ON THE MEASUREMENT OF MUSICAL INTERVALS *

IN a series of communications to the *Académie des Sciences* (February 8 and 22, 1869, July 17, 1871, and January 29, 1872), M. Cornu and I have shown that musical impressions are based upon several systems of musical intervals. We were also able to announce, as a preliminary result of experiments not yet completed, the following propositions, which, while they show clearly the origin of discussions that have gone on for more than two thousand years, appear to be capable of putting an end to these discussions, by reconciling the two contrary opinions which have always been entertained upon this subject.

1. The musical intervals formed by the successive sounds of a melody without modulation, belong to the Pythagorean scale, the degrees of which are represented by the following ratios, containing only the factors 2 and 3†:—

do re mi fa sol la si do

$$1 \quad \frac{3^2}{2^3} \quad \frac{3^4}{2^6} \quad \frac{2^2}{3} \quad \frac{3}{2} \quad \frac{3^3}{2^4} \quad \frac{3^5}{2^7} \quad 2.$$

2. The intervals formed by the simultaneous sounds of the concords, which are the basis of harmony, belong to very different systems, depending upon the complexity of the cords. Those which form part of the simpler concords of two or three sounds, thirds, sixths, perfect concords, &c., may be included in the scale given in all treatises on physics, the degrees of which are represented by the following ratios formed by the factors 2, 3, and 5:

do re mi fa sol la si do

$$1 \quad \frac{9}{8} \quad \frac{5}{4} \quad \frac{4}{3} \quad \frac{3}{2} \quad \frac{5}{3} \quad \frac{15}{8} \quad 2.$$

To demonstrate these propositions several conditions require to be fulfilled.

In the first place, in the two scales above given, the three different intervals, viz., the major third *do—mi*, the sixth *do—la*, and the seventh *do—si*, differ from one another by the interval called a "comma," the value of which is $\frac{81}{80}$, as will be found on dividing one by the other the fractions which represent these intervals on the two scales. Now this value of the comma is very small, though very perceptible to the ear; to demonstrate it we must, therefore, seek the assistance of skilled musicians, and employ apparatus of considerable delicacy.

Secondly, to measure the intervals formed by successive sounds it is best to study these intervals not separately, but as they occur in the actual course of a melody. Consequently, if we employ as our means of measurement the process which consists in causing the sounding body to trace out its own vibrations (and in the present state

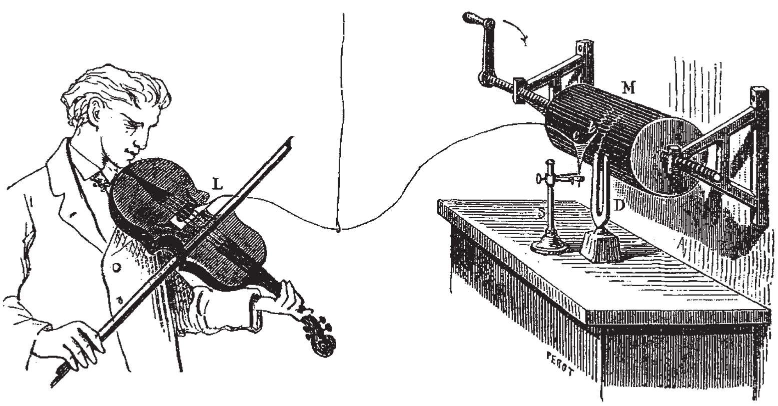

FIG. 1

of science no better method can be adopted), we must have the means of inscribing continuously the sounds which constitute fragments of melodies as they are executed upon an instrument.

Lastly, it is clearly necessary that the registration of the vibrations shall be automatic, and independent of the volition of the observers. The player must have nothing to do with it; he must not even see it going on, so that his attention may be entirely devoted to the music which he is playing.

After many trials we have succeeded in fulfilling these conditions. The apparatus which we use is very simple, the elements of it being found in every physical cabinet. It will, therefore, be useful to describe it.

Experiment shows that a metallic wire of steel, copper, brass, &c., without tension, and merely supported in such a manner that its vibrations may be executed freely, transmits to one of its extremities, by transverse vibrations, the sounds emitted by a sonorous body fixed to the other extremity. To show this it is sufficient to take two tuning-forks having mirrors attached to them, and tuned exactly in unison. Fix the end of a wire to one of them, and attach to the other end a feather carrying a shining point placed in front of the mirror of the second tuning-fork. On setting one of the forks in vibration, and properly adjusting the feather, the shining point is seen to describe an ellipse characteristic of the unison, and varying in form when a weight however small (a little piece of wax for example) is attached to the tuning-fork fixed to the wire.

A wire five, six, eight, ten, &c., metres long, suspended by narrow strips of caoutchouc, is soldered at one end to a small plate of brass, L, placed between the sounding-board of a stringed instrument and the foot of the bridge, the other end being slightly clasped to a heavy stand S. Near the fixed point a small piece of tinsel (*c*) is soldered on, and to this is attached a feather (*b*), by means of a little soft wax (by this arrangement a greater amplitude of vibration is attained than if the feather were directly attached to the wire). The musician stands in such a position that the wire may not impede the movements of his bow, and plays fragments of simple melodies in slow time (each note lasting at least a second). The vibrations of the strings are transmitted to the bridge, the metal plate, the wire, and, lastly, to the feather, which vibrates synchronously. It only remains to trace these vibrations.

The registering instrument is composed of a metal cylinder, M, the axis of which is furnished with a screw moving a double nut, firmly fixed to a table or to the wall. This cylinder is covered with a sheet of paper, which is blackened by making it revolve over the smoky flame of an oil-lamp. A tuning-fork, D, making from 300 to 500

* Translated from the *Journal de Physique*.

