accounted for, to be in a position to prove the fact. Now all this can be accomplished by anxious, minute observation and correct calculation!" He then appends (we don't see the connection): "Length of an arc of one degree, ·017 . . . to twenty-seven places final." We got the last communication a few days ago; it is, "Construction of the Perfect Ellipse" (Dec. 22, 1874). This is a fine large figure on a sheet of paper some eighteen inches by fourteen. He finds that the true ellipse is only to be described on the perpendicular of the equilateral triangle. Mr. Harbord has evidently an idea, and that is, that the equilateral triangle is the key to unlock many geometrical mysteries.

Mr. Michael Callanan, of Cork (September 1874), "is in a position to demonstrate before any appointed number of scientific gentlemen, the perfect quadrature of the circle, rendering it as clear as the most simple, plain (*sic*) rectilineal figure. The Circle, that colossal mystery, to prove the area of which has been looked upon as the climax of geometrical science; and, although the object of search by the mathematicians of all nations, their greatest efforts have failed; every attempt, as yet, to square the circle being undemonstrable, and offering no reward to the anxious investigator beyond mechanical or approximate measurement—a manipulation of the great problem. My solution will be found original and thoroughly demonstrative in all its details, without having anything whatever to do with the given or polygonal rules for approximation. Entirely new ground is opened up in the path of science which I have chosen, guided only by positive mathematical laws, combined in the most strict logical arrangement, and thus *proved to demonstration*. I now proclaim the absolute fact of being able to set aside for ever any further doubt as to the complete quadrature of the circle, and thereby confound those scientific prophets who pronounced it an impossibility." Local circumstances offer many impediments in bringing the matter before the scientific world, and "being a geometrical secret, the law of Patent cannot be applied." He then puts himself in the same position with other inventors and discoverers, but he asks for an accredited tribunal "from which I would ask an impartial hearing, so as to verify these statements, and also to be identified and protected as the discoverer." For this end he is willing to attend at any selected place in England, Ireland, or Scotland. He then glances at some of the immediate results in the realisation of this problem. "At the proper time will be published a comprehensive work, including all the new diagrams necessary to carry out and complete the demonstration." And this is all we know of Mr. Callanan's "Secret of 'the Circle' solved."

Our next claimant for notice is not a Circle-squarer, but he would certainly have got a warm corner in the "Budget." Middleton's "new process of measuring the height of the sun," an observation for latitude demonstrated by geometry, proving the sun's height less than the latitude of observer. On this leaflet our paradoxer says, "the sun's height is under 3,000 miles." The principles of this discovery are published in the *West Londoner*. Mr. Empson E. Middleton, *Poet* (Naval and Military Club), sends us a further document (May 5th, 1873): "£100 Reward to the first who disproves the following Diagram—Middleton's Geometrical Proof that the Earth is Flat." Proof is said to turn upon the SPHINX SOLUTION,—"a globe demands six cardinal points." Having disposed of this point to his satisfaction, he "challenges all the mathematicians to support their statement that a perpendicular line and a line at right angles are the same; one is *flat*, the other *upright*. I undertake to prove that the perpendicular line is *not* the same as a line at right angles, though the two are utterly confused in every school-book of the day. I undertake to meet in public and to confute any of our mathematical professors who may have the manliness to come forward and discuss this question of the perpendicular, a question which forms the fundamental basis of the whole science of geometry, and is of the very first importance. I remain faithfully, to the Majesty of Truth." Mr. Middleton has published a translation of "the first two books of the Æneid of Virgil" to supersede Mr. Connington's (*sic*): he has a work "On Space" unpublished, and one "On Man" awaiting demand.

Naturally, after this we should turn to Parallax, or to

* Concluded from vol. xii. p. 560.

Mr. John Hampden, but we have preserved nothing from either of these paradoxers. The former has gained notice in the *Budget* (we are sorry to record the recent death of another able opponent of these views, Mr. T. T. Wilkinson, F.R.A.S.); the latter has figured before the public in the daily papers. A consequence of Mr. Wallace's acceptance of Mr. Hampden's wager is that the former gentleman has for nearly five years been the subject of continuous libels (see letters in *Daily News*, March 11, also March 9). It is to be hoped that an enforced retirement of a twelvemonth will result in Mr. Hampden's learning wisdom and the keeping of the peace towards Mr. Wallace and all others.

