## POLARISATION OF LIGHT * VII.

AMONG the phenomena of polarised light which may be observed either with a Nicol's prism or even with the naked eye, one of the most curious, and perhaps not yet fully explained, is that of Haidinger's brushes. If the eye receives a beam of polarised light a pale yellow patch in the form of an hour-glass, the axis of which is perpendicular to the plane of vibration, is perceived. On either side of the neck of the figure two protuberances of a violet tint are also seen to extend. After a little practice these figures or "brushes" may readily be observed. If the day be cloudy a Nicol must be used and directed to a tolerably bright cloud. The brushes are better defined in one position than in others; but if the Nicol be turned round, the brushes will be seen to revolve with it. If on a clear day we look in a direction at $90^{\circ}$ from that of the sun, where the skylight is most completely polarised, the brushes may be seen with a naked eye. Jamin has suggested in explanation of this phenomena that the substances of the eye act like a pile of glass plates, or rather spheres, which affect in different degrees ( r ) the rays of the same colour whose vibrations are differently inclined to the plane of incidence, and (2) the rays of different colours whose vibrations are similarly inclined. This will cause one colour to predominate in a general direction parallel, and its complementary to predominate in a plane perpendicular, to that of vibration. Helmholtz, however, connects the phenomenon with some double refraction due to the yellow spot in the eye, with the area of which that of the brushes is coincident.
It was explained above that in Iceland spar there is a particular direction, viz. that of the line joining the two opposite obtuse angles of the natural crystal, in which there is no double refraction, and in which all rays travel with the same velocity. This direction (that is to say, this line and all lines parallel to it) bears the name of the optic axis. There are many other crysta's having the same property in one and only one direction, in other words having a single optic axis. There is, moreover, another class of crystals having two such axes. Crystals of the first class or uni-axal crystals are again divided into two groups, viz. positive, in which the extraordinary ray is more refracted than the ordinary, and negative, in which the ordinary ray is the more refracted. It will be remembered that the ray which travels slowest is the most refracted. Among the former may be mentioned

Uni-axal Crystals.
Positive.

Apophyllite.
Boracite.
Ditopaz.
Hydrate of magnesia.
Hyposulphate of lead. Ice.
Quartz.
Red Silver.
Apatite.
Arseniate of copper.
Arseniate of lead.
Arseniate of potash.
Beryl.
Carbonate of lime and magnesia.
Carbonate of lime and iron.
Chloride of calcium.
Chloride of strontium.
Cinnabar.
Corundum.
Emerald.

Stannite.
Superacetate of copper and lime.
Sulphate of potash and iron.
Tungstate of zinc.
Zircon.

Negative.
Honey stone.
Idocrase.
Miellite.
Mica.
Molybdate of lead.
Nepheline.
Octaedrite.
Phosphate of lime.
Phosphate of lead,
Rubellite.
Ruby.
Sapphire.

Crystals are usually divided into six systems, in each of which there is a fundamental aad a variety of derived forms. The fundamental form of each system is based upon the number, magnitude, and inclination of the crystallographic axes or lines drawn through a point in the interior of the crystal, and terminating in its angles. The optic axes do not of necessity coincide with any of these.
(r.) The regular system, which is based upon a system of three equal rectangular axes. Any form derived from this will be perfectly symmetrical with reference to the three axes, and will present no distinguishing feature in relation to any of them. Crystals belonging to this system have no optic axis, nor any doubly refracting property.
(2.) The quadratic system, based upon a system of three rectangular axes, whereof two are equal, but the third greater or less than the other two. Crystals belonging to this system have one optic axis coinciding with the last-mentioned crystallographic axis.
(3.) The hexagonal system, having three equal axes lying in one plane inclined at $60^{\circ}$ to one another, and a fourth axis at right angles to the other three. Crystals of this system have one optic axis coinciding with the fourth crystallographic axis. Iceland spar belongs to one of the derived forms of this system.
(4.) The rhombic system, having three rectangular but unequal axes.
(5.) The monoclinic system, which differs from the rhombic in this, that one of the three axes is oblique to the other two, which are still rectangular to one another.
(6.) The triclinic system in which all the axes are oblique.

All crystals belonging to the last three systems have two optic axes. In the rhombic system they lie in a plane containing two of the three crystallographic axes; in the monoclinic, they lie either in the plane containing the oblique axes, or in a plane at right angles thereto. In the triclinic no assignable relation between the optic and the crystallographic axes has been determined.

The phenomena of colours and their variations hitherto described have been produced by a beam of light, all the rays of which were parallel in their passage through the crystal or other substance under examination. There is, however, another class of phenomena due to the transmission of a convergent or divergent beam of polarised light, which we now proceed to consider.

