

as before, has utilised the opportunities presented in the interest of science. Captain Giraud surveyed a large portion of the so-called "volcanic region" of the Atlantic Ocean, finding the water very deep in that vicinity. Specimens brought up from the bottom appeared to be of undoubted volcanic origin. The Casella-Miller deep-sea thermometer was used on one occasion at a depth of 2,040 fathoms, two miles north of the equator, in longitude $22^{\circ} 16'$ west, and indicated a temperature of 35° F., at 1,000 fathoms 38° , and at the surface 81° , the air being 80° . During the voyage from the Canary Islands to Rio the temperature at uniform depths was found to vary only about two degrees.

THE Iron-Steel Institute conclude their meeting at Willis's Rooms to-day.

PRIZES for papers on the "Elvan Courses" of Cornwall, are offered by Mr. J. A. Phillips, F.C.S., to the present and former pupils of the Miners' Association of Cornwall and Devon. The papers and illustrative specimens are to be deposited with Mr. J. H. Collins, F.G.S., Hon. Assistant Secretary of the Miners' Association, Polytechnic Hall, Falmouth, on or before Sept. 1, 1873. The author of the best paper will be entitled to a prize (in books, selected by himself) of the value of 5*l.* A second prize, also in books, of the value of 3*l.*, will be given to the author of the paper next in order of merit.

WE have received the first number of a new American journal, started last month, *The Sanitarian*, edited by Dr. A. N. Bell, of New York. It aims at presenting the results of the various inquiries which have been, and which hereafter may be made, for the preservation of health and the expectations of human life, so as to make them most advantageous to the public and to the medical profession. Among the most important articles is one by the editor, on "The New York Quarantine Establishment," which is illustrated with two maps. This is preceded by one on "Infant Mortality, with suggestions for improving the condition of Foundlings;" and followed by another on "The necessity of Re-Vaccination." We strongly recommend this excellently conducted journal to those interested in sanitary science.

AMONG the rarer and more interesting remains found in the mounds of the west of America, are plates of mica cut into different shapes, and evidently preserved as objects of great rarity and value; and, in the absence of this mineral in the Mississippi Valley, the question has frequently arisen whence the material could have been derived. A recent communication from Prof. W. C. Kerr, the State Geologist of North Carolina, tends to throw some light on this subject, and to open an interesting chapter in regard to the American prehistoric man. The work of collecting mica is at present carried on upon the largest scale in the high and rugged region between the Black Mountain, the Roanoke, and the head waters of the Nolachucky, principally in Mitchell County, North Carolina. The region in question has long been known for the existence of numerous open works and tunnels, which, at first sight, were supposed to have been made in the search for silver or some other valuable metal. Prof. Kerr, in his capacity of State Geologist, was led to investigate this question, and very soon found, in every instance, that the excavations referred to were much older than the earliest discovery of the country by the Spaniards, and that in all cases they were found in ledges of coarse granite, which contained nothing but large patches of mica. Prof. Kerr has been satisfied for some time that in these mines we have the work of the contemporaries of the mound-builders, and the localities whence they derived the mica. What use they made of it we cannot say; but it is suggested that it may have served the purpose of mirrors, or possibly have been used as windows, as well as for

ornament. The number and size of these mines is remarkable, some of the open cuts being more than 100 ft. in diameter, and 20 ft. or 30 ft. in depth, even after the caving in and filling up of centuries of weathering. The tunnels often extend inwards several yards, but are said to be too small for a man of ordinary size to work in. These show distinct marks of the tool in the granitic wall, as if made by a chisel-shaped instrument about an inch broad. Numerous plates of mica are found in these tunnels and excavations, some of them trimmed to particular shapes. These facts open up a new chapter in the history of the American aborigines, illustrating the character of the commerce carried on at a very remote period, and showing the magnitude of the operations, and the extended period of time over which they must have been prosecuted, to enable a people furnished with nothing better than wooden and stone tools to produce excavations of so great magnitude.

Sirius, a journal of popular astronomy published at Leipzig and Vienna, contains, in its fourth number for this year, a lecture by Prof. Oppolzer, on "The Importance of Astronomy in connection with Ancient History," the continuation of an article on "Copernicus and his Anniversary," one of a series of articles on the "Topography of the Heavens," the present treating of the constellation Gemini, besides a few notes.

