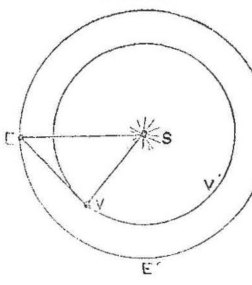
*THE COMING TRANSIT OF VENUS**

II.

THERE is perhaps no problem which has been so constant a source of interest to the learned in all ages as the solving of the mystery of the solar system. The labours of Copernicus, Tycho Brahé, Kepler, and Newton have given us a general knowledge of the nature of the planetary motions; and the investigations of later mathematicians have enabled us to predict, with wonderful accuracy, the future positions of the planets. But the dimensions of the solar system are not known with the same precision.

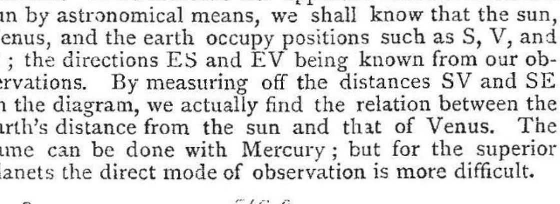
It is true that we know the *relative* distances of all the planets from the sun with tolerable exactness. This problem has been attacked in two totally different methods. The first is by measuring directly the changes that are

FIG. 8



produced in the motions of the planets when the earth has moved through a certain portion of its orbit. In the case of the planets Mercury and Venus, which move in smaller orbits than that of the earth, the direct observation can easily be made. For let us suppose VV' and EE' (Fig. 8) to be the orbits of Venus and the earth, and S to be the sun. Let us watch the position of Venus night after night until she is as far away from the sun as possible. If we measure her apparent distance from the sun by astronomical means, we shall know that the sun, Venus, and the earth occupy positions such as S, V, and E; the directions ES and EV being known from our observations. By measuring off the distances SV and SE on the diagram, we actually find the relation between the earth's distance from the sun and that of Venus. The same can be done with Mercury; but for the superior planets the direct mode of observation is more difficult.

FIG. 9.



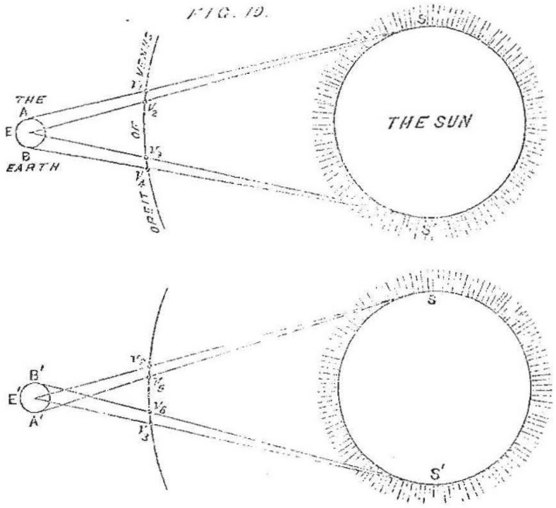
But there is an indirect method which is much more easy to apply. Kepler's three laws have been shown to be necessary consequences of Newton's theory of gravitation. Now Kepler's third law tells us how to find the relative distances of two planets from the sun when we know the relation between their periods of revolution. The exact law is this:—Multiply the number of years taken by a planet to go round the sun, by the same number. This gives us a first number. Then find a second number which, multiplied by itself twice, gives us the first number; this second number is the distance of the planet from the sun (the earth's distance being called 1). To take an example: Jupiter takes about 11 years to go round the sun; 11 multiplied by 11 gives us a first number, 121. Now if 5 be multiplied by 5 we get 25, and if this be again multiplied by 5 we get 125, which is almost the same as the first number, 121. Hence we are right in saying that Jupiter is about five times as far from the sun as the earth. If we had used the exact number of years we should have got the exact distance. Now it is very easy to find the period of revolution of a planet. For we can easily measure the interval between two dates when Jupiter and the earth, for example, are in the same line with the sun; in other words, we can measure the "synodical revolution" of Jupiter; and from this it is easy to calculate the time of Jupiter's revolution round the sun.

* Continued from p. 449.

By applying these methods to all the planets we can lay down their orbits upon a plan; *all we wish now is to find the scale upon which our plan is drawn.* If we knew the distance of the earth from the sun, or if we knew the distance between any two of the planetary orbits, we should know the scale upon which our plan is laid down. Various methods have been adopted for this, but the one which makes use of a transit of Venus has generally been considered to be the most accurate.