It was shown above that the retardation due to any doubly refracting crystal, and consequently the colour produced by it is dependent on the thickness; and that with a crystal of constantly increasing thickness, the colours go through a complete cycle, and then begin again. Suppose then a divergent beam to fall perpendicularly upon a uni-axal crystal plate cut at right angles to the optic axis ; the central rays will fall perpendicularly to the surface; but the rays which form conical shells about that central ray will fall obliquely. The rays forming each shell will fall with the same degree of obliquity on different sides of the central ray, those forming the outer shells having greater obliquity than the inner. Now the more obliquely any ray falls upon the surface the greater will be the thickness of the crystal which it traverses; and this will still be the case even though it suffers refraction, or bending towards the perpendicular on entering the crystal. Each incident cone of rays will consequently still form a cone when refracted within the crystal, although less divergent than at incidence, in its passage through the plate; and the successive refracted cones will be more and more oblique, as were the incident cones, but in a less degree, as we pass from the more central to the more external members of the assemblage forming the beam of light.

Let A B C D (Fig. 2I) represent the crystal plate, O P the
direction of the optic axis and of the central ray, $O n, O n^{\prime}$ those of any two other rays. The ray O P will not be divided; but O n will be separated by the double refraction of the plate into two, $\mathrm{n} \mathrm{s}, \mathrm{n} \mathrm{r}$, the one ordinary, the other extraordinary; and these will emerge parallel to one another, and may be represented by the lines $s t, r v$. Similarly the effect of double refraction on $O n^{\prime}$ may be represented by $n^{\prime} s^{\prime}, n^{\prime} r^{\prime}, s^{\prime} t^{\prime}, r^{\prime} v^{\prime}$. Suppose now that the process were reversed, and that two monochromatic rays, one ordinary, the other extraordinary, reach the plate at S and r in the directions $\mathrm{t} \mathrm{s}, \mathrm{v} \mathrm{r}$, respectively ; these would meet at n and travel together to O . Suppose, further, that the difference in length of $s n$ and $r n$ is equal to one wave-length, then, since one of them is an ordinary and the other an extraordinary ray, their vibrations will be perpendicular to one another, and if the polariser and analyser be crossed, the point $n$ viewed from $O$ will appear dark. Similarly, if two rays arrive in the directions $t^{\prime} s^{\prime}$, $v^{\prime} r^{\prime}$, at the points $s^{\prime} r^{\prime}$ respectively, they will meet at $n^{\prime}$ and proceed together to O ; and if the difference of the paths $s^{\prime} n^{\prime}, r^{\prime} n^{\prime}$ be two wave-lengths, the point $n^{\prime}$ will also appear dark. A pair of rays reaching the crystal at points between the pairs before-mentioned, will emerge at a point $n^{\prime \prime}$ between $n$ and $n^{\prime}$, and will present a difference of phase equal to a half wave-length. On principles explained in an earlier part of these lectures, such a point $n^{\prime \prime}$ will appear bright. On either side of $n^{\prime \prime}$, that is towards n and n ', the light will gradually fade. The same alternations of light and darkness will recur at intervals as we proceed along any straight line drawn outwards from the central ray. And inasmuch as the obliquity of the ray is the same for every point equidistant from the centre $O$, it follows that the phenomena of light and darkness will be the same throughout each circle drawn about the centre 0 . In otber words, the centre will be surrounded by rings alternately bright and dark. The diameters of the ring depend, as was seen above, on the wave-length of the particular light used, and will consequently be different for different coloured rays. If, therefore, white light be used, the different coloured rings would not coincide, but would be disposed in recurring series as we proceed outwards from the centre.

Another effect would, however, also be produced. Suppose the polariser and analyser to be so placed that, the field being regarded as a map, the vibrations in the one being E. and W., those in the other N. and S. ; then of the two rays emerging at the most nothern or the most southern point of any ring the vibrations of one would be towards the axis, or N. and S. ; those of the other would be across it, or E. and W. And of these one would be extinguished by the polariser, the other by the analyser; and the same will be the case for every ring. Hence, throughout a N . and S. line crossing the entire field the light will be extinguished; and a similar effect will obviously occur along an E and a W. line. Hence, when the polariser and analyser are crossed, the entire system of rings will be intersected by a black cross, two of whose arms are parallel to the plane of vibration of the polariser and two to that of the analyser, and the rings in the quadrants on each side of an arm are of complementary tints. When the analyser is turned round through a right angle from its former position, only one set of vibrations (say those executed in a direction E. and W.) will be extinguished, and consequently along one pair of arms of the cross the ordinary rays will pass undisturbed, along the other the extraordinary; that is to say, the cross will be white. When the polariser and analyser occupy any other position than those noticed above, there are two crosses inclined at an angle equal to that between the planes of vibration, each arm of which separates complementary rings.

Various forms of polariscopes have been devised for showing the crystal rings. The simplest of these is the tourmalin forceps, which consists of two plates of tour-
malin fixed in cork discs; the latter are encircled in wire in such a way that they may be turned round in their own planes. The wire after encircling one disc is bent round so as to form a handle; it then encircles the other; and the elasticity of the wire allows the pair of discs to be opened and shut like a pair of pincers. If a crystal plate be inserted between the two, and the whole held close to the eye, the rays from parts of the field at different distances from the centre will reach the eye, having traversed the crystal with different degrees of obliquity; and a system of rings and brushes will be formed.

Another method consists in applying to Norremberg's polariscope a pair of lenses, one below the crystal with the crystal in the focus, the other above it. The first ensures that the rays shall traverse the crystal with different degrees of obliquity; the second brings within the range of vision rays which would otherwise fail to reach the eye, and at the same time converging them into a cone with a smaller vertical range, renders the ring smaller than when seen with the simple tourmalins. An additional lens of greater focal length, i.e. of less power, is

often added in order to adjust the whole to individual eyesight.

Fig. 22 gives the general appearance of the addition to the apparatus of Norremberg described above, and Fig. 23 the course of a system of rays brought to a focus on the lens $a \mathrm{~b}$, and again converged by a second lens $\mathrm{c} d$.
But by far the most successful arrangement for enlarging the field of view so as to comprise the complete system of rings even with bi-axal crystals having widely inclined optic axes, is the system of lenses due in the first instance to Norremberg. The disposition of the parts is shown in Fig. 24; and the general appearance of the instrument as constructed by Hofmann of Paris, and called by him the "Polarimicroscope," is also given, Fig. 25. In this instrument the lenses which converge the rays upon the crystal plate can be taken out, and replaced by others giving parallel light ; it can then be used as an ordinary polariscope.

Mention has been made above of the effect of the circular polarisation of quartz in the colours produced by a beam of parallel rays of polarised light. It remains for us to examine the modification which the rings and-
brushes undergo from the same cause. It has been explained that a ray of plane-polarised light in traversing a crystal of quartz in the direction of its axis is divided into two, the vibrations of which are circular, one righthanded, the other left. If the ray traverses the crystal in a direction perpendicular to the axis, and if the original vibrations are neither parallel nor perpendicular to the axis it is also divided into two, whereof the vibrations are not circular but rectilinear. It was suggested, first by Sir G. Airy, that these circular and rectilinear vibrations are limiting cases of elliptical; and both theory and experiment tend to confirm the suggestion, by showing that if the ray be incident on the crystal in any direction


F c. 23 .
oblique to the axis, it is divided into two, the vibrations of which are similar ellipses having the longer diameter of the disc coincident with the shorter of the other, and the motion in the two oppositely directed. The longer diameters of the ellipses coincide with the directions of vibration of the ordinary and extraordinary rays in the case of an ordinary positives crystal ; and are consequently directed, the one toward the centre of the figure, the other in a direction at right angles to the first.

The exact, or even appproximate determination of the figures produced is a complicated question, and requires mathematical analysis for its solution, but a general idea of their nature may nevertheless be easily formed.

When the polariser and analyser are either parallel or crossed, circular rings are formed, and towards the outer parts of the field traces of the black cross are seen, which grow stronger as we proceed outwards from the centre, that is, towards the parts where the rays are more oblique, and where the polarisation more nearly approaches to rectilinear. But in the centre, and near to it, where the polarisation is circular, or nearly so, the effects will resemble those produced by parallel rays, viz. the rays of different colours will emerge plane-polarised in different planes, and will be variously affected by the angle between polariser and analyser. In no position can they all be extinguished, and consequently in the centre all traces of the black cross will disappear.

When the planes of vibration of the polariser and the analyser are inclined at any other angle than $0^{\circ}$ or $90^{\circ}$, the arms of the cross are less strongly marked, and the curvature of the rings becomes less uniform, increasing in the four points where they are crossed by the arms, and diminishing in the intermediate quadrants. When the angle between the planes of vibration is $45^{\circ}$, the rings assume a nearly square form, the corners of the square lying upon the lines which bisect internally and externally the angles between the planes. If the figures are produced


Fig. 25.
with the analyser at $45^{\circ}$ by two quartz plates of equal thicknesses, one right-handed, the other left, it will be found that the diagonals of the squares are at right angles to one another, the remains of the black cross occupying the same position in the field in both cases.
If two plates of the same thickness, the one righthanded and the other left, are placed one over the other, a beautiful effect, called from their discoverer Airy's spirals, is produced. In the centre of the field the rotatory powers of the plates neutralise one another, and a black cross commences. As we proceed outwards, the arms of the cross cease to be black, and become tinged with red on one side, and with blue on the other. At the same time they are bent round in a spiral form, in the direction of the hands of a watch if the first plate be right-handed, and in the opposite direction if the first plate be left-handed. These spirals intersect at intervals the circular rings; the points of intersection lie in four rectangular directions, which terminate towards the outer margin of the field in four arms of a shadowy cross. The colours of the rings and spirals are more brilliant and better defined than in most other phenomena of chromatic polarisation.

> W. SpOtriswoode
(To be continued.)