THE additions to the Zoological Society's Gardens during the last week include a Ring-necked Parakeet (*Palaornis torquata*) from India, presented by Mr. W. E. Johnson; a long-eared Owl (*Otus vulgaris*) from Europe, presented by Dr. Bree; a Wood Owl (*Syrnium aluco*), presented by Mr. H. W. L. Browne; a Chinese Harrier (*Circus spilonotus*); a grey Eagle Owl (*Bubo cinereus*) and a Bosman's Potto (*Perodicticus potto*) from W. Africa; a horned Tragopan (*Cerionis satyra*) from the Himalayas; a black-tailed Hawfinch (*Coccothraustes melanurus*) from Japan; two crested Buntings (*Melophus melanicterus*); two red-eared Bulbuls (*Pycnonotus jocosus*), and a red-vented Bulbul (*P. hamorrhous*) from India; a red-headed Bunting (*Emberiza rutila*), and a yellow-browed Bunting (*E. chrysophrys*) from Japan; a black Tanager (*Tachyphonus melaleucus*) from S. America, purchased; two Emus (*Dromaeus nova-hollandiæ*) from Australia, deposited; a great Kangaroo (*Macropus giganteus*), and a Derbian Wallaby (*Halmaturus derbianus*), born in the gardens.

ON THE HYPOTHESES WHICH LIE AT THE BASES OF GEOMETRY*

Plan of the Investigation

IT is known that geometry assumes, as things given, both the notion of space and the first principles of constructions in space. She gives definitions of them which are merely nominal, while the true determinations appear in the form of axioms. The relation of these assumptions remains consequently in darkness; we neither perceive whether and how far their connection is necessary, nor, *a priori*, whether it is possible.

From Euclid to Legendre (to name the most famous of modern reforming geometers) this darkness was cleared up neither by mathematicians nor by such philosophers as concerned themselves with it. The reason of this is doubtless that the general notion of multiply extended magnitudes (in which space-magnitudes are included) remained entirely unworked. I have in the first place, therefore, set myself the task of constructing the notion of a multiply extended magnitude out of general notions of magnitude. It will follow from this that a multiply extended magnitude is capable of different measure-relations, and consequently that space is only a particular case of a triply extended magnitude. But hence flows as a necessary consequence that the propositions of geometry cannot be derived from general notions of magnitude, but that the properties which distinguish space from other conceivable triply extended magnitudes are only to be

* By Bernhard Riemann. (Translated by Prof. W. K. Clifford, from vol. xiii. of the Göttingen Abhandlungen.)

deduced from experience. Thus arises the problem, to discover the simplest matters of fact from which the measure-relations of space may be determined; a problem which from the nature of the case is not completely determinate, since there may be several systems of matters of fact which suffice to determine the measure-relations of space—the most important system for our present purpose being that which Euclid has laid down as a foundation. These matters of fact are—like all matters of fact—not necessary, but only of empirical certainty; they are hypotheses. We may therefore investigate their probability, which within the limits of observation is of course very great, and inquire about the justice of their extension beyond the limits of observation, on the side both of the infinitely great and of the infinitely small.

I.—Notion of an n -ply extended magnitude

In proceeding to attempt the solution of the first of these problems, the development of the notion of a multiply extended magnitude, I think I may the more claim indulgent criticism in that I am not practised in such undertakings of a philosophical nature where the difficulty lies more in the notions themselves than in the construction; and that besides some very short hints on the matter given by Privy Councillor Gauss in his second memoir on Biquadratic Residues, in the "Göttingen Gelehrte Anzeige," and in his Jubilee-book, and some philosophical researches of Herbart, I could make use of no previous labours.