One method which has successfully been applied to measuring the moon's distance is that used by surveyors. The surveyor chooses two spots, B, C, whose distance he measures. Suppose it to be one mile. He draws this distance, say, to one inch on a sheet of paper. He then

FIG. 10.



takes a telescope, mounted so as to enable him to measure any angle through which it is turned. He places the telescope at B, pointing towards C. He then turns it till it points at the distant object, and finds what the angle of B is. He then draws the line BA upon the paper, and he knows that the distant object lies somewhere on the line BA. He then does the same with C, and thus he knows that the remote object lies on CA. But A is the only point lying both on BA and CA; hence A corresponds to the distant object. If on measuring CA he finds it to be 30 inches, then since CB, which is 1 inch, means one mile; CA, which is 30 inches, means 30 miles, and this is what he wanted to find out.

If, instead of taking a base-line (as it is called) of one mile, the diameter of the earth, or 8,000 miles, be taken; then, if the moon be the distant object, we can determine its distance in almost the same way. It is in this manner that the moon's distance has been measured. It is easy to see that if the angle at A (Fig. 9) were very small, a slight error in measuring either of the angles B or C would make a great difference in the distance deduced for the remote object. Hence, if the moon's parallax were very small, this method would be unsuitable. But the parallax of the sun is very small, and hence we cannot find the sun's distance with any exactness by this method.

But if any one of the planets ever came so close to the earth as to make its parallax tolerably large, then we could determine the scale upon which the solar system is built up. Now Venus and Mars are two planets which at certain times come closer to the earth than any other planet. But, unfortunately, when Venus is most near to the earth she is generally invisible, because the whole of her illuminated side is turned away from us. Mars, however, is a planet that gives us a very favourable opportunity for determining its distance. The advantage is increased by this peculiarity, that every fifteen years Mars is at its shortest distance from the sun, at the same time that the earth is at its greatest distance, the two planets being also in the same line with the sun, so that they are closer than we might have thought possible. In fact, on these occasions Mars is nearer to the earth by $\frac{1}{25}$th part than she is if the conjunction take place when both the earth and Mars are at about their mean distances from the sun. Suppose then that under such circumstances two observers, one at Greenwich and the other at the Cape of Good Hope (where there is a fine observatory), observe the position of Mars as compared with that of a star at the same time. The position of Mars will be displaced by parallax; and by comparing the apparent distance of the planet from the fixed star at these two places we can find the sum of the parallaxes in these cases. Hence we can find the distance of Mars, as already explained.

This was the first method to give a value of the solar parallax with anything like accuracy. At the suggestion of Cassini, the French sent out an expedition to the Cape, under the astronomer Picard. The value obtained for the sun's parallax was $9''\cdot 5$. Prof. Henderson in 1836, and Mr. Stone, in 1862, utilised this method. Another opportunity will occur in 1878.

Before proceeding to the method of the Transits of Venus, it will be well briefly to allude to some other methods by means of which the solar parallax, or the sun's distance, has been estimated.

It has been found that light takes a sensible time to propagate itself through space. Hence, when one of Jupiter's satellites passes into the shadow of the planet, this fact is not communicated to our vision for something like 38 minutes, the time taken by light to pass from Jupiter to the earth. Now, when we are on the same side of the sun as Jupiter, this distance is shorter by the whole diameter of the earth's orbit than when we are at the opposite side of the sun. Hence, in the former case, the eclipses will seem to take place sooner than the predicted time, and in the latter case later. The difference in either case is about 8 minutes, and as we know that light travels over 298,500 kilometres per second,* this tells us that our distance from the sun is about 91,000,000 miles.

But our knowledge of the velocity of light has been utilised in another manner to solve the same problem. You see that if we know the earth's velocity in miles, we can find its distance from the sun. For if it goes $1\frac{1}{2}$ million miles in one day, it must go over 365 times that in a year, and *that measures in miles the circumference of our earth's orbit*, and hence we can get our distance from the sun. How then are we to find the velocity of the earth in miles. This depends on a curious property of light. In a steady down-pour of rain you hold your umbrella upright if you are standing still, but incline it forward if you are walking fast. This is to make the umbrella catch the rain-drops. The amount of inclination you give it depends upon the rate at which you are walking compared with the velocity with which the drops fall. The same thing happens with light. We have to incline our telescopes forward a little in the direction in which the earth is moving to catch the rays of light; and at opposite seasons of the year the earth is moving in contrary directions, and the telescope has to be pointed in sensibly different directions. The inclination that a telescope receives is known, and the velocity of light being known, we can find the velocity of the earth, and hence, as I have shown, the distance of the earth from the sun.