† See the same volume, p. 109, Sur l'Histoire de l'Acoustique Musicale.

double vibrations per second, and carrying a strip of tinsel to serve as an index, is firmly fixed in a vice or in the wall, and arranged so that its index may vibrate in the direction of the generating lines of the cylinder. These vibrations serve to mark the time, and the tuning-fork serves as a chronograph, obviating the necessity of giving to the cylinder a regular and uniform motion. Further, the feather is moved forward, so that its point may just touch the blackened paper, and that it may vibrate quite close to the index, and, like the latter, in the direction of the generating lines of the cylinder.

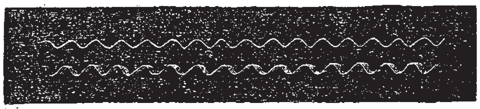

FIG. 2

These arrangements being made, the tuning-fork is set in vibration, either with a bow or by striking it with a stick covered with leather, and the musician plays, while the cylinder is turned at a suitable rate either by the hand or by any convenient motive power.

In this manner a tracing is obtained like that of which a fragment is shown in Fig. 2, each note of the melody being represented by a form of vibration peculiar to itself. The number of vibrations for each note, corresponding to 100 vibrations, for example, of the tuning-forks, is counted, and the ratio of the numbers thus obtained gives the values of the intervals. The vibrations are sometimes complicated with harmonics (Fig. 3), but they are almost always octaves, rarely fifths, very rarely thirds; moreover, it is not possible to make a mistake on this point.

To preserve the tracing after it is detached from the cylinder, it is split longitudinally, dipped for an instant into a 4 per cent. solution of shellac in alcohol, whereby it becomes covered with a very thin layer of unalterable varnish.

If, instead of measuring intervals of melody, we wish to measure the harmonic intervals of two sounds, two strings of the instrument are tuned simultaneously (in the ordinary way), to the third, fifth, sixth, &c., till beats are no longer perceptible, and the ear is perfectly satisfied; the sounds of the two strings thus tuned are then separately traced.

We have made numerous experiments with several persons, in particular melody experiments with M. Léonard, the Belgian violinist, and M. Séligmann, the violoncellist. The mean values of the results obtained with the assistance of these eminent artists are given in the following table; other experiments gave octaves equal to 2.

	Do.	Re.	Mi.	Fa.	Sol.	La.	Si.	Do.
Mean of the results.	1·000	1·128	1·265	1·330	1·500	1·686	1·917	...
Pythagorean Scale.	1·000	1·125	1·256	1·333	1·500	1·687	1·898	2·000
Ordinary Scale.	1·000	1·125	1·250	1·333	1·500	1·666	1·875	2·000
Values of the Comma.*	0·013	0·014	0·016	0·017	0·019	0·021	0·024	0·025

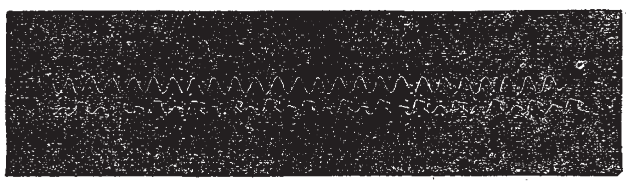

FIG. 3

It is necessary to add that the mean deviations of the experiments rarely exceed the third of a comma. As to the differences between the mean values of the results and the values of the intervals of the Pythagorean Scale, it is a mere fraction of the comma, insensible to the ear. For the seventh alone the difference amounts to five-sixths of the comma; but this result is remarkable, inasmuch as it exhibits a fact well known to musicians, namely, that in the case where the sensitive note Si is resolved upon the tonic Do (which is precisely what occurs in the four cases in which we obtained sevenths), it is perceptibly higher than in the inverse movement.

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* These numbers are the *differences* between the numerical value of each interval I, and the mean of this same interval raised by a comma, that is to say,

$$\left(\frac{81}{80} - 1\right) \times I = I \times 0·015$$

DISCOVERY OF A LARGE BONE CAVE IN BAVARIA

DURING the cutting of the railway from Nuremberg to Regensburg by the Bavarian Eastern Railway Co., it was necessary to cut directly through a piece of mountain chain in Schelmengraben near Regensburg. It was owing to this that this bone cave was discovered, and its miscellaneous contents were able to be examined and arranged. Since the railway cut right across the middle of the cave, it allowed it to be very thoroughly examined, and under the most favourable circumstances and in daylight, as has been the case in very few other instances. The railway company have given every facility in their power that the cave should be thoroughly examined, and under the direction of Profs. Fraas and Zittel, a gang of men were actively employed for many days, and the objects so obtained were carefully preserved. From the local German papers the following particulars have been obtained, which, allowing for a little local colouring and exaggeration, show the find to have been a most important one, and one that may well come under the notice of the International Congress of Archæology and Anthropology at their meeting this year, where the whole question of bone caves and their contents is to form a prominent subject for discussion.

The cave in question was originally, when first discovered about two years ago, 28 metres (about 91 ft.) long, and was simply a fissure in the Jura limestone which had been enlarged by running water. Its opening was visible half way up the mountain side, partly hidden in dense woods. It stretched from North to South, with a slight turn towards the West of about 15°. The new line of railway cut deeply into the hill side, and during the course of this year has already cut away one half of the cave, but unfortunately the contents were employed on the line. On this account, only the part not touched was able to be excavated and examined, and this was 11 metres (36 ft.) long, 2 metres (6½ ft.) wide, and in the middle 3 metres (9½ ft.) deep. Wood ashes and pieces of coal, together with pieces of pottery, had accumulated to about the height