§ 1.—Magnitude-notions are only possible where there is an antecedent general notion which admits of different specialisations. According as there exists among these specialisations a continuous path from one to another or not, they form a *continuous* or *discrete* manifoldness: the individual specialisations are called in the first case points, in the second case elements, of the manifoldness. Notions whose specialisations form a *discrete* manifoldness are so common that at least in the cultivated languages any things being given it is always possible to find a notion in which they are included. (Hence mathematicians might unhesitatingly found the theory of discrete magnitudes upon the postulate that certain given things are to be regarded as equivalent.) On the other hand, so few and far between are the occasions for forming notions whose specialisations make up a *continuous* manifoldness, that the only simple notions whose specialisations form a multiply extended manifoldness are the positions of perceived objects and colours. More frequent occasions for the creation and development of these notions occur first in the higher mathematics.

Definite portions of a manifoldness, distinguished by a mark or by a boundary, are called Quanta. Their comparison with regard to quantity is accomplished in the case of discrete magnitudes by counting, in the case of continuous magnitudes by measuring. Measure consists in the superposition of the magnitudes to be compared; it therefore requires a means of using one magnitude as the standard for another. In the absence of this two magnitudes can only be compared when one is a part of the other; in which case also we can only determine the more or less and not the how much. The researches which can in this case be instituted about them form a general division of the science of magnitude in which magnitudes are regarded not as existing independently of position and not as expressible in terms of a unit, but as regions in a manifoldness. Such researches have become a necessity for many parts of mathematics, e.g., for the treatment of many-valued analytical functions; and the want of them is no doubt a chief cause why the celebrated theorem of Abel and the achievements of Lagrange, Pfaff, Jacobi for the general theory of differential equations, have so long remained unfruitful. Out of this general part of the science of extended magnitude in which nothing is assumed but what is contained in the notion of it, it will suffice for the present purpose to bring into prominence two points; the first of which relates to the construction of the notion of a multiply extended manifoldness, the second relates to the reduction of determinations of place in a given manifoldness to determinations of quantity, and will make clear the true character of an n -fold extent.

§ 2.—If in the case of a notion whose specialisations form a continuous manifoldness, one passes from a certain specialisation in a definite way to another, the specialisations passed over form a simply extended manifoldness, whose true character is that in it a continuous progress from a point is possible only on two sides, forwards or backwards. If one now supposes that this manifoldness in its turn passes over into another entirely different, and again in a definite way, namely so that each point passes

over into a definite point of the other, then all the specialisations so obtained form a doubly extended manifoldness. In a similar manner one obtains a triply extended manifoldness, if one imagines a doubly extended one passing over in a definite way to another entirely different; and it is easy to see how this construction may be continued. If one regards the variable object instead of the determinable notion of it, this construction may be described as a composition of a variability of $n+1$ dimensions out of a variability of n dimensions and a variability of one dimension.

§ 3.—I shall now show how conversely one may resolve a variability whose region is given into a variability of one dimension and a variability of fewer dimensions. To this end let us suppose a variable piece of a manifoldness of one dimension—reckoned from a fixed origin, that the values of it may be comparable with one another—which has for every point of the given manifoldness a definite value, varying continuously with the point; or, in other words, let us take a continuous function of position within the given manifoldness, which, moreover, is not constant throughout any part of that manifoldness. Every system of points where the function has a constant value, forms then a continuous manifoldness of fewer dimensions than the given one. These manifoldnesses pass over continuously into one another as the function changes; we may therefore assume that out of one of them the others proceed, and speaking generally this may occur in such a way that each point passes over into a definite point of the other; the cases of exception (the study of which is important) may here be left unconsidered. Hereby the determination of position in the given manifoldness is reduced to a determination of quantity and to a determination of position in a manifoldness of less dimensions. It is now easy to show that this manifoldness has $n-1$ dimensions when the given manifoldness is n -ply extended. By repeating then this operation n times, the determination of position in an n -ply extended manifoldness is reduced to n determinations of quantity, and therefore the determination of position in a given manifoldness is reduced to a finite number of determinations of quantity *when this is possible*. There are manifoldnesses in which the determination of position requires not a finite number, but either an endless series or a continuous manifoldness of determinations of quantity. Such manifoldnesses are, for example, the possible determinations of a function for a given region, the possible shapes of a solid figure, &c.