There is another method of peculiar interest depending upon the motions of the moon. The law of gravitation says that the attraction of each body for each other one depends upon the distance between them. The moon is attracted to the earth by a force, depending upon the distance of the moon, which is known in miles. But the moon is caused to deviate from its natural course on account of the sun's attraction. This depends upon the distance of the sun from the earth, and if this be not known exactly in miles we shall see that it is impossible to apply calculation to foretell the motions of the moon; for, if upon any scale we attempt to lay down upon paper the relative positions of the sun, earth, and moon, we shall place the moon at its proper distance, and the sun, though in its proper direction, will not be placed at the proper distance, and we shall not know the direction in which it attracts the moon, nor the magnitude of this attraction, and we shall make our calculation wrongly, and the moon's observed place will differ considerably from its calculated place.

Such a difference was actually detected by the illustrious Hansen, whose tables of the moon are the best we possess. Hansen saw that this must be due to a wrong assumption as to the distance of the sun, and communicated his doubts to the Astronomer Royal* in the year 1854. This led to a re-discussion of our knowledge of the subject which has confirmed Hansen's views, and which leads us to see the importance of knowing accurately the sun's distance, if we wish ever to have our tables of the moon so accurate that we may determine the longitude by their aid. This method for investigating the solar parallax was first used by Laplace.†

More recently, M. le Verrier has suggested a new method that promises in time to be the best.‡ In the lunar theory, an equation appears connecting the relative masses of the earth and sun with the solar parallax, so that if we know the one we can find the other; and from a peculiarity in the equations, a small error in determining the relative masses will affect only very slightly the deduced parallax. Le Verrier finds the ratio of the masses of the earth and sun by determining the effect of the earth's attraction upon Venus and Mars. This being applied to the lunar theory, a value of the solar parallax is obtained.

The method, however, which has found most favour up to the present time, is the employing of transits of Venus to measure the sun's distance. When a transit of Venus occurs, the first evidence of the phenomenon is given by a slight notch being made in the contour of the sun's edge at a certain spot. This notch increases until the full form of the planet is seen. The first appearance of a notch is called the time of first external contact. But when the planet appears to be wholly on the sun, her black figure is still connected with the sun's limb by a sort of black ligament, of which we shall say more hereafter. When the whole of the planet is just inside the sun's edge, the time of first internal contact has arrived. The breaking of the ligament is a very definite occurrence, and was, until lately, taken to indicate the true moment of internal contact. The second internal and external contacts take place as the planet leaves the sun.

In 1663, the celebrated James Gregory, in his famous work the "Optica Promota," *prop.* 87, *Scholium*, alludes

* As determined by Foucault, *Comptes Rendus de l'Acad. des Sciences*, vol. lv. p. 502; also by Cornu, *Comptes Rendus*, Feb. 10, 1873.

* *Monthly Notices, R.A.S.*, vol. xv., Nov. 1854.
† *Système du Monde*, t. ii. p. 91.
‡ *Comptes Rendus*, July 22, 1872.

to the possibility of determining the sun's parallax by means of the transit of an inferior planet. He has been showing methods of finding the parallax of a planet by comparison of observations made at different parts of the earth upon the position of the planet compared with that of a star. He then takes, in place of a fixed star, another planet, the two being in one line, as seen from the earth. The application of this to the case of Mercury or Venus and the sun, was obvious.

But Halley was the first to see clearly what a powerful means of determining the sun's parallax an observation of contact really is. So far as I can discover, he first mentions the method in a letter to Sir Jonas Moore, written at St. Helena in 1677,* just after having seen a transit of Mercury. The exactness with which he believed the time of contact to be determinable, led him frequently afterwards to urge his countrymen to make every effort to utilise the method on the occasion of the transits of 1761 and 1769, when he should be dead.† And thus, in addition to his celebrated prediction of a comet, he left a second legacy to his successors, who, as Englishmen, might be entitled to be proud of his foresight though he could not live to reap the glory of it.

It is a matter of some difficulty to show, in an elementary manner, the way in which the value of the sun's parallax can be found from observations of contact. We will try, however, to put it in a light which anyone, with a little attention, will understand.

1. It must be thoroughly understood, from what has already been said, that if we know the amount of the sun's parallax we know its distance. In other words, if we know the angle subtended by any known distance on the earth's surface at the distance of the sun.

2. We know that the relative positions of the earth, Venus, and the sun, are given by supposing the earth to go round the sun in 365 days, and Venus in 224 days. Or, if we please, we may take no account of the earth's revolution, but suppose it fixed, in which case the revolution of Venus *relatively* to the earth (*i e.* the synodical revolution) is 584 days.

3. If, then, Venus moves round the sun through 360° relatively to the earth in 584 days, she moves through $\frac{1}{584}$ of that in one day, and through $\frac{360}{584 \times 24}$ of a degree in one hour; which is at the rate of about $1\frac{1}{5}$ seconds of arc in a minute of time.