In De Morgan's account of Taylor the Platonist (*B* of *P*. pp. 182, &c.) there is nothing said of an early work of his, "The Elements of a new method of Reasoning in Geometry applied to the rectification of the Circle" (1780), "a juvenile performance lost or suppressed" (biographer in *Penny Cyclopædia*). We have examined this work, but it is impossible to give an account of it here; the solution is approximative.

The malady (*Malus cyclometricus*) is not confined to the Old World; our concluding instances will be drawn from a Geometry published at New York, and from a treatise specially devoted to the subject and printed at Montreal. We have not a copy of Mr. Lawrence S. Benson's "The Elements of Euclid and Legendre, with Elements of Plane and Spherical Trigonometry," but he has sent us "A Reply to Criticisms on Benson's Geometry." This will answer our purpose better, for the defence shows that the malady is confirmed. The symptoms are even more exaggerated than in Mr. J. Smith's case, for whereas his circumference ("Budget," p. 318) shrank into exactly $3\frac{1}{8}$ times his diameter, Mr. Benson's has shrunk to only 3 times! Where all this will end if the malady increases it is hard to say; perhaps the unfortunate circle will shrink up into its own centre! Opponents had pointed out "that when the areas of polygons inscribed in the circle are computed by means of plane triangles, a result is obtained for the inscribed polygons greater than $3R^2$," and they reasoned (it seems to us irresistibly) "that it is impossible for a circle to be less than a figure inscribed in the circle." Mr. Benson trusts, however, that after fourteen years' application to mathematics he will not be thought to have committed so egregious a blunder as to bring himself into direct contradiction of the self-evident proposition, "A part is less than the whole." He commences his defence with the statement that Torelli contends that the circle will be proved to be the square on its diameter exactly as 3 to 4. He then goes on to instance that Playfair ("Euclid," p. 307) demonstrates that Torelli's proposition is true on *two conditions*. Is it credible that Mr. Benson should proceed to say: "The fact that the proposition is true 'on two conditions' prevents the proposition from being *false*, for a false proposition can be true on *no* condition." The conclusion of the whole matter is that he replies to the inquiry, "How is it that reasoning from plane triangles for the computation of the areas of polygons, and reasoning from the ratios of rectangles, when they are all rectilinear magnitudes, that different and conflicting results are obtained?" that "the reasoning on the *ratios* and *rotation* of surfaces involves their *relation* to each other; whereas the computation of the plane triangles involves their *boundaries*: and since for the QUADRATURE OF THE CIRCLE *the relation between the circle and a certain rectangular space is required*, it is evident that the proper mode of reasoning is by means of the relation of the ratios of the small rectangles inscribed in the circular spaces to the ratios of the sums of those rectangles, or of the whole rectilinear figures; or by means of the rotation of rectilinear and curvilinear surfaces around a common axis—and not by the process of continually doubling the number of sides of the polygons described about the circle; since the sides do not reach the circumference, this process gives an approximate result only, which is inconsistent with the strictness of geometrical reasoning." We do not profess to follow the writer's reasoning, but hold fast by the *terra firma* which he appears to discard.

"The Circle and Straight Line" is a work got up in an elaborate and elegant dress: it consists of Parts I., II., III., and a supplement in brown binding, and a duplicate of the supplement in green (there is a portion of a flyleaf additional in the former supplement, or else the two copies appear to be identical). Further, there is with each a book of plates, all most clearly drawn, and the diagrams protected by slips of tissue paper. Evidently the author, John Harris, or Kuklos, is not a needy man. Let us gather from Mr. Harris's preface the object he has in view. Deeming the solution of the geometrical problem which demonstrates the relation of the circle to the straight line to be peculiarly of public importance, he gives a statement of what he has done in the matter. "The discovery of the solution was communicated by letter, dated 29th of December, 1870, accompanied with demonstration, &c., to the Astronomer Royal." There was, the author admits, imperfection and error in the case as then presented. The Astronomer Royal declined to examine the case. In January 1873 the papers were presented to the President of the Royal Society (still Sir G. B. Airy), "with a request (claim) in writing to have the case judicially examined by that Society." The documents were returned; they met with a similar fate at the hands of the Professors of M'Gill College. The subject is to describe a circle (or circumference) equal in length to a given straight line, and to draw a straight line equal in length to the arc of a circle, "accompanied with demonstration that the conditions of the requisition have been mathematically fulfilled. We publish our solution with the distinct statement that it is essentially in strict accordance with that scientific system known as Euclid's. We claim to have our demonstration admitted or disproved, and we challenge objection or adverse argument on that system." We shall first convince our mathematical readers, on Kuklos's own summing up ("Corollary," p. 34), that he is wrong, and then, on the charitable supposition that he is willing to be convinced, point out where we consider he has failed. We shall take the last sentence of the Corollary cited above: "Wherefore, if a square be inscribed in a circle, the ratio of the inscribed square to the circle is the ratio of nine to ten." It will be seen that this gives for the value of π, $\frac{20\sqrt{2}}{9}$, that is 3·142696; not a very close approximation to the accepted value. But, of course, in arguing with Mr. Harris we must go over his work and point out, if possible, where he has tripped. We commence with enunciating his Theorem A.: "If an arc containing one-eighth of a circle be applied upon a straight line, and from the terminal extremity of the arc a perpendicular be drawn intercepting the straight line, and if from the arc one-tenth thereof be cut off, then, if the remaining arc (to wit, the arc containing nine-tenths of the whole arc) be rolled upon the straight line, the point of contact shall be the same point on the straight line intercepted by the perpendicular drawn from the terminal extremity of the whole arc." $B\,M$, $B\,n$ are taken to be the two arcs, and O, d are taken to be the corresponding points to M, n, on the tangent at B, also D is the foot of the perpendicular from M on the same tangent. Mr. Harris's object is to show that $D\,d$ coincide: *if* they did, then we would admit that he has proved his point; but on p. 22, line 13 (all his previous working having been sound, though somewhat tediously put), he has "$c\,d$" instead of $C\,D$ (his $c\,d$ is a misprint, we presume, for $C\,d$), and then easily gets to his desired conclusion. We would ask him *how* he gets "$C\,d$." Again, on p. 24, third line from bottom of page, we tell him that "$D\,O$ is one-tenth of $B\,O$" is a cool assumption, and we also ask him how he gets the last line on p. 27. These crucial points occur in "independent proofs" of the same theorem; they are pure "beggings of the question," we believe. This is all we have to say on Part I. Part II. opens with an admirable motto (reminding us herein of Mr. James Smith), "Prove *all* things; hold fast *that* which is *good*." Having *proved* then the previous theorem, he *holds* fast to *that*, and proceeds to the "construction of the circle;" his object being "to make manifest the great importance of the circle as one of the fundamental facts belonging to the Plan of Creation." As we consider the foundation wrong, until Prop. A is proved, we shall not follow the writer through the twenty-four pages of rather obscure mathematics devoted to this subject. We come next to "Mathematics and the Art of Computation." Starting from what he has (as he thinks, we will say) just proved, viz., that "the difference of the quadrant and the chord of the quadrant is an aliquot part of the quadrant and of the chord, and that the number of those equal parts contained in the chord being nine—the quadrant contains ten": because he finds in this "conclusive evidence that the (so-called) Arabic system of notation is is not an artificial human contrivance, but a great natural fact of a primary character, a fundamental part of the Science of Creation." Further down he speaks of many persons preferring "with a strange, and, as it would seem, with an increasing perversity, to cultivate the *thorns and thistles*, leaving the *good seed* as not worth utilising." He is then careful to state that by "thorns and thistles" he does not mean the modern methods of mathematical analysis. Still, "is it, or is it not, true that the language of mathematics is fast becoming an unknown tongue to ordinarily educated men, and that those to whom it is known can scarcely hold converse with their fellows (on any scientific subject) in ordinary language without a feeling of condescension, and scarcely without a feeling of impropriety? . . . . Is it true that the mathematician does now, *in some degree*, regard his fellow-worker who is unpractised in the calculus and non-conversant with differential methods as but little better than a publican and heathen?" We will not undertake to answer this question, but perhaps our author's ground for this opinion is the reputed division of the human species by the "Cambridge Wrangler" into those who understand the differential calculus and those who do not. He himself goes on to say, "If it be true that such a result does manifest itself in any considerable degree, it may be pronounced decidedly unwholesome and bad—bad for science and bad for civilisation—because mathematical knowledge is a necessity to science and a necessity to civilisation." This we admit. He then reiterates the statement that he knows that examination will show his demonstration of the quantitive (*sic*) ratio of the perimeters of the circle to the diameters is "mathematically incontestable." He then goes into an examination of Prop. XIII., Book V., of Brewster's Legendre: "The surface of a regular inscribed polygon and that of a similar polygon circumscribed, being given, to find the surface of the regular inscribed and circumscribed polygons having double the number of sides." Among other objections, he objects to the italicised statement (Prop. XIV., "Legendre"), "We shall infer that the last result expresses the area of the circle, which, *since it must always lie between the inscribed and circumscribed polygon*, and since these polygons agree as far as a certain place of decimals, must also agree with both as far as the same place." His objection to the whole method is "in the omission to observe that comparison has to be made between a continuous curved line (the circle) and a continuous straight line (the diameter)." And then, as elsewhere, he indulges in metaphysics. Part III. begins with Curvature and ends with Theology. "A human science which does not distinctly recognise the primary truths of theology as its ultimate basis, is not based on reality; it has not and cannot have any actual and secure foundation. If the science of England is not so based, no matter what seeming progress may for a time be made, whenever the trial comes it will be as the house built on the shifting sand, and, if not destroyed by sudden catastrophe, will eventually become a ruin, together with the civilisation which rests upon it." Our safety then, we presume, Kuklos would have us believe, is to believe in $\pi = \frac{20\sqrt{2}}{9}$. The supplement has "Supplementary Illustrations" and Tables. The work is printed at Montreal.

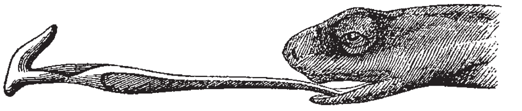
The conclusion of the matter is, that there are Cyclometers and Cyclometers. We have endeavoured to give a fair presentment of the several kinds by giving as far as possible their views in their own words. The majority of their writings evidence great waste of ingenuity, which, had it been otherwise directed, might have resulted in works of utility instead of in such utterly trivial work as it has done.

To any who may be thinking of taking up this "curiosity of literature," not having done so hitherto, we say emphatically, "Don't."

SCIENCE IN GERMANY

(*From our own Correspondent.*)

In Wiedersheim's recently published book, "*Salamandrina perspicillata* und *Geotriton fuscus*," two very little-known tailed amphibians (Urodela) are described and compared anatomically, which, by their entire organisation, stand at the two opposite limits of the Salamandrinæ that are known to us, representing the highest and the lowest form of these. *Salamandrina perspicillata*, which is rather a land than a water animal, seems to be found only in the western half of Italy; it is a prettily coloured, small, and slender animal, which lives on insects, and during the dry summer months continues in a kind of summer sleep, but in winter it is found in full vital activity. In its skull are almost entirely wanting the cartilaginous parts denoted as the "primordial cranium," so that in this it rises above all other Salamandrinæ, and comes near the Reptiles. In accordance with this, also, is the existence of a cavity in the base of the skull (sella turcica), the prolongation of the frontal bone (frontale) into the eye cavity, and a roofing-over of the latter; lastly, the absence of a special nose-partition (which, again, quite characterises the Reptiles). On account also of the course of development of its vertebræ, and the numerous bones of its carpus and tarsus, *Salamandrina perspicillata* must stand at the top of the Salamandrinæ; its divided kidneys, again, suggest the reptile, so that we must look on this animal as a form rendering



Tongue of *Geotriton fuscus*.

possible the transition from the Amphibia to the Reptilia, and which, on account of its peculiarities, might represent a separate family. *Geotriton fuscus*, on the other hand, holds quite a different position. If, in view of the numerous anatomical relations adduced, we are able, commencing with *Salamandrina perspicillata*, and passing through the various water salamanders (Tritons), to the land salamander (*Salamandra maculata*), to form a descending series of ever less-developed forms, *Geotriton fuscus* comes at the lower end of the series, for in many respects it ranks with the lowest Amphibia generally, the Perennibranchiata. Indications of this appear in the fewness of bones in the skull and the tarsus, the extended double cone form of the soft-cartilaged vertebræ;