II.—*Measure-relations of which a manifoldness of n dimensions is capable on the assumption that lines have a length independent of position, and consequently that every line may be measured by every other.*

Having constructed the notion of a manifoldness of n dimensions, and found that its true character consists in the property that the determination of position in it may be reduced to n determinations of magnitude, we come to the second of the problems proposed above, viz., the study of the measure-relations of which such a manifoldness is capable, and of the conditions which suffice to determine them. These measure-relations can only be studied in abstract notions of quantity, and their dependence on one another can only be represented by formulæ. On certain assumptions, however, they are decomposable into relations which, taken separately, are capable of geometric representation; and thus it becomes possible to express geometrically the calculated results. In this way, to come to solid ground, we cannot, it is true, avoid abstract considerations in our formulæ, but at least the results of calculation may subsequently be presented in a geometric form. The foundations of these two parts of the question are established in the celebrated memoir of Gauss—"Disquisitiones generales circa superficies curvas."

§ 1.—Measure-determinations require that quantity should be independent of position, which may happen in various ways. The hypothesis which first presents itself, and which I shall here develop, is that according to which the length of lines is independent of their position, and consequently every line is measurable by means of every other. Position-fixing being reduced to quantity-fixings, and the position of a point in the n -dimensioned manifoldness being consequently expressed by means of n variables $x_1, x_2, x_3, \dots, x_n$, the determination of a line comes to the giving of these quantities as functions of one variable. The problem consists then in establishing a mathematical expression for the length of a line, and to this end we must consider the quantities x as expressible in terms of certain units. I

shall treat this problem only under certain restrictions, and I shall confine myself in the first place to lines in which the ratios of the increments dx of the respective variables vary continuously. We may then conceive these lines broken up into elements, within which the ratios of the quantities dx may be regarded as constant; and the problem is then reduced to establishing for each point a general expression for the linear element ds starting from that point, an expression which will thus contain the quantities x and the quantities dx . I shall suppose, secondly, that the length of the linear element, to the first order, is unaltered when all the points of this element undergo the same infinitesimal displacement, which implies at the same time that if all the quantities dx are increased in the same ratio, the linear element will vary also in the same ratio. On these suppositions, the linear element may be any homogeneous function of the first degree of the quantities dx , which is unchanged when we change the signs of all the dx , and in which the arbitrary constants are continuous functions of the quantities x . To find the simplest cases, I shall seek first an expression for manifoldnesses of $n-1$ dimensions which are everywhere equidistant from the origin of the linear element; that is, I shall seek a continuous function of position whose values distinguish them from one another. In going outwards from the origin, this must either increase in all directions or decrease in all directions; I assume that it increases in all directions, and therefore has a minimum at that point. If, then, the first and second differential coefficients of this function are finite, its first differential must vanish, and the second differential cannot become negative; I assume that it is always positive. This differential expression, then, of the second order remains constant when ds remains constant, and increases in the duplicate ratio when the dx , and therefore also ds , increase in the same ratio; it must therefore be ds^2 multiplied by a constant, and consequently ds is the square root of an always positive integral homogeneous function of the second order of the quantities dx , in which the coefficients are continuous functions of the quantities x . For Space, when the position of points is expressed by rectilinear co-ordinates, $ds = \sqrt{\sum(dx)^2}$; Space is therefore included in this simplest case. The next case in simplicity includes those manifoldnesses in which the line-element may be expressed as the fourth root of a quartic differential expression. The investigation of this more general kind would require no really different principles, but would take considerable time and throw little new light on the theory of space, especially as the results cannot be geometrically expressed; I restrict myself, therefore, to those manifoldnesses in which the line-element is expressed as the square root of a quadric differential expression. Such an expression we can transform into another similar one if we substitute for the n independent variables functions of n new independent variables. In this way, however, we cannot transform any expression into any other; since the expression contains $n \frac{n+1}{2}$ coefficients which are arbitrary functions of the independent variables; now by the introduction of new variables we can only satisfy n conditions, and therefore make no more than n of the coefficients equal to given quantities. The remaining $n \frac{n-1}{2}$ are then entirely determined by the nature of the continuum to be represented, and consequently $n \frac{n-1}{2}$ functions of positions are required for the determination of its measure-relations. Manifoldnesses in which, as in the Plane and in Space, the line-element may be reduced to the form $\sqrt{\sum dx^2}$, are therefore only a particular case of the manifoldnesses to be here investigated; they require a special name, and therefore these manifoldnesses in which the square of the line-element may be expressed as the sum of the squares of complete differentials I will call *flat*. In order now to review the true varieties of all the continua which may be represented in the assumed form, it is necessary to get rid of difficulties arising from the mode of representation, which is accomplished by choosing the variables in accordance with a certain principle.

§. 2.—For this purpose let us imagine that from any given point the system of shortest lines going out from it is constructed; the position of an arbitrary point may then be determined by the initial direction of the geodesic in which it lies, and by its distance measured along that line from the origin. It can therefore be expressed in terms of the ratios dx_0 of the quantities dx in this geodesic, and of the length s of this line. Let us intro-

duce now instead of the dx_0 linear functions dx of them, such that the initial value of the square of the line-element shall equal the sum of the squares of these expressions, so that the independent variables are now the length s and the ratios of the quantities dx . Lastly, take instead of the dx quantities $x_1, x_2, x_3, \dots, x_n$ proportional to them, but such that the sum of their squares = s^2 . When we introduce these quantities, the square of the line-element is $\sum dx^2$ for infinitesimal values of the x , but the term of next order in it is equal to a homogeneous function of the second order of the $n \frac{n-1}{2}$ quantities $(x_1 dx_2 - x_2 dx_1), (x_1 dx_3 - x_3 dx_1), \dots$ an infinitesimal, therefore, of the fourth order; so that we obtain a finite quantity on dividing this by the square of the infinitesimal triangle, whose vertices are $(0, 0, 0, \dots), (x_1, x_2, x_3, \dots), (dx_1, dx_2, dx_3, \dots)$. This quantity retains the same value so long as the x and the dx are included in the same binary linear form, or so long as the two geodesics from 0 to x and from 0 to dx remain in the same surface-element; it depends therefore only on place and direction. It is obviously zero when the manifold represented is flat, *i.e.* when the squared line-element is reducible to $\sum dx^2$, and may therefore be regarded as the measure of the deviation of the manifoldness from flatness at the given point in the given surface-direction. Multiplied by $-\frac{1}{2}$ it becomes equal to the quantity which Privy-councillor Gauss has called the total curvature of a surface. For the determination of the measure-relations of a manifoldness capable of representation in the assumed form we found that $n \frac{n-1}{2}$ place-functions were necessary; if, therefore, the curvature at each point in $n \frac{n-1}{2}$ surface-directions is given, the measure-relations of the continuum may be determined from them—provided there be no identical relations among these values, which in fact, to speak generally, is not the case. In this way the measure-relations of a manifoldness in which the line-element is the square root of a quadric differential may be expressed in a manner wholly independent of the choice of independent variables. A method entirely similar may for this purpose be applied also to the manifoldness in which the line element has a less simple expression, *eg.*, the fourth root of a quartic differential. In this case the line-element, generally speaking, is no longer reducible to the form of the square root of a sum of squares, and therefore the deviation from flatness in the squared line-element is an infinitesimal of the second order, while in those manifoldnesses it was of the fourth order. This property of the last-named continua may thus be called flatness of the smallest parts. The most important property of these continua for our present purpose, for whose sake alone they are here investigated, is that the relations of the twofold ones may be geometrically represented by surfaces, and of the morefold ones may be reduced to those of the surfaces included in them; which now requires a short further discussion.

§. 3.—In the idea of surfaces, together with the intrinsic measure-relations in which only the length of lines on the surfaces is considered, there is always mixed up the position of points lying out of the surface. We may, however, abstract from external relations if we consider such deformations as leave unaltered the length of lines—*i.e.* if we regard the surface as bent in any way without stretching, and treat all surfaces so related to each other as equivalent. Thus, for example, any cylindrical or conical surface counts as equivalent to a plane, since it may be made out of one by mere bending, in which the intrinsic measure-relations remain, and all theorems about a plane—therefore the whole of planimetry—retain their validity. On the other hand they count as essentially different from the sphere, which cannot be changed into a plane without stretching. According to our previous investigation the intrinsic measure-relations of a twofold extent in which the line-element may be expressed as the square root of a quadric differential, which is the case with surfaces, are characterised by the total curvature. Now this quantity in the case of surfaces is capable of a visible interpretation, *viz.* it is the product of the two curvatures of the surface, or multiplied by the area of a small geodesic triangle, it is equal to the spherical excess of the same. The first definition assumes the proposition that the product of the two radii of curvature is unaltered by mere bending; the second, that in the same place the area of a small triangle is proportional to its spherical excess. To give an intelligible meaning to the curvature of an n -fold extent at a given point and in a given surface-direction through it, we must start from the fact that a geodesic proceeding