Now we are ready to understand Halley's reasoning.

Let A (Fig. 10) be the position of an observer on the earth at the time of 1st internal contact. S is the sun, and V_1 is now the position of Venus. This observer sees the contact earlier than a hypothetical observer at the earth's centre would see it, by the time Venus takes to move over V_1V_2. If we knew by calculation the instant when an observer at E would see it, and the observer at A saw it 8 minutes sooner, then, since Venus moves over $1\frac{1}{5}''$ in a minute, she has moved over $8 \times 1\frac{1}{5}$ or $9\frac{3}{5}''$ of arc in this time, and hence we learn that the angle A S E = $9\frac{3}{5}''$.

Suppose that by the time of the last contact the point A on the earth's surface has been carried by her rotation to B: the time of the last contact will now be too late by 8′; since the whole duration of the transit as seen by this observer is 16′ too long, and the angle moved over by Venus in 16′ is the sum of the sun's parallax as seen from A and from B.

But we cannot calculate with absolute accuracy the duration a transit would have when seen from E, because we should require to know more accurately than we do the values of Venus' and the sun's diameters.

Halley got rid of this by taking another station which should be in the position A at the beginning of the transit. In the case we have been considering the time of the first contact would here be too late by 8 minutes; and if this place had reached B′ by the end of the transit, the time of contact would be too soon by 8 minutes. Hence in this case the whole duration would be shortened by 16 minutes; but in the former case it was lengthened by 16 minutes. Hence 32 minutes is the time taken by Venus to pass over an angle equal to the sum of the parallaxes in the four cases considered. This difference of duration, whether it be 32 minutes or anything else, is a quantity which can be observed. Now Venus moves over about $1\frac{1}{5}''$ of arc in a minute, or $38\frac{2}{5}''$ in these 32 minutes. Hence one-fourth of $38\frac{2}{5}''$ or $9\frac{3}{5}''$ would appear, from the above hypothetical observation, to be the value of Venus's parallax.

It must be noticed that we have here supposed that the transit takes exactly twelve hours, whereas the longest transit cannot exceed 8 hours. We have also supposed that two stations had been selected which were exactly situated so as to bring out the full effect of parallax at the time of each observation. These suppositions have been introduced only to simplify the understanding of the method. Anyone who has followed the above explanation will see how the method may be applied to actual cases that may occur.

Halley saw (what many people fail to see even now) that the great accuracy of the method consists in this, that in one second of time Venus moves over about $0''\cdot02$; and if we can determine the time of contact, with an error of no more than a second, we are measuring the sun's parallax with an error of no more than ·02 of a second of arc.

Halley even pointed out the best stations for observation. We may consider the earth to be at rest if we suppose Venus to move with the velocity she has relative to the earth. He supposed that the planet would cross near the sun's centre, and that the transit would occupy about eight hours. An observer in India would see the commencement of the transit four hours before mid-day, and the end of the transit four hours after mid-day. But, in the meantime, the part of the earth where he is has been moving from west to east, and Venus has moved from east to west, hence the duration of transit will have been shortened. But at Hudson's Bay the transit begins just before sunset and ends just after sunrise, that part of the earth having moved in mean time from east to west so as to lengthen the transit; and thus at one place the duration of transit is lengthened, and at the other shortened, and the difference of time depends upon the parallaxes of Venus and the sun * at the two stations, and after finding these parallaxes we can calculate the equatorial horizontal parallax.

GEORGE FORBES

(*To be continued.*)

THE LECTURES AT THE ZOOLOGICAL SOCIETY'S GARDENS

I.

ON Tuesday, April 14, Mr. P. L. Sclater, F.R.S., gave the Introductory of the twelve lectures which are to be continued during the spring. His remarks on that occasion were chiefly confined to the subject of Zoological Gardens in general. After an interesting account of the most important continental gardens, including those of Paris, Amsterdam, Antwerp, Berlin, and Hamburg, he

* Hooke's "Lectures and Collections," 1678.

† "Catalogus Stellarum Australium;" also "Phil. Trans.," 1694 and 1715.

* This lengthening or shortening of the time of transit will be rendered more evident by an analogy. A person standing still sees a carriage pass between him and a distant house. The carriage will take a certain time to pass the house. But if he be also moving, and in the same direction with the carriage, the transit of the carriage will take longer; but if he move in the opposite direction to the carriage, the transit will take a shorter time. If, then, two persons be seated at opposite sides of a merry-go-round, so that at the time the carriage seems to be passing the distant house, one observer is moving with the carriage and the other in the opposite direction; then one observer will see the time lengthened, and the other shortened. Now, the world is such a merry-go-round, and the positions of these two people correspond to the positions of India and Hudson's Bay, as pointed out by Halley.