acceptance which his endeavour to solve the climatal problems of past epochs by astronomical computation has very deservedly met with on the part of Geologists, his denial of the possibility of a thermal circulation in the ocean is utterly repudiated alike on mathematical and on experiential grounds, by those whose authority as physicists ought to make him feel less confident in his own conception of the question. W. B. CARPENTER

Source of Volcanic Energy

A FEW words of explanation are necessary concerning my letter which appeared in NATURE, vol. xii. p. 396. Mr. Mallet's *prime* source of energy for producing tangential pressures is the force of cohesion developed in a cooling globe, gravitation giving only partial assistance; and when I spoke of "gravitation of the whole mass to itself," I wished to convey that, setting aside altogether the force of cohesion and its accompanying motions, there still remains the force of gravitation, which, acting in a globe of such size as the earth, and composed of heterogeneous materials, must of itself produce enormous local pressures.

Mr. Fisher objects to my supposing the possibility of the development of heat without room being left for motion, but so far as I understand the doctrine of energy, it is only necessary to have *force* for the production of heat when motion is impossible.

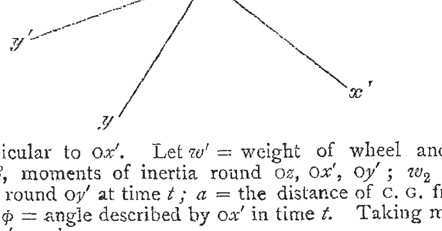
In Mr. Fisher's interesting paper his objection appears to be to the localisation of *fusing*, and not to the localisation of heat, fusing in some cases being prevented by the accompanying pressure. But in my little diagram I attempted to explain that the forces producing the high temperature might act in one set of strata, the neighbouring strata above and below at the same time being under much lower pressure, the pressure upon them being equal to the pressure of the rocks doing the work, minus the cohesion of said rocks; this difference of pressure being sufficient to allow one set of rocks to melt while others are crushed.

Kenmare, Co. Kerry WM. S. GREEN

Gyrostat Problem: Spinning-top Problem

IN vol. xi. p. 424 is given the solution, by Sir W. Thomson, of his gyrostat problem at p. 385. I venture to send a slightly different method * of obtaining the result (far inferior to Sir W. Thomson's in elegance and simplicity), in which Euler's equations for the motion of a rigid body about a fixed point are employed.

1. Take point of suspension for origin; the string for axis of z. The axis of the wheel Ox' revolves in horizontal plane xOy with uniform angular velocity Ω, and the wheel revolves round its axis Ox' with angular velocity w_1. The weight of wheel and axis will have moments round an axis Oy' in horizontal plane



perpendicular to Ox'. Let w' = weight of wheel and axis; A, B, B, moments of inertia round Oz, Ox', Oy'; w_2 angular velocity round Oy' at time t; a = the distance of C. G. from Oz, $xOx' = \phi$ = angle described by Ox' in time t. Taking moments about Oy', we have

$$B\frac{d\,w_2}{d\,t} + \overline{A - B}\,w_1\,\Omega = w' a g \quad . \quad . \quad . \quad (1)$$

(Pratt, "Mech. Phil." 446). Also since there is no velocity about an axis in horizontal plane perpendicular to resultant axis of $w_1\,w_2$,

$$w_1 \sin.\,\phi - w_2 \cos.\,\phi = 0 \quad . \quad . \quad . \quad . \quad (2)$$

where $$\phi = \Omega\,t.$$

$\therefore \frac{d\,w_2}{d\,t} = w_1\,\Omega \sec.\,^2\phi = w_1\,\Omega$ for $t = 0$ in (1), since w_1, Ω are independent of the time; whence (1) becomes

$$A\,w_1\,\Omega = w'\,a\,g,$$

where $$A = w\,k^2, \Omega = \tfrac{1}{4} \quad . \quad . \quad . \quad . \quad \text{Q.E.D.}$$

2. A similar question (concerning a spinning top) was proposed in the Senate House, Cambridge, in 1859, of which indeed the preceding is a particular case.

A uniform top spins upon a perfectly rough horizontal plane, its axis being inclined to the vertical at a constant angle α, and revolving about it with constant angular velocity Ω. Prove that the velocity of rotation of the top about its axis must be $\frac{(a^2 + k^2)\,\Omega^2 \cos.\,\alpha + g\,a}{k_1{}^2\,\Omega}$, where a is the distance of the centre of gravity from the extremity of the peg, k' k the radii of gyration about the axis of figure, and about an axis through C. G. perpendicular to it respectively. Take O, the extremity of the peg, which remains fixed, as origin, and let OZ' be position of axis at any time t; $OG = a$; $ZOZ' = \alpha$. Let M = mass of the top; A, C, C, moments of inertia about Ox', Oy', OZ' (rectangular axes moving with the top); $w_1\,w_2\,w_3$, angular velocities about Ox', Oy', OZ' at time t.

The intersection of planes xOy, $x'Oy'$ will move round OZ with angular velocity Ω. Let ϕ = angle which Ox' makes with this line.

If we take moments about Ox', we have by Euler's equations (Pratt, art. 446)—

$$\frac{A\,d\,w_1}{d\,t} + \overline{C - A}\,w_2 w_3 = M g a \cos.\,z y^1 \quad . \quad . \quad (1)$$

Also $w_1 = \Omega \sin.\,\phi \sin.\,\alpha$, $w_2 = \Omega \cos.\,\phi \sin.\,\alpha$, $w_3 = \frac{d\,\phi}{d\,t} + \Omega \cos.\,\alpha$

$$\cos.\,z\,y^1 = \cos.\,\phi \sin.\,\alpha \text{ (ibid. 447)};$$

$$\therefore \frac{d\,w_1}{d\,t} = \Omega \cos.\,\phi \sin.\,\alpha \frac{d\,\phi}{d\,t} = \Omega \cos.\,\phi \sin.\,\alpha\,(w_3 - \Omega \cos.\,\alpha).$$

Substituting in (1) and reducing, we get—

$$C\,\Omega\,w_3 = M g\,a + A\,\Omega^2 \cos.\,\alpha. \quad . \quad . \quad . \quad (2)$$

But $A = M\,(k^2 + a^2)$, $C = M k_1{}^2$;

$$\therefore w_3 = \frac{g a + (a^2 + k^2)\,\Omega^2 \cos.\,\alpha}{k_1{}^2\,\Omega}.$$

If $\alpha = 90°$ in equation (2), we get the solution of the preceding question as a particular case. F. M. S.

Arnesby

* A comparison of this method with Sir W. Thomson's (which is virtually the same as that adopted by Airy in his tract on Precession and Nutation) is instructive as illustrating the dynamical meaning of Euler's equations.—ED. NATURE.

OUR ASTRONOMICAL COLUMN

THE MASS OF JUPITER.—M. Leverrier has made a special communication to the Paris Academy of Sciences with reference to the bearing of his researches on the motion of Saturn, in a period of 120 years, on the value of Jupiter's mass. Laplace, in the *Mécanique Céleste*, had fixed $\frac{1}{1067{\cdot}09}$ making use of the elongation of the fourth satellite as determined by the observations of Pound, the contemporary of Newton, observations of which it appears we have no knowledge, except from the reference to them in the "Principia;" subsequently Bouvard, comparing Laplace's formulæ with a great number of observations, discussed with particular care, constructed new Tables of Jupiter, Saturn, and Uranus, in which important work he formed equations of condition, wherein the masses of the planets entered as indeterminates, and by the solution of which their values adopted in the Tables were obtained. The denominator for Jupiter's mass, expressed as a fraction of the sun's taken as unity, is 1070·0, and Laplace stated that on applying his theory of probabilities to Bouvard's equations it appeared to be nearly a million to one against the error of the mass thus deduced, amounting to one-hundredth part of the whole. M. Leverrier then