from a point is entirely determined when its initial direction is given. According to this we obtain a determinate surface if we prolong all the geodesics proceeding from the given point and lying initially in the given surface-direction; this surface has at the given point a definite curvature, which is also the curvature of the n -fold continuum at the given point in the given surface-direction.

§ 4.—Before we make the application to space, some considerations about flat manifoldnesses in general are necessary; *i.e.* about those in which the square of the line-element is expressible as a sum of squares of complete differentials.

In a flat n -fold extent the total curvature is zero at all points in every direction; it is sufficient, however (according to the preceding investigation), for the determination of measure-relations, to know that at each point the curvature is zero in $\frac{n-1}{2}$ independent surface directions. Manifoldnesses whose curvature is constantly zero may be treated as a special case of those whose curvature is constant. The common character of these continua whose curvature is constant may be also expressed thus, that figures may be moved in them without stretching. For clearly figures could not be arbitrarily shifted and turned round in them if the curvature at each point were not the same in all directions. On the other hand, however, the measure-relations of the manifoldness are entirely determined by the curvature; they are therefore exactly the same in all directions at one point as at another, and consequently the same constructions can be made from it: whence it follows that in aggregates with constant curvature figures may have any arbitrary position given them. The measure-relations of these manifoldnesses depend only on the value of the curvature, and in relation to the analytic expression it may be remarked that if this value is denoted by α , the expression for the line-element may be written

$$\frac{1}{1 + \frac{\alpha}{4} \sum x^2} \sqrt{\sum dx^2}$$

§ 5.—The theory of surfaces of constant curvature will serve for a geometric illustration. It is easy to see that surfaces whose curvature is positive may always be rolled on a sphere whose radius is unity divided by the square root of the curvature; but to review the entire manifoldness of these surfaces, let one of them have the form of a sphere and the rest the form of surfaces of revolution touching it at the equator. The surfaces with greater curvature than this sphere will then touch the sphere internally, and take a form like the outer portion (from the axis) of the surface of a ring; they may be rolled upon zones of spheres having less radii, but will go round more than once. The surfaces with less positive curvature are obtained from spheres of larger radii, by cutting out the lune bounded by two great half-circles and bringing the section-lines together. The surface with curvature zero will be a cylinder standing on the equator; the surfaces with negative curvature will touch the cylinder externally and be formed like the inner portion (towards the axis) of the surface of a ring. If we regard these surfaces as *locus in quo* for surface-regions moving in them, as Space is *locus in quo* for bodies, the surface regions can be moved in all these surfaces without stretching. The surfaces with positive curvature can always be so formed that surface regions may also be moved arbitrarily about upon them without bending, namely (they may be formed) into sphere-surfaces; but not those with negative curvature. Besides this independence of surface regions from position there is in surfaces of zero curvature also an independence of direction from position, which in the former surfaces does not exist.

(To be continued.)

SCIENTIFIC SERIALS

Zeitschrift für Ethnologie, No. 6.—The present number gives a compendium of useful suggestions, which might advantageously be acted on in other countries besides Germany, addressed by the Anthropological Society of Berlin to all persons engaged in exploring, or other expeditions to distant regions. In those directions for observing and collecting whatever is most adapted to extend and rectify our actual knowledge, information is given in regard to the various races with whom travellers may come in contact, and the special geographical, linguistic, social and other conditions, which more particularly require further elucidation.—Prof. A. Bastian gives us in this number with his habitual

completeness an exposition of the worship of the heavenly bodies among different nations, and the extent to which local conditions of climate and ethnological differences have influenced the character of the adoration offered to the sun and the moon and the stars. According to him a true worship of the sun—except in the polar regions—is only to be found on elevated plateaux, where the return of the orb of day was welcomed with gratitude after the colder night, while in low-lying tropical lands the aborigines looked with dread at the glowing ball of fire which each summer seemed to threaten their world with annihilation. We can strongly commend this paper as a most comprehensive, although not specially novel exposition of Aryan and other mythological systems.—The German engineer, Herr H. Keplin, has drawn attention to the mussel-hills (*Casquiros sambaquis*) of Brazil in the district of the Rio do San Francisco do Sol. The position of these deposits appears to refute the idea of their being mere Kjøkkenmødings, while the great respect shown by the natives for the dead, and their care to provide them proper sepulture, would seem to afford further evidence that these elevations, which often rise to a height of 50 feet, cannot be due to the hand of man. In reference to the above, it may interest our own archeologists to know that Herr Walter Kauffman draws attention in the same number to his discovery in the neighbourhood of Hull, at a spot known as Castle Hill, near Holderness, of a burial place belonging, as he conjectures, to the transition period between the Stone and Bronze ages. Herr Kauffman found on the western side of the hill, where the ground had been cut for building purposes, a fragment of some loam vessel, a compact mass of oyster shells, some flint flakes, and a human rib. After carefully removing the earth, Herr K. discovered at from 4 to 4½ feet below the surface the vertebrae of another skeleton, and finally collected nearly all the bones of two skeletons, completely enclosed in a mass of oyster shells.—Dr. A. B. Meyer, of Manilla, in the course of a short visit in the Philippines, found skulls which presented that peculiar appearance of sharpening or filing of the teeth, described by the old traveller, Thévenot, and the accuracy of which has often been called in question. The Negrito skulls from the Philippines, examined by Dr. Meyer, also exhibited the artificial flattening of the heads noticed by Thévenot.—Herr Virchow drew attention last summer to the fact that occasional deviations present themselves from the normal cranial configuration of a race, which ought to teach us extreme caution in regarding any single specimen as a typical form. He was led to make this remark by his observation in the Anatomical Museum of Copenhagen of the skull of Kay Lykke, a man of the noblest Danish descent, who had flourished two hundred years ago, and been celebrated in his day for his personal beauty, his effeminacy, and the sensual bias of his disposition. Yet the skull of this once elegant, accomplished, and self-indulgent courtier of the 17th century, belonging to an otherwise brachycephalic race, is more strikingly dolichocephalic and depressed than the Neanderthal head, and might readily be supposed to have belonged to an Australian savage. The cranial capacity which is given by Professor Panum, of Copenhagen, as 1,250 cubic centm., is, moreover, below the amount that is conjecturally assumed for the Neanderthal skull.

The supplement to the vol. of the "Zeits. f. Ethnologie," for 1872, is exclusively occupied with the Linguistic Notes of Dr. G. Schweinfurth, drawn up as the result of his travel in Central Africa, and gives numerous vocabularies and specimens of the languages of the different tribes who occupy the district of the Bahr-el-Ghasal, among whom Dr. Schweinfurth lived more than two years.

Nuovo Giornale Botanico Italiano, vol. iv. Nos. 1—4, Jan.—Dec., 1872. The volume for 1872 of this journal, edited by one of the most accomplished of Italian botanists, Prof. Caruel, contains evidence of considerable scientific activity in the Peninsula. A large space of these four numbers is devoted to cryptogamic botany; we have papers on the mosses of Abyssinia, by De Venturi, and of Ceylon and Borneo, by Hampe; on the fungi of Parma, by Passerini; on Diatoms, by Ardissonne, and on a new classification of cryptogams, proposed by Prof. Cohn. Besides several papers on systematic, descriptive, and geographical botany, one of the most interesting on physiological and histological subjects is by Saccardo, on the amyloid corpuscles contained within the fovilla of pollen, illustrated by a plate. Prof. Caruel contributes a very valuable biographical notice of the Italian botanist, Andrea Cesalpino, born at Arezzo in 1519, and a summary of the contents